Optimal Capacitor Placement for Voltage Stability Enhancement in Distribution Systems using BBO

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ABSTRACT
The operating conditions of the present day distribution systems are closer to the voltage stability boundaries due to the ever increasing load demand. This article presents a biogeography based optimization (BBO) algorithm for determining the optimal locations and sizing of static and/or switched shunt capacitors with a view to enhance voltage stability of distribution systems. Biogeography deals with geographical distribution of biological species. Mathematical models of biogeography portray how species arises, migrates from one habitat to another and gets smeared out. The BBO algorithm searches for global solution through migration and mutation. The superiority of this approach is demonstrated by testing the algorithm on 15, 33 and 69-node distribution systems.

General Terms
Distribution systems

Keywords
voltage stability, radial distribution systems, capacitor placement

NOMENCLATURE
BBO biogeography based optimization
CC capacitor cost
CFC cost of the fixed capacitor × no. of fixed capacitors
CSC cost of the switched capacitor × no. of switched capacitors
$E_{\text{max}}$ maximum emigration rate
FIC fixed installation cost at each node
$H_{SI}$ habitat suitability index
$i^{\text{max}}$ maximum immigration rate
iter(max) maximum number of iterations
$K_e$ $K_p$ $K_v$ constants for energy loss cost, peak load loss cost, capacitor cost and $VSC$
$K_q$ penalty factor for reactive support limit constraint
$mnc_j$ maximum number of capacitors banks to be connected at node-$j$
NCB number of capacitor banks
$nn$ number of nodes in the system
$nh$ number of habitats
$n$ total number of species in the habitat
$neh$ number of elite habitats
$nl$ number of load levels

$N_{c_{i,j}}$ number of capacitor banks for $i$-th load level at $j$-th ranked node.
PCPA proposed capacitor placement algorithm
$P^t(i)$ probability that the habitat contains exactly $S$ species at time $t$
$p^s$ species count probability
$p_{\text{mod}}$ habitat modification probability
$p^m$ mutation probability
$p^l_i$ net power loss in the system at $i$-th load level
$p^p$ peak power loss in the system
$P_m + jQ_m$ real and reactive powers at the receiving end of branch-$m$
$Q^R_j$ available rating of a capacitor
$Q_{c_{i,j}}$ reactive power support required for $i$-th load level at node-$j$
$r_m + jx_m$ resistance and reactance of branch-$m$ connected between nodes-$k$ and $m$
$S$ species in the habitat
$SIV$ suitability index variable
TCB type of capacitor banks
$T_i$ duration of $i$-th load level
$t$ and $\Delta t$ time and change in time respectively
VM voltage magnitude
VS voltage stability
$VM$ lowest value of VM in the system
$VSC$ voltage stability cost
$VSI_m$ VSI at node-$m$
$VSI_{L}$ lowest value of VSI in the system
$\Psi$ objective function to be minimized
$\lambda$ and $\mu$ immigration and emigration rates respectively
$\eta$ a set of nodes considered for CP

1. INTRODUCTION
The phenomenon of voltage instability in power systems is characterized by a monotonic voltage drop, which is slow at first, becomes abrupt after some time and is generally
triggered by some form of disturbance or change in the operating conditions that create an increased demand for reactive power, which is in excess of what the system is capable of supplying. The other factors contributing to voltage collapse are generator reactive power/voltage control limits, load characteristics, characteristics of reactive compensation devices and the action of voltage control devices such as transformer under load tap changers [1-2]. In recent years, the distribution systems experience a sharp increase in load demand on account of the extensive growth of the utilities. Besides, with the advent of deregulation in the power industry, there is a greater focus on managing the network assets efficiently rather than reinforcing the network’s capacity. The operating conditions are thus more and more closer to the voltage stability (VS) boundaries. In addition, distribution networks are subjected to distinct load changes everyday. In certain industrial areas, it is observed that under certain critical loading conditions, the distribution system suffers from voltage collapse [3]. Over the years, a number of approaches besides various indices for studying the VS are suggested in the literature [4-6].

Shunt capacitors supply reactive power and boost local voltages. Though several attempts have emerged to use capacitor banks in distribution systems for power factor correction, feeder voltage control [1-2], loss minimization [7], reliability enhancement [8] and profitability enhancement [9], hardly any work is found involving them with a view of enhancing VS of distribution systems. Recently algorithms for enhancing the VS of transmission system by optimal CP have been discussed in [10]. A relationship between VS and loss minimization that facilitates maximizing VS through loss minimization is outlined in [11]. An elegant CP algorithm to enhance VS of distribution systems through obtaining a condition for maximizing VS is suggested in [12]. An analytical CP algorithm to enhance VS of distribution system by treating the reactive line flow of the weakest line as the decision variable is outlined in [13]. A CP algorithm based on a VS index (VSI) for determining the optimal locations and sizing of static and/or switched shunt capacitors in order to enhance VS in addition to improving the VP is developed in [14]. A simple CP technique with a view to avoid voltage collapse through linearizing a VSI and reactive power flow of the weakest line is proposed for distribution systems in [15]. However, there is still a need for efficient and economically justified solution strategies for enhancing the VS of the present day stressed distribution systems.

Recently, a Biogeography-Based Optimization (BBO) modeled on the theory of biogeography, which is the study of the geographical distribution of biological organisms, has been proposed for solving optimization problems by Simon [16]. Like other evolutionary algorithms, BBO is a population based stochastic optimization technique, sharing information between candidate solutions based on their fitness values with a view of obtaining the global best solution. Since its introduction, it has been applied to a variety of problems including sensor selection [16], power system optimization [17-18], ground water detection [19] and satellite image classification [20]. A new BBO algorithm that uses the VSI suggested in [5], for optimal placement of static and/or switched shunt capacitors in radial distribution system for VS enhancement is proposed in this article. This method improves the voltage profile and reduces system losses in addition to enhancing VS. The method is tested on 15-, 33- and 69-node radial distribution systems and the results presented.
\[ P' = \begin{cases} \frac{-(\lambda + \mu)P + P^{s+1} \lambda_{s+1}}{S = 0} \\ \frac{-(\lambda + \mu)P' + P^{s+1} \lambda_{s+1} + P - 1 \lambda_{s+1}}{1 \leq S \leq S_{\max}} \\ \frac{-(\lambda + \mu)P' + P^{s+1} \lambda_{s+1}}{S = S_{\max}} \end{cases} \]

The equation for emigration rate \( \mu_k \) and immigration rate \( \lambda_k \) for \( k \) -number of species is developed from Fig.1 as

\[ \mu_k = \frac{E_{\max}}{n} \]

\[ \lambda_k = I_{\max} \left(1 - \frac{k}{n}\right) \]

When \( E_{\max} = I_{\max} \), the immigration and emigration rates can be related as

\[ \lambda_k + \mu_k = E_{\max} \]

The concept of BBO is based on the mechanisms of migration and mutation as discussed below.

2.1 Migration
A population of candidate solutions can be represented as vectors of real numbers in BBO algorithm. Each real number in the array is considered as an SIV, which is then used to evaluate the fitness of each candidate solution, denoted by HSI. High HSI represents a better quality solution and low HSI denotes an inferior solution. The emigration and immigration rates of each solution are probabilistically used to control the sharing of features between habitats through a habitat modification probability, \( P_{\text{mod}} \). If a given solution \( S_i \) is selected for modification, then its \( \lambda \) is used to probabilistically decide whether or not to modify each SIV in that solution. After selecting the SIV for modification, \( \mu \) of other solutions are used to select which of the solutions of the habitat set will migrate randomly chosen SIVs to the selected solution \( S_i \). Some kind of elitism, which retains the best habitat having highest HSI without performing migration operation, is used in order to prevent the best solutions from being corrupted.

2.2 Mutation
The cataclysmic events that drastically change the HSI of a habitat is represented by mutation of SIV and species count probabilities are used to determine mutation rates. The probability of each species count, \( P^{s'} \) given by Eq. (2), indicates the likelihood that it exists as a solution for a given problem. If the probability of a given solution is very low, then that solution is likely to mutate to some other solution. Similarly if the probability of some solution is high, then that solution has very little chance to mutate. So, it can be said that very high HSI solutions and very low HSI solutions have less chance to create more improved SIV in the later stage. But the medium HSI solutions have better chance to create much better solutions after mutation operation. Mutation rate of each set of solution can be calculated in terms of species count probability using the following equation:

\[ m(S) = m_{\max} \left(1 - \frac{P^s}{P_{\max}}\right) \]

This mutation scheme tends to increase diversity among the population, avoids the dominance of highly probable solutions and provides a chance of improving the low HSI solutions even more than they already have.

3. PROPOSED ALGORITHM

![Fig. 2 Sample Distribution Line](image)

The philosophy of Proposed Capacitor Placement Algorithm (PCPA) is to identify a set of weaker nodes from VS point of view and determine the locations, types, number and sizes of fixed and switchable capacitors to be installed through BBO with an objective to reducing the energy losses, decreasing the peak load loss, lowering the capacitor cost and improving the VS. It uses a VSI, suggested in [5], that varies between unity at no load and zero at voltage collapse point for determining the weaker nodes. The VSI at node-\( m \) of Fig.2 is determined by

\[ VSI_m = V_{\text{m}} - 4 \left( P_{\text{m}} \frac{x_{\text{m}}}{r_{\text{m}}} - Q \frac{r_{\text{m}}}{x_{\text{m}}} \right) - 4 \left( P_{\text{Lm}} + Q_{\text{Lm}} \right) \]

The node with lowest VSI value is identified as the weakest node. In addition nodes can be ranked based on the VSI values.

There are two components of losses in any distribution line. The first one is the losses associated with real current flow and the other associated with reactive current flow. It is known that the real current flow cannot be eliminated for a given loading pattern. But the reactive currents can be supplied at appropriate nodes and their flow through the distribution lines can be reduced, thereby minimizing the associated reactive power losses and enhancing VS. It is obvious that the losses due to reactive current flow can be completely eliminated by placing capacitor banks at all the nodes to supply local reactive power. However, it may not be practical to place capacitor banks all the nodes in the system.

3.1 Representation BBO Variables

The location for CP may be identified by computing the VSI, given by Eq. (7), and ranking the nodes by arranging the VSI values in ascending order. The node that is ranked first is the most sensitive node from VS point of view and requires priority in providing compensation. The first few nodes in the ranking list are selected for CP. Let us assume \( ncn \) - nodes are selected for compensation. It should be noted that the BBO algorithm determines the amount of VAR support required at each load level only at the selected nodes. The number of decision variables of the problem thus depends on the number of load levels considered over the planning period and the number of nodes considered for CP. The SIV of each habitat can therefore be represented as

\[ \text{Habitat} = \]
\begin{align*}
N_{C}^{1,1} & \quad N_{C}^{1,2} & \quad \cdots & \quad N_{C}^{1,j} & \quad \cdots & \quad N_{C}^{1,n_{cn}} \\
\cdots & \quad \cdots & \quad \cdots & \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \\
N_{C}^{i,1} & \quad N_{C}^{i,2} & \quad \cdots & \quad N_{C}^{i,j} & \quad \cdots & \quad N_{C}^{i,n_{cn}} \\
\cdots & \quad \cdots & \quad \cdots & \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \\
N_{C}^{n_{cl},1} & \quad N_{C}^{n_{cl},2} & \quad \cdots & \quad N_{C}^{n_{cl},j} & \quad \cdots & \quad N_{C}^{n_{cl},n_{cn}}
\end{align*}

(8)

The capacitive support required for \( i \)-th load level at \( j \)-th ranked node can be calculated by

\[ Q_{C,i,j} = N_{C,i,j} \times Q^{R} \tag{9} \]

At each node, there is always a maximum limit for providing the reactive power support for practical and economic reasons. Therefore, the maximum number of capacitors banks to be connected at node \( j \) is limited to \( mnc_{j} \) and the limit constraints on the decision variables are

\[ 0 \leq N_{C,i,j} \leq mnc_{j} \quad i = 1, 2, \ldots, n_{ll} \tag{10} \]

3.2 Problem Objectives and Constraints

The objective of the proposed formulation is to minimize the energy losses, decrease peak load loss and lower the capacitor cost and improve the VS by providing appropriate amount of reactive power support at the selected nodes. It can be written as

\[ \text{Min} \quad \Psi = K \sum_{i=1}^{n_{ll}} TP_{i,j} + K_{P} P_{i,j} + K_{CC} CC + K_{VSC} VSC \tag{11} \]

where

\[ VSC = \frac{1}{\prod_{n=1}^{m} VSI_{n}} \]

\[ CC = \text{FIC} + \text{CFC} + \text{CSC} \]

At any load level, the net reactive power support should not exceed the total reactive power load in the system. The reactive power limit constraint can thus be modeled as

\[ \sum_{j=0}^{m} Q_{C,i,j} - \sum_{j=2}^{m} Q_{j,i} = i = 1, 2, \cdots, n_{ll} \tag{13} \]

3.3 Habitat Suitability Index

The BBO searches for optimal solution by maximizing a \( HSI \) function, which is formulated from the objective function of Eq. (11) and reactive power limit constraint of Eq. (13) as

\[ \text{Max} \quad HSI = \frac{1}{1 + \Phi} \tag{14} \]

Where

\[ \text{Min} \quad \Phi = K \sum_{i=1}^{n_{ll}} TP_{i,j} + K_{P} P_{i,j} + K_{CC} CC + K_{VSC} VSC \]

3.4 Repair Algorithm

The number of capacitor banks required at any node for lower load level should be less than that of higher load levels and vice versa. Let the first row of each habitat denotes the lowest load level, the second row next higher load level and so on for convenience. The following repair mechanism is performed for each habitat.

for \( j = 1 : n_{cn} \)

\[ M = N_{C,k,j} \]

if \( N_{C,k,j} \geq M \)

replace \( N_{C,k,j} \) by a random number in the range \((0, M)\)

end

end

3.5 Stopping Criteria

The process of generating new habitats can be terminated either after a fixed number of iterations or if there is no further significant improvement in the global best solution.

3.6 Algorithm

1. Read the line data and load data at different load levels.
2. Choose maximum number of capacitors banks, \( mnc_{j} \) to be connected at each node-i.
3. Choose heuristically the number of nodes, \( n_{cn} \), for VAR support
4. Carryout distribution power flow [23] and compute VSI using Eq. (7)
5. Rank the VSI values in ascending order and choose nodes corresponding to the first \( n_{cn} \)-values in the ranked list.
6. Set the iteration counter \( iter = 1 \)
7. Choose the BBO parameters such as \( nh \), \( neh \), \( I_{max}^{n} \), \( E_{max}^{n} \), \( P_{mod}^{n} \), \( P_{w}^{n} \) and \( Iter_{max}^{n} \).
8. Randomly generate integer values to denote \( SIVs \) of each habitat in the range of \([0, mnc_{j}]\) so as to form habitat matrix containing \( nh \)-habitats.
9. Repair the habitats using the procedure described in section 3.3
10. Perform the following for each habitat
For each load level,
- Add the reactive power support equal to \( \left\{ N_{\text{c}k} \times Q_{\text{c}k} \right\} \) at the respective node for \( k = 1,2,\cdots, ncn \)
- Carry out Distribution Power Flow
- Evaluate VSI using Eq. (7)

- Evaluate HSI through Eq. (14)
  11. Identify \( neh \) habitats having highest HSI and retain them as it is without making any modifications
  12. Evaluate \( \mu \) and \( \lambda \) for each habitat.
  13. Perform migration probabilistically on those SIVs of non-elite habitats based on \( P^{\text{mod}} \).
  14. Perform mutation probabilistically on those SIVs of non-elite habitats based on \( P^{\text{m}} \).
  15. Check for convergence, i.e., check whether any one of the following criteria is satisfied.
    - the number of iterations since the last change of the best solution is greater than a pre-specified number
    - the number of iterations reaches \( \text{iter}^{\max} \).
  16. If converged, the solution corresponding to the elite habitat is the optimal solution; else, increment iteration counter \( [\text{iter} = \text{iter} + 1] \) and go to step (9)

4. RESULTS AND DISCUSSIONS

The PCPA is tested on 15-, 33-, and 69-node distribution systems. The line and load data for these three systems are obtained from the references [21] and [22]. The size of the capacitor bank considered in this study is 150 kVAR and its multiples. The cost of fixed and switched capacitors is considered as $900 and $1200 respectively. The results are obtained for light, medium, full, and over load conditions by multiplying the base-load by a factor 0.5, 0.8, 1.0, and 1.1 respectively. The duration of each interval is taken as 1000, 3000, 3760, and 1000 hours. It uses the distribution power flow suggested in [23].

15 node test system: The algorithm is run several times by changing values from 5 to 10 and the results obtained. It is observed that the losses are reduced to reasonably low level when \( \mu \) equals ten. The net reactive power load in the system and minimum VAR support required at each selected nodes for different loading conditions are given in Table 1. It is to be noted that the net VAR support at each loading level is less than the reactive power load in the system. The capacitor banks to provide VAR support at low load level is chosen as fixed type and the rest as switched type. The classification of fixed and switch type are given in Table 2.

33 & 69 node test systems: The net reactive power load in the system and minimum VAR support required at each selected nodes for different loading conditions when \( \mu \) equals 15 and 14 respectively for 33 and 69-node systems respectively are given in Tables 3 and 6. The classification of fixed and switch type are given for 33 and 69 node systems are given in Tables 4 and 5 respectively.

The losses, the lowest VM and the lowest VSI for all the systems under study before and after CP are graphically displayed in Figs. 3, 4 and 5 respectively. It is clear from these figures that the proposed method minimizes the losses, improves the voltage profile and enhances the VS.

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<th>Table 1 Requirement of VAR compensation for 15-node system</th>
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Table 5  Requirement of VAR compensation for 69-node system

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Table 6  Type and Size of Capacitor placed for 69 node system

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5. CONCLUSIONS

A CP algorithm using BBO for radial distribution system has been developed. This method has been framed with a view to find the optimal locations and determine the size and type of capacitor banks to be placed in order to minimize losses besides enhancing the VS and improving the voltage profile.

The algorithm has been coined to place capacitors at a set of weaker nodes. The algorithm will be suitable for practical implementation on systems of any size on account of the ease of the computational procedure and the flexibility with which it can be incorporated. The developed method can be extended to place static var compensators (SVC) for better...
performances besides adaptively adjusting the BBO parameters.

6. ACKNOWLEDGMENTS
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7. REFERENCES