An Advanced Method to Solve Fuzzy Linear Programming Problem

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ABSTRACT
Aim of this paper is to present an advanced method to solve Linear programming problem (LPP) in which decision variables, cost coefficients involving in objective function and right hand side coefficients in the constraints are trapezoidal fuzzy numbers. Using multiplication, addition operators of trapezoidal fuzzy numbers (TrFNs) and linear ranking function, Fuzzy Linear programming problem (FLPP) is converted into crisp LPP. Eventually solved it by simplex method and compared results with the results of existing method.

Keywords
Fuzzy linear programming problem, Trapezoidal fuzzy numbers, ranking function.

1. INTRODUCTION
Linear programming (LP) is one of the best optimization methods for solving real world problems. However, the conventional LPP methods fail to deal with imprecise data/information in real world problems. In order to address such issues, the fuzzy sets theory has been extensively used to represent imprecise data in LPP [2], [3]. If either coefficients in the objective function or/and coefficients involved in the constraints are imprecise in nature then those LPPs are referred as fuzzy linear programming problem (FLPP).

The FLPP has become popular when it has gained the success in solving real world problems associated with game theory [7], transportation problem, supply chain management, project scheduling etc. Wan and Dong [5] developed a new method to solve FLPP with trapezoidal fuzzy numbers (TrFNs). In this method author constructed an auxiliary multiobjective programming to solve the corresponding possibility linear programming with TrFNs. Ebrahimnejad at el. [12] proposed the method which is simple and computationally more efficient method for solving FLPP in which the coefficients of the objective function and the values of the right hand side are represented by symmetric trapezoidal fuzzy numbers while the elements of the coefficients matrix are represented by real numbers. They converted the FLPP into an equivalent crisp LPP and solved it by simplex method. The FLPP in which all the decision variables and the all coefficients of constraints are represented by fuzzy numbers is known as fully fuzzy linear programming problem (FFLPP). R.Ezzati at el. [13] proposed an algorithm to solve the FFLPP which is based on a new lexicographic ordering on triangular fuzzy numbers. In their study the FFLPP is converted to its equivalent multiobjective LPP. Eventually compared with Kumar's method [8]. Many authors have found the fuzzy optimal solution of FFLPP with equality constraints [10]. J.Kaur et al. [11] also proposed a method to solve FLPP in which some or all the parameters are represented by unrestricted L-R fuzzy numbers. They presented an algorithm to find the product of unrestricted L-R fuzzy numbers, and then with the help of proposed product, a new method namely Mehar's method is proposed for solving FLPP. It is also shown that the existing methods fail to solve FLPP but the Mehar's method is suited to do so.

This paper propose an improved method to solve FLPP in which symmetrical or non-symmetrical trapezoidal fuzzy numbers are employed and showed that this method is more efficient than that of Ebrahimnejad [12].

2. DEFINITIONS AND BASIC OPERATIONS
Definition 2.1. A subset A of universal set X is said to be a crisp set if it is defined by its characteristic function χ_A as follows:

$$\chi_A(x) = \begin{cases} 1 \text{ for } x \in A \\ 0 \text{ for } x \notin A \end{cases}$$

Means the characteristics function of a crisp set assigns a value either 1 or 0 to each member of the universal set X.

Definition 2.2. A set A is said to be a fuzzy set of the universal set X if each element of set A has a membership function or the degree of belongingness in X. Here we denote the membership function of a fuzzy set by μ_A

$$\mu_A\colon X \to [0,1]$$

Definition 2.3.A fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ is said to be a trapezoidal if it is characterized by the following membership function:

$$\mu_A(x) = \begin{cases} 0\,, & x < a_1 \\ \dfrac{x-a_1}{a_2-a_1}\,, & a_1 \le x \le a_2 \\ 1\,, & a_3 \le x \le a_4 \\ \dfrac{x-a_4}{a_3-a_4}\,, & x > a_4 \end{cases}$$

Definition 2.4. $\propto -$ cut of a TrFN is an interval $[(a_2 - a_1) \propto + a_1\ , -(a_4 - a_3) \propto + a_4]$ and it is denoted by $A_\propto$. Note that for triangular fuzzy number $a_2 = a_3$.

Definition 2.5. Let $\mathcal{R}\colon F(A) \longrightarrow R$, where *F(A)* be the set of all TrFNs and R be the set of real numbers. The ranking of TrFN $\tilde{A}$ is defined and denoted as

$$\mathcal{R}(\tilde{A}) = \frac{a_1+a_2}{2} + \frac{1}{4}\,(a_4 - a_3)$$

Basic operations 2.6.

Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers.

1. **Addition**:

$\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$

2. **Subtraction:**

$\tilde{A} \ominus \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$

3. **Multiplication:** To calculate the product of two trapezoidal fuzzy numbers, we first take product of $\propto -$ cuts of $\tilde{A}$ and $\tilde{B}$ using interval arithmetic

Let $A_\propto = [(a_2 - a_1) \propto + a_1 \ , -(a_4 - a_3) \propto + a_4]$

and

$B_\propto = [(b_2 - b_1) \propto + b_1 \ , -(b_4 - b_3) \propto + b_4]$

be the $\propto$-cut intervals of TrFNs $\tilde{A}$ and $\tilde{B}$

respectively then

$$A_\propto \otimes B_\propto = [(a_2 - a_1)(b_2 - b_1) \propto^2 + b_1(a_2 - a_1) \propto + a_1(b_2 - b_1) \propto + a_1 b_1 \, , (a_4 - a_3)(b_4 - b_3) \propto^2 - b_4(a_4 - a_3) \propto - a_4(b_4 - b_3) \propto + a_4 b_4 \,]$$

Case 1: For $\propto = 0$, $A_0 \otimes B_0 = [a_1 b_1 \, , \, a_4 b_4 \,]$

Case 2: For $\propto = 1$ $A_1 \otimes B_1 = [(a_2 - a_1)(b_2 - b_1) + b_1(a_2 - a_1) + a_1(b_2 - b_1) + a_1 b_1 \, , (a_4 - a_3)(b_4 - b_3) - b_4(a_4 - a_3) - a_4(b_4 - b_3) + a_4 b_4 \,]$

Therefore using above definition, multiplication of two trapezoidal fuzzy numbers $\tilde{A}$ and $\tilde{B}$ is can be written as $\tilde{A} \otimes \tilde{B} = [a_1 b_1 \, , (a_2 - a_1)(b_2 - b_1) + b_1(a_2 - a_1) + a_1(b_2 - b_1) + a_1 b_1 \, , \ (a_4 - a_3)(b_4 - b_3) - b_4(a_4 - a_3) - a_4(b_4 - b_3) + a_4 b_4 \, , \, a_4 b_4 \,]$

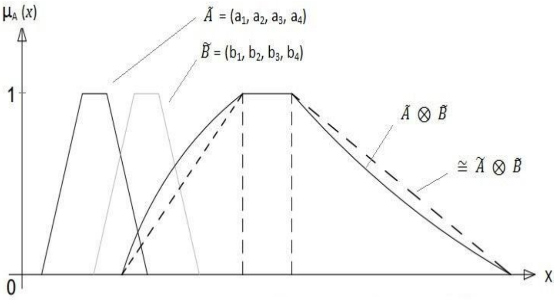

Fig 1: Graphical representation of trapezoidal Fuzzy numbers and their multiplication

3. PROPOSED METHOD TO SOLVE LINEAR PROGRAMMING PROBLEM WITH TrFNs

In some pioneer works of Bellman and Zadeh [1], Negoita et al.[2], Tanaka et al.[3], and Zimmermann[4], they have proposed fuzzy mathematical programming models to incorporate fuzziness of objective and constraint. However, in the last decade, many researchers contributed a lot in FLPP in which all parameters are fuzzy. In this study, decision variable $\tilde{x}$, cost coefficients $\tilde{c}_j$ and the right side coefficients b_i are trapezoidal fuzzy numbers. FLPP is to determine $\tilde{x}_j$ which maximizes the objective function z under the constraints.

A FLPP is given by

Objective function: $Max\ \tilde{z} \cong \sum \tilde{c}_j \tilde{x}_j$ (1)

Constraint: $\sum_{i=1}^{m} A\ \tilde{x}_j \leq \tilde{b}_i$

$\tilde{x} \geq \tilde{0} = (0,0,0,0)$

i.e $Max\ \tilde{z} \cong \sum_{j=1}^{n} \tilde{c}_j \otimes \tilde{x}_j$

such that $\sum_{i=1}^{m} a_{ij} \otimes \tilde{x}_j \leq \tilde{b}_i$

$\tilde{x} \geq \tilde{0} = (0,0,0,0)$, where $i = 1,2,3, \dots, m$ and

$j = 1,2,3, \dots, n$

Following steps are used to solve above FLPP.

Step I: Multiply corresponding TrFNs.

Step II: Using ranking formula, convert fuzzy objective function in to crisp objective function.

Step III: Covert fuzzy constraints in to crisp constraints with the help of ranking function.

Step IV: Solve the crisp linear programming problem obtained in step III, Find the optimal solution.

Numerical Example:

Example 1: Consider the following FLPP to find the maximum value of an objective function in which cost coefficient, decision variables and right hand side coefficients are TrFNs

$$\text{Max}\{(11,13,15,17) \otimes (x_1, x_2, x_3, x_4) + (9,12,14,17) \otimes (x_5, x_6, x_7, x_8) + (13,15,17,19) \otimes (x_9, x_{10}, x_{11}, x_{12})\}$$

Such that

$$12\tilde{x}_1 + 13\tilde{x}_2 + 12\tilde{x}_3 \leq (469,475,505,511)$$
$$\tilde{x}_1 + 13\tilde{x}_3 \leq (452,460,480,488)$$
$$12\tilde{x}_1 + 15\tilde{x}_2 \leq (460,465,495,500) \quad (2)$$

We can write above FLPP as

$$\text{Max}\{(11x_1 + 9x_5 + 13x_9), (13x_2 + 12x_6 + 5x_{10}), (15x_3 + 14x_7 + 17x_{11}), (17x_4 + 17x_8 + 19x_{12})\} \quad (3)$$

such that $12(x_1, x_2, x_3, x_4) + 13(x_5, x_6, x_7, x_8) + 12(x_9, x_{10}, x_{11}, x_{12}) \leq (469,475,505,511)$

$\boldsymbol{(x_1, x_2, x_3, x_4) + 13(x_9, x_{10}, x_{11}, x_{12}) \leq (452, 460, 480, 488)}$

$12\boldsymbol{(x_1, x_2, x_3, x_4) + 15(x_5, x_6, x_7, x_8) \leq (460, 465, 495, 500)}$

By using definition 2.5, above is converted into the following crisps LPP problem as

$$Max\ z = 5.5x_1 + 6.5x_2 - 3.75x_3 + 4.25x_4 + 4.5x_5 + 6x_6 - 3.5x_7 + 4.25x_8 + 6.5x_9 + 7.5x_{10} - 4.25x_{11} + 4.25x_{12} \quad (4)$$

such that
$$12x_2 + 13x_6 + 12x_{10} \leq 475$$
$$12x_3 + 13x_7 + 12x_{11} \leq 505$$
$$12x_4 + 13x_8 + 12x_{12} \leq 511$$
$$x_1 + 13x_9 \leq 452$$
$$x_2 + 13x_{10} \leq 460$$
$$x_3 + 13x_{11} \leq 480$$
$$x_4 + 13x_{12} \leq 488$$
$$12x_1 + 15x_5 \leq 460$$
$$12x_2 + 15x_6 \leq 465$$
$$12x_3 + 15x_7 \leq 495$$

$$12x_4 + 15x_8 \leq 500$$
$$x_i \geq 0 , i = 1,2,3, \dots \dots ,12$$

Therefore

$x_1 = 38.3333$, $x_2 = 4.5486$, $x_3 = 0.0000$, $x_4 = 5.4653$
$x_5 = 38.3333$, $x_6 = 4.5486$, $x_7 = 0.0000$, $x_8 = 5.4653$
$x_9 = 0.60000, x_{10} = 35.0347$,
$x_{11} = 36.9231$, $x_{12} = 37.1181$

Putting these values in
$\tilde{x}_1 = (x_1, x_2, x_3, x_4)$, $\tilde{x}_2 = (x_5, x_6, x_7, x_8)$,
$\tilde{x}_3 = (x_9, x_{10}, x_{11}, x_{12})$

The fuzzy optimal solution is

$$\tilde{x}_1 = (38.3333, 4.5486, 0.0000, 5.4653)$$
$$\tilde{x}_2 = (\ 0.0000, 0.0000, 4.7633, 0.0000)$$
$$\tilde{x}_3 = (\ 0.6000, 35.0347, 36.9231, 37.1181)$$

The optimal value is 880.1926

4. CONCLUSION

This paper presents an advanced method to solve fuzzy linear programming problem in which the decision variables, cost coefficient and right hand side coefficients in (1) are trapezoidal fuzzy numbers. Ebrahimnejad [12] has presented method to solve fuzzy LPP which is computationally more efficient than that of Ganeshan-Veermani [14] and Amit Kumar. However the optimum values obtained by existing methods are equivalent i.e. 634.6153 approximately. But using proposed method the optimum value is coming to be 880.1926.

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