Studies the Reliability and Availability Characteristics of Two Different System under Preventive Maintenance

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ABSTRACT
The research studies the availability and reliability to two systems which different due to the effect of preventive maintenance (PM), a two system analyses a two state with two type failure. The rate of failure is exponential distribution but the rate of repair is general distribution. The system resolved by supplementary technique and Laplace transforms depend on Complex imagine roots. Several measures of availability and (MTTF) of system are obtained, we analysis graphically to watch the impress of several system parameters in mean time failure and availability.

Keywords
Availability, Reliability, (MTTF), Laplace transforms (L.T), supplementary technique, preventive maintenance (PM).

1. INTRODUCTION
Reliability study of the repair problem of device is importance in our lifetime where it is used widely in the manufacturing system. So the systems repairable study is an important component in reliability analysis. Many authors study availability, reliability and (MTTF) under P.M. Like [1] deals with cost-benefit of a two-unit cold standby system with two-phase repair of the failed unit and preventive maintenance. The rate to failure and time to go PM are exponential while rate of repair and the time till go PM are general. [2] The author presents a two system having single unit in parallel which different because to the additional of the preventive maintenance. [3] Studied two parallel systems which every unit has two failure type where unit fails due to operating characteristics, so, the system go under preventive maintenance randomly. Rate of Failure are constant while the rate of PM & repair are general. [4] This paper deals with an aero plane model; namely, a two-unit (non-identical) parallel system with dual mode of failures which preventive maintenance happen at random epochs. [5] Deals with a 2-state repairable complex system of two failure types which solved by Laplace transform which rate of failure and repair of [type1, type2] are assumed as exponential distribution. [6] Study the availability and profit analysis of a repairable redundant 3-out-of-4 system with preventive [7] this paper show complex system where fail in n-mutually exclusive ways of total failure. [8] Study some reliability parameters of three states with failure environmental [9] the author talk about complex system consisting two subsystems where use supplementary technique & Laplace Transform (L.T). [10] Talk about the system reliability where transform the basic equations of the model into integro-differential eq. and solve it using Supplementary variables. This research show comparison between two systems where they different because of the additional of PM, the two systems resolved by partial differential eq. & Laplace Transform where depend on Complex imagine roots then we show numerical results to analyze the impress of the various system parameters on reliability and system availability.

2. SYSTEM DESCRIPTION
The system is analyzed under following practical assumptions:
- The system unit contain of two state repairable units with two failure types.
- The system has two repair facilities, first for repairing type 1 & the second for type 2.
- After repairing a unit, it will work like a new state.

We suppose that preventive maintenance is provided to this system at random epochs when the system at state So. Through the P.M the system remains operating.

3. NOTATIONS AND SYSTEM STATES:
- $\lambda_1$ & $\lambda_2$: Rate of failure of type 1 & type 2
- $\mu_1$ & $\mu_2$: Rate of repair of type 1 & type 2
- $\mu_3$: Constant rate for taking a unit into preventive maintenance
- $\mu_4(x), \mu_5(y)$: general repair of $S_1, S_2$ elapsed time of repair $x$ and $y$.
- $\mu_4(x)$: General repair end of PM
- $p_j(\tau)$: Probability that the system in state $S_j$ at time $\tau$, $j=0, 1, 2, 3$
- $0\rightarrow$ normal state
- $1\rightarrow$ failed state of type1.
- $2\rightarrow$ failed state of type2.
- $3\rightarrow$ normal state and preventive maintenance.
- $P_1(x, \tau) \& P_2(y, \tau)$: Probability that the system in state $S_1, S_2$ at time $\tau$, and under repair, elapsed time of repair is $x$ & $y$.
- $P_3(x, \tau)$: Probability that the system in state $S_3$ at time $\tau$, and under Preventive Maintenance, elapsed time of repair is $x$.
- $p_j^*(s)$: Laplace transform (L.T) of $p_j(\tau)$
- $A(\tau)$: functions of availability.

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functions of reliability.

MTTF: mean time failure.

Where Laplace transforms (L.T) of $p_j(t)$ is:

$$p_j^*(s) = \int_0^\infty e^{-st} p_j(t) \, dt$$

There are a relation between repair rate $\mu_1(x)$, $\mu_2(y)$ and their cumulative functions $F(x)$, $F(y)$, i.e.

First system with P.M

**4. MATHEMATICAL MODEL DESCRIPTION**

This part showing the differential eq. for the system of figure (1) Transition states

$$\frac{\partial}{\partial t} + \lambda_1 + \lambda_2 + \mu_2) P_0(t) =$$

$$\int_0^\infty \mu_1(x) P_1(x, t) \, dx + \int_0^\infty \mu_2(y) P_2(y, t) \, dy +$$

$$\int_0^\infty \mu_4(x) P_4(x, t) \, dx$$

(4-1)

$$\int_0^\infty \frac{\partial}{\partial x} [\mu_1(x)] P_1(x, t) \, dx = 0$$

(4-2)

$$\int_0^\infty \frac{\partial}{\partial y} [\mu_2(y)] P_2(x, t) \, dy = 0$$

(4-3)

$$\int_0^\infty \frac{\partial}{\partial x} [\mu_4(x)] P_4(x, t) \, dx = 0$$

(4-4)

Initial conditions:

$$P_j(0) = \begin{cases} 1 & \text{if } j > 0 \\ 0 & \text{else} \end{cases}$$

Boundary conditions

$$P_1(0, t) = \lambda_1 P_0(t)$$

$$P_2(0, t) = \lambda_2 P_1(x, t)$$

$$P_3(0, t) = \mu_3 P_0(t)$$

**5. MODEL SOLUTION**

Use Laplace Transform (L.T) For Eq. (4-1) to (4-4) and Boundary conditions

$$\left(s + \lambda_1 + \lambda_2 + \mu_2\right) P_0(s) =$$

$$1 +$$

$$\int_0^\infty \mu_1(x) P_1^*(x, s) \, dx + \int_0^\infty \mu_2(y) P_2^*(y, s) \, dy +$$

$$\int_0^\infty \mu_4(x) P_4^*(x, s) \, dx$$

(5-1)

$$F_1(x) = \mu_1(x) e^{-\int_0^x \mu_1(u) \, du}$$

$$F_2(y) = \mu_2(y) e^{-\int_0^y \mu_2(u) \, du}$$

$$F_3(x) = \mu_3(x) e^{-\int_0^x \mu_3(u) \, du}$$

**Figure 1. System configuration diagram**
Setting the system probability in operable (up) and failed (down) state availability by Laplace transforms

\[
P_0'(s) = \frac{\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_3}{s + \lambda_1 + \lambda_2 + \mu_3}
\]

From equation (7-1), (7-4), one may get

\[
P_{up}'(s) = P_0'(s) + P_2'(s)
\]

\[
P_{up}(s) = \frac{s^2 + A_1s^2 + BS + m}{s^2 + (s + A_1)w + w_1 + \sqrt{w_1}}
\]

Where

\[
q = \frac{2\sqrt{\lambda_1}}{\mu_1}, \quad r = \frac{9\lambda_1\lambda_2 - 2\lambda_1 - 2\lambda_2}{54}
\]

\[
D = q^3 + r^2, \quad u = (r + \sqrt{D})^3/2, \quad \tau = (r - \sqrt{D})^3/2
\]

\[
w_1 = \frac{(u + \tau)}{2}, \quad w = (u + \tau)
\]

\[
v_1 = \frac{(u - \tau)}{2}, \quad A_1 = \frac{v_1}{3}
\]

Applying inverse Laplace Transform for the eq., we obtain

\[
P(r) = \frac{m}{(A_1 - w)(A_1 + w + w_1 + 3\tau)}
\]

\[
+ \frac{[2(\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_3) + 3\tau]}{X^2 + \lambda_1^2} \frac{e^{(A_1 - w_1)\tau}}{X^2 + \lambda_1^2}
\]

\[
H = (6A_1v_1w_1 - 3v_1^3 + 3w_1^2v_1 + 3A_1^2v_1 - AA_1v - A_1v + Bv)
\]
\[ X = 3v^2w + 6v_1^2A_1 \]
\[ T = 3vwyA_1 + 3w_1^2v - 6v_1^3 \]

- Reliability System and availability

System availability:

\[
A(t) = \frac{m}{(A_1-w)(A_1^2+A_1w+w_1^2+3w_1^3)} + \frac{(-A_1+w)^2+(A_1-w)^2+B(-A_1+w)+m}{(-A_1+w)(9w_1^2+3w_1^3)} + \frac{2[(X+3HT)\cos^2(v_1)]-2\sqrt{2}[(H-X)-(TP)][\sin \sqrt{3}(v_1)t]}{X^2+3T^2} e^{(-A_1-w)t} \tau
\]

So we can obtain the steady-state availability \( A \) from the following relation

\[
A = \lim_{t \to 0} A(t) = \frac{m}{(A_1-w)(A_1^2+A_1w+w_1^2+3w_1^3)}
\]

- The mean time failure (MTTF):

Taking all repairs zero in (7-4), mean time to failure of the system is obtained as

\[
MTTF = \lim_{s \to 0} p_{up}(s)
\]

7.1.2. Second case \( D < 0 \) [All roots are real and unequal]

\[
p_{up}(s) = \frac{s^2+A_1^2+BS+m}{s(s+A_1-w)(s+A_1-w_1)(s+A_1-w_2)}
\]

Where

\[
s_1 = w_0 - \frac{2A_1}{3}, \quad s_2 = w_2 - \frac{2A_1}{3}, \quad s_3 = w_2 - \frac{2A_1}{3}, \quad \theta = \cos^{-1} \frac{r}{\sqrt{q^2}}
\]

\[
w_0 = 2\sqrt{-q} \cos \left( \frac{\theta}{2} \right), \quad w_2 = 2\sqrt{-q} \cos \left( \frac{\theta}{2} + 120^\circ \right)
\]

\[ v_2 = 2\sqrt{-q} \cos \left( \frac{\theta}{2} + 120^\circ \right), \quad A_2 = \frac{A_1}{3}
\]

By applying inverse Laplace transform

\[
p_{up}(t) = \frac{m}{(A_1-w_0)(A_1^2-A_1A_2-A_1w_2-w_2w_2)} + \frac{(-A_1+w_0)^2+(A_1-w_0)^2+B(-A_1+w_0)+m}{(-A_1+w_0)(w_0-w_2-w_2w_2-w_2w_2)} e^{(-A_1+w_0)t}
\]

\[
+ \frac{(-A_1+w_2)^2+(A_1-w_2)^2+B(-A_1+w_2)+m}{(-A_1+w_2)(w_2-w_0-w_2w_2-w_2w_2)} e^{(-A_1+w_2)t}
\]

\[
+ \frac{(-A_1+w_2)^2+(A_1-w_2)^2+B(-A_1+w_2)+m}{(-A_1+w_2)(w_2-w_0-w_2w_2-w_2w_2)} e^{(-A_1+w_2)t}
\]

System availability: \( A(t) = \frac{m}{(A_1-w_0)(A_1^2-A_1A_2-A_1w_2-w_2w_2)} \)

And the steady-state availability are:

\[
A = \lim_{t \to \infty} A(t) = \frac{m}{(A_1-w_0)(A_1^2-A_1A_2-A_1w_2-w_2w_2)}
\]

- The mean time failure (MTTF):

Taking all repairs zero in (7-4), mean time to failure of the system is obtained as

\[
MTTF = \lim_{s \to 0} p_{up}(s)
\]

\[
MTTF = \frac{(\mu_1+\mu_2)(\lambda_1+\lambda_2)}{\lambda_1+\lambda_2}
\]

MTTF

\[
= \lim_{s \to 0} \left(s+A_1-A_1w_1-A_1w_2-A_1w_2w_2\right) = \lambda_1+\lambda_2+\mu_1+\mu_2
\]

\[ \lambda_1+\lambda_2+\mu_1+\mu_2 = \lambda_1+\lambda_2+\mu_1+\mu_2
\]

\[ S_1 \quad f a i l e d \ 1 \]

\[ S_0 \quad S_2 \quad f a i l e d \ 2 \]

\[ \mu_2 \]

\[ \lambda_1 \]

\[ P_0(t) \]

\[ S_2 \quad f a i l e d \ 2 \]

\[ S_0 \]

\[ P_0(t) \]

\[ S_1 \quad f a i l e d \ 1 \]

Figure 2. System configuration diagram

8. MATHEMATICAL MODEL DESCRIPTION

This part showing the differential eq. for the system of Table (1) Transition states

\[
\frac{\partial}{\partial t} + \lambda_1 + \lambda_2 \] \( P_0(t) = \int_0^\infty \mu_1(x) P_2(x,t) dx + \int_0^\infty \mu_2(x) P_1(y,t) dy \)

\[
= \frac{\partial}{\partial x} (\mu_1 + \mu_2) P_0(x,t) = 0
\]

\[
\frac{\partial}{\partial y} (\mu_1 + \mu_2) P_0(y,t) = 0
\]

Initial conditions:

\[
P_0(0) = 0 \quad \text{where } j = 0
\]

else

Boundary conditions

\[
P_1(0,t) = 3\lambda_1 P_0(t) \quad , \quad P_2(0,t) = \lambda_2 P_0(t)
\]
9. SOLUTION OF THE MODEL
By taking Laplace transform for (8-1) to (8-3) and Boundary conditions

\[ (s + \lambda_1 + \lambda_2) P_1^s(s) = 1 + \int_0^\infty \mu_1(x) P_1^s(x, s) \, dx + \int_0^\infty \mu_2(y) P_2^s(y, s) \, dy \]  
\[ (s + \mu_1 \lambda_2 + \lambda_2) P_1^s(0, s) = 0 \]  
\[ (s + \frac{\mu_1}{\lambda_2}) P_1^s(x, s) = 0 \]  
\[ (s + \frac{\mu_2}{\lambda_2}) P_2^s(y, s) = 0 \]  

With Boundary conditions

\[ P_1^s(0, s) = \lambda_1 P^0_1(s) \]  
\[ P_2^s(0, s) = \lambda_2 P_0^s(s) \]  

Integrating equations (9-2) & (9-3)

\[ P_1^s(x, s) = P_1^s(0, s) e^{-s x - \int_0^x \mu_1(u) \, du} \]  
\[ P_2^s(y, s) = P_2^s(0, s) e^{-s y - \int_0^y \mu_2(u) \, du} \]  

Again integrating by parts equations (9-6) & (9-7) using (9-4) & (9-5)

\[ P_1^s(s) = \frac{1}{\lambda_1} \int_0^\infty \mu_1(x) \, e^{-s x - \int_0^x \mu_1(u) \, du} \, dx \]  
\[ P_2^s(s) = \frac{1}{\lambda_2} \int_0^\infty \mu_2(y) \, e^{-s y - \int_0^y \mu_2(u) \, du} \, dy \]  

Also we have from equations (9-6) & (9-7) using equations (9-4) & (9-5)

\[ \int_0^\infty P_1^s(x, s) \, \mu_1(x) \, dx = \lambda_1 P_0^s(s) F_1^s(s) \]  
\[ \int_0^\infty P_2^s(y, s) \, \mu_2(y) \, dy = \lambda_2 P_0^s(s) F_2^s(s) \]  

Now from equations (9-10) & (9-11) in (8-1) we get

\[ P_0^s(s) = \frac{1}{S + \lambda_1 + \lambda_2 - \lambda_1 F_1^s(s) - \lambda_2 F_2^s(s)} = \frac{1}{A(s)} \]  

Where

\[ A(s) = S + \lambda_1 + \lambda_2 - \lambda_1 \lambda_2 F_1^s(s) - \lambda_2 F_2^s(s) \]

10. EVALUATION DOWN AND UP STATE AVAILABILITY BY LAPLACE TRANSFORMS
The system probability in operable (up) and failed (down) state at time \( \tau \) can be obtained by Laplace transform as:

\[ P_{up}\star(s) = P_0^s(s) \]  
\[ P_{down}\star(s) = 1 - P_{up}\star(s) \]

11. PARTICULAR CASE
In this section the up and down state availability, MTTF, the steady-state availability of the system have been evaluated, when repair times will be exponential distribution.

\[ F_1^s(s) = \frac{\lambda_1}{\lambda_1 + \mu_2} \]  
\[ F_2^s(s) = \frac{\mu_2}{\lambda_1 + \mu_2} \]  

\[ P_0^s(s) = \frac{\lambda_1 \lambda_2 (s + \mu_2)}{s(s + \lambda_1 + \lambda_2 + \mu_2)} \]  
\[ P_1^s(s) = \frac{\lambda_1 \lambda_2 (s + \mu_2)}{s(s + \lambda_1 + \lambda_2 + \mu_2)} \]  
\[ P_2^s(s) = \frac{\lambda_1 \lambda_2 (s + \mu_2)}{s(s + \lambda_1 + \lambda_2 + \mu_2)} \]

We know that

\[ P_{up}\star(s) = P_0^s(s) \]  
\[ P_{up}\star(s) = \frac{\lambda_1 \lambda_2 (s + \mu_2)}{s(s + \lambda_1 + \lambda_2 + \mu_2)} \]

Where

\[ b = \mu_1 \mu_2 \]  
\[ a = \mu_1 + \mu_2 \]  
\[ c = \mu_1 + \mu_2 + \lambda_1 + \lambda_2 \]  
\[ d = \mu_1 \mu_2 + \mu_1 \lambda_2 + \mu_2 \lambda_1 \]

By using inverse of Laplace Transform (I.L.T) of eq., we obtain

\[ p_{up} (t) = \frac{b + e^{-a t}}{d} \left( d - b \right) \cosh \left( \frac{1}{2} \sqrt{c^2 - 4 d} \right) + \frac{1}{\sqrt{c^2 - 4 d}} \]  

- Reliability System and availability

System availability:

Availability of the system can be get from the relation

\[ A_{up} (t) = \frac{1}{d} \left( b + e^{-d t} - d \right) \cosh \left( \frac{1}{2} \sqrt{c^2 - 4 d} \right) + \frac{1}{\sqrt{c^2 - 4 d}} \]  

The steady – state availability can be obtained from the following relation

\[ A = \lim_{s \to \infty} A_{up} (t) = \frac{b}{a + b + \lambda_1 \mu_2 + \lambda_2 \mu_1} \]

- Mean time to system failure
Taking all repairs zero in (11-4), mean time to failure of the system is obtained as

\[ \text{MTTF} = \lim_{s \to 0} P_{up}\star(s) \]

\[ \text{MTTF} = \lim_{s \to 0} \frac{1}{s(s + \lambda_1 + s \lambda_2 + s \mu_2 + \lambda_2 \mu_1 + \lambda_1 \mu_2 + \lambda_2 \mu_1 + \mu_1 \mu_2)} \]

\[ \text{MTTF} = \frac{1}{(\lambda_1 + \lambda_2)} \]

12. NUMERICAL EXAMPLE
To see the system behavior, we plot the steady-state availability for the models, against \( \lambda_1 \) keeping the other parameters fixed at

\[ \lambda_2 = 0.25, \mu_2 = 0.3, \mu_1 = 0.1 \text{, } \mu_3 = \mu_4 = 0.7 \]
13. CONCLUSIONS

- We use computer software, to plot system availability and MTTF in figure 1 and 2 respectively. It is noted that A decrease as $\lambda_1$ increases and MTTF decreases as $\lambda_1$ increases also the system with (P.M) is better than the system without (P.M).

14. REFERENCES


