Soret Effect on n^{th} Order Chemically Reactive MHD Flow through Porous Medium Bounded by Two Vertical Plates with Heat Source

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ABSTRACT
The effects of Soret number, heat source parameter and order of chemical reaction on separation of a binary fluid mixture for the case of a two dimensional steady free convective and mass transfer flow of an incompressible, viscous, chemically reacting, electrically conducting fluid through a porous medium bounded by two stationary infinite vertical porous plates in the presence of transverse magnetic field and heat generation/ absorption are investigated. The governing non linear partial differential equations are transformed into coupled non dimensional non linear ordinary differential equations by using non-dimensional quantities and are solved numerically by using MATLAB's built in solver bvp4c. The influence of the Dufour number, Soret number, heat source parameter and order of chemical reaction on velocity, temperature and concentration profiles as well as on skin friction, Nusselt number and Sherwood number are illustrated graphically. It is concluded that the Soret number, Dufour number and the chemical reaction parameter play a crucial role on the heat and mass transfer.

Keywords
Heat and Mass transfer; MHD; porous medium; two vertical plates; Dufour and Soret effects; chemical reaction.

1. INTRODUCTION
Many researchers have shown a great amount of interest in the field of heat and mass transfer over years due to their applications in reservoir engineering and extending theory of separation processes. The binary mixture in MHD through porous medium has many applications in nuclear reactors, problems dealing with liquid metals, enhanced oil recovery, cooling of nuclear reactors and underground energy transport, MHD pumps, MHD bearings, existence of geomagnetic field, hydromagnetic flow and heat transfer in the earth's liquid core and others. Chemical reactions in combined heat and mass transfer problems play a significant role. Chemical reactions are either homogeneous or heterogeneous processes. The reaction is homogeneous, if it occurs uniformly through a given phase. In well mixed system, it takes place in the solution while a heterogeneous reaction occurs at the interface i.e. in a restricted region or within the boundary of a phase. If the rate of reaction is proportional to the n^{th} power of concentration then the chemical reaction is said to be of order 'n'. Chemical reaction between a foreign mass and the fluid occurs in many industrial applications such as polymer production, the manufacturing of ceramics or glassware, damage of crops due to freezing, distribution of temperature and moisture over groves of fruit trees and so on. Separation processes of components of a fluid mixture have many applications in science and technology, environmental engineering, chemical industry and separation of isotopes from their naturally occurring mixture. Soret effect is the tendency of a convection-free fluid mixture to separate under a temperature gradient. Soret effect also plays a crucial role in the hydrodynamic instability of mixtures, mineral migrations and mass transport in living matters.

2. RELATED WORK
Researchers such as Nield and Bejan [1] and Ingham and Pop [2, 3] have made comprehensive reviews in the field of heat and mass transfer by natural convection in a fluid-saturated porous medium. Chambre and Young [4] have analyzed a first order chemical reaction in the neighbourhood of a horizontal plate. Apelblat [5] studied analytical solution for mass with a chemical reaction of first order. Das et al. [6] have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Osterle and Young [7] have discussed the natural convection between heated vertical plates in a horizontal magnetic field. Heat transfer between two horizontal plates in presence of a uniform magnetic field has been analyzed by Bodosa et. al. [8]. Sharma et. al. [9, 10] investigated analytically the Soret effect on de-mixing of species in hydro-magnetic flow of a binary mixture of incompressible viscous fluid between two parallel plates, first taking the plates horizontal and second by taking the plates vertical. H. A. Attia [11] discussed the effect of variable properties on the unsteady Couette flow with heat transfer considering the Hall Effect. MHD free convective flow through a porous medium bounded by two infinite vertical porous plates was studied by Ahmed et. al.[12]. Chamkha et. al.[13] analyzed MHD double diffusive and chemically reactive flow through porous medium bounded by two vertical plates. Sharma et al.[14] discussed the influence of chemical reaction, heat source, Soret and Dufour effects on separation of a binary fluid mixture in MHD natural convection flow in porous media. Sharma and Konwar [15] have discussed the effect of chemical reaction in unsteady MHD Couette Flow.

The present paper deals with the two dimensional steady free convection flow through two stationary infinite vertical porous plates embedded in a porous medium in the presence of heat generation or absorption, n^{th} order chemical reaction and transverse magnetic field. The aim of this paper is to study the influence of Soret effect, Dufour effect, heat source parameter and order of chemical reaction. The present study is an extension of the work of Chamkha et al. [13] by considering the chemical reaction of order n, Dufour effect and heat source.

3. FORMULATION OF THE PROBLEM

Consider the steady laminar incompressible free convective flow of a viscous, chemically reacting, electrically conducting and heat generating/ absorbing fluid through two stationary infinite vertical porous plates embedded in a porous medium in the presence of a transverse uniform magnetic field B_0. Homogeneous chemical reaction of order n takes place in the flow. Let h be the distance between the two plates. The x-axis is taken in the direction vertically upward i.e. along the plate and y-axis normal to it i.e. directed into the fluid region. Figure 1 shows the physical model of the problem. The plates are maintained at constant temperatures T_0 at y = 0 and T_1 at y = h. The concentration of the rarer and lighter components on the surface of the plates are considered to be C_0 at y = 0 and C_1 at y = h. The magnetic Reynolds number is considered to be very small and therefore Hall current is negligible. Also the induced magnetic field is assumed to be very small in comparison to the applied magnetic field and hence is neglected. Since the plates are of infinite length therefore all the physical quantities are independent of x.

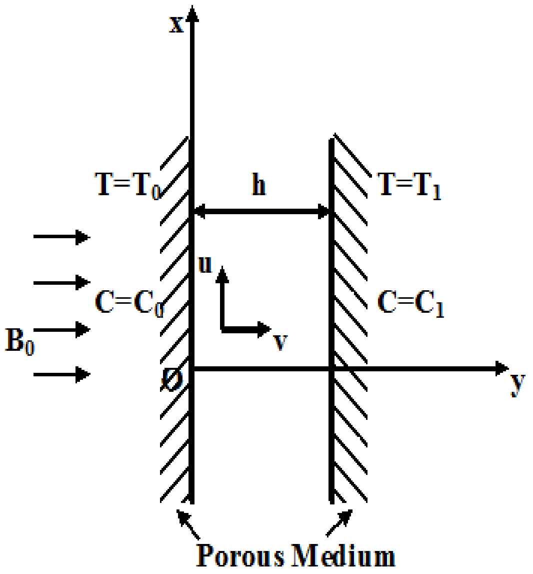


Figure 1 : Physical model of the problem

Under these assumptions, the governing equations of continuity, momentum, energy and diffusion are given by

$$\frac{\partial v}{\partial y} = 0 \quad (1)$$

$$v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta_T(T - T_\infty) + g\beta_C(C - C_\infty) - \frac{\nu u}{\kappa} - \frac{\sigma B_0^2 u}{\rho} \quad (2)$$

$$v\frac{\partial T}{\partial y} = \alpha_m\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{C_P}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{D_m K_T}{C_S C_P}\frac{\partial^2 C}{\partial y^2} + \frac{Q_0}{\rho C_P}(T - T_\infty) \quad (3)$$

$$v\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y^2} + \frac{D_m K_T}{T_m}\frac{\partial^2 T}{\partial y^2} - K_1(C - C_\infty)^n \quad (4)$$

where u and v are fluid velocities along x and y direction respectively, $v = -v_0$ is constant suction/ injection velocity, T is temperature, T_∞ is temperature at static condition, C is concentration, C_∞ is concentration at static condition, ν is the kinematic viscosity, σ is the electrical conductivity, B_0 is the applied magnetic field, ρ is the fluid density, β_T is the thermal expansion coefficient, β_C is compositional expansion coefficient, α_m is the thermal conductivity, g is the acceleration due to gravity, K_1 is the dimensional chemical reaction parameter, C_P is the specific heat of the fluid at constant pressure, D_m is the mass diffusion coefficient, Q_0 is the heat generation/ absorption coefficient, K_T is the thermal diffusion ratio, C_S is the concentration susceptibility , T_m is the mean fluid temperature, κ is the permeability of porous medium and n is the order of the chemical reaction.

The boundary conditions are

$$\left.\begin{aligned} u = 0, T = T_0, C = C_0 \quad &\text{at} \quad y = 0 \\ u = 0, T = T_1, C = C_1 \quad &\text{at} \quad y = h \end{aligned}\right\} \quad (5)$$

Introduce the following non-dimensional quantities:

$$Y = \frac{y}{h}, U = \frac{u}{v_0}, \theta = \frac{T - T_\infty}{T_0 - T_\infty}, \phi = \frac{C - C_\infty}{C_0 - C_\infty} \quad (6)$$

Using (6), equations (2) – (4) reduce to

$$\frac{1}{Re}\frac{d^2U}{dY^2} + \frac{dU}{dY} + G_T\theta + G_C\phi - \frac{1}{Re\ Da}U - Re\ M\ U = 0 \quad (7)$$

$$\frac{1}{PrRe}\frac{d^2\theta}{dY^2} + \frac{Ec}{Re}\left(\frac{dU}{dY}\right)^2 + \frac{D_f}{Re}\frac{d^2\phi}{dY^2} + \frac{d\theta}{dY} + \delta\ Re\ \theta = 0 \quad (8)$$

$$\frac{1}{ScRe}\frac{d^2\phi}{dY^2} + \frac{Sr}{Re}\frac{d^2\theta}{dY^2} + \frac{d\phi}{dY} - \gamma\ Re\ \phi^n = 0 \quad (9)$$

where Re is Reynolds number, G_T is the thermal Grashof number, G_C is the solutal Grashof number, M is the Hartmann number, Da is the permeability parameter, Pr is the Prandtl number, Ec is the Eckert number, δ is the heat source parameter, D_f is the Dufour number, Sc is the Schimdt number, Sr is the Soret number and γ is the dimensionless chemical reaction parameter and are defined as

$$\left.\begin{aligned} &Re = \frac{v_0 h}{\nu}, G_T = \frac{g\beta_T(T_0 - T_\infty)h}{v_0^2}, M = \frac{\sigma B_0^2 \nu}{\rho v_0^2}, Da = \frac{\kappa}{h^2}, \\ &G_C = \frac{g\beta_C(C_0 - C_\infty)h}{v_0^2}, Pr = \frac{\nu}{\alpha_m}, Sc = \frac{\nu}{D_m}, D_f = \\ &\frac{D_m K_T}{C_S C_P \nu}\frac{(C_0 - C_\infty)}{(T_0 - T_\infty)}, \delta = \frac{Q_0 \nu}{\rho C_P v_0^2}, Ec = \frac{v_0^2}{C_P(T_0 - T_\infty)}, Sr = \\ &\frac{D_m K_T}{T_m \nu}\frac{(T_0 - T_\infty)}{(C_0 - C_\infty)}, \gamma = \frac{K_1 \nu}{v_0^2} \end{aligned}\right\} \quad (10)$$

The boundary conditions (5) in terms of dimensionless quantities reduce to

$$\left.\begin{aligned} U = 0, \theta = 1, \phi = 1 \quad &\text{at} \quad Y = 0 \\ U = 0, \theta = a, \phi = b \quad &\text{at} \quad Y = 1 \end{aligned}\right\} \quad (11)$$

where $a = \frac{(T_1 - T_\infty)}{(T_0 - T_\infty)}$ and $b = \frac{(C_1 - C_\infty)}{(C_0 - C_\infty)}$ are constants.

The dimensionless coefficients of skin friction, Nusselt number and Sherwood number near the plates can be written as

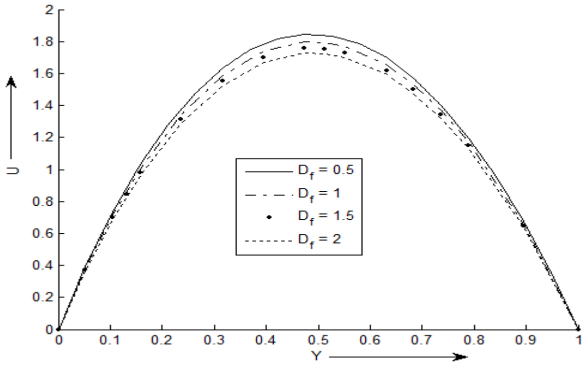
$$\left.\begin{aligned} &\tau_0 = \left[\frac{dU}{dY}\right]_{Y=0}, \ \tau_1 = \left[\frac{dU}{dY}\right]_{Y=1}, \ Nu_0 = \left[\frac{d\theta}{dY}\right]_{Y=0}, \\ &Nu_1 = \left[\frac{d\theta}{dY}\right]_{Y=1}, Sh_0 = \left[\frac{d\phi}{dY}\right]_{Y=0}, Sh_1 = \left[\frac{d\phi}{dY}\right]_{Y=1} \end{aligned}\right\} \quad (12)$$

The system of ordinary differential equations (7)-(9) under the boundary conditions (11) have been solved numerically by using MATLAB's built in solver bvp4c.

4. RESULTS AND DISCUSSION

The numerical results for velocity, temperature and concentration profiles of various values of the parameters D_f, Sr, Da, δ and n are carried out and displayed graphically in Figures (2) - (6).

Figures 2 (a)-(c) to 6 (a)-(c) depict velocity, temperature and concentration profiles for various values of D_f, Sr, Da, δ, n respectively. It is noticed that with an increase in the values of Dufour number D_f, Soret number Sr and order of chemical reaction n, the velocity and temperature decrease while the result is opposite with the variation of dimensionless heat source parameter δ and porosity parameter Da. Also the concentration of the rarer and lighter components increases with the increase of Dufour number but the effect get reversed with the increase of Sr, Da, δ and n. It can be depicted that as distance between the plates increases the velocity increase gradually near the first plate, attains the maximum value at the centre and then decreases towards the other plate. The temperature increases from the plate at Y = 0, attains its maximum value near the first plate and then decreases towards the other plate. The concentration seems to decrease from the plate at Y = 0, attains its minimum value near that plate and then increases towards the other plate. It is observed that the velocity and temperature increase with the decrease in Dufour number, Soret number and order of chemical reaction. Also the Dufour effect restricts the separation of the species of the rarer and lighter components. The separation process is enhanced by the Soret effect, heat source and order of the chemical reaction.



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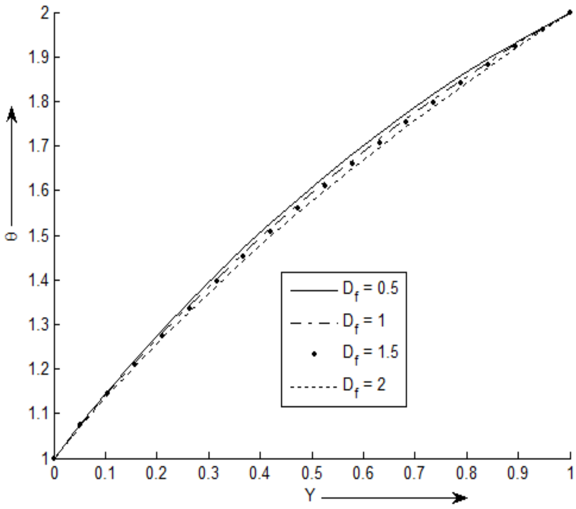

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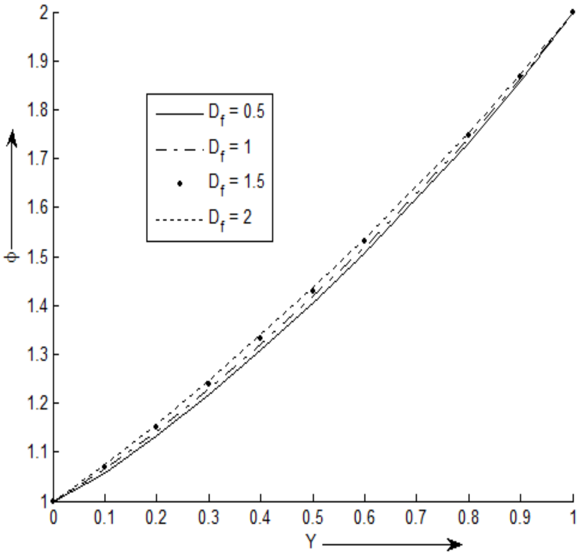

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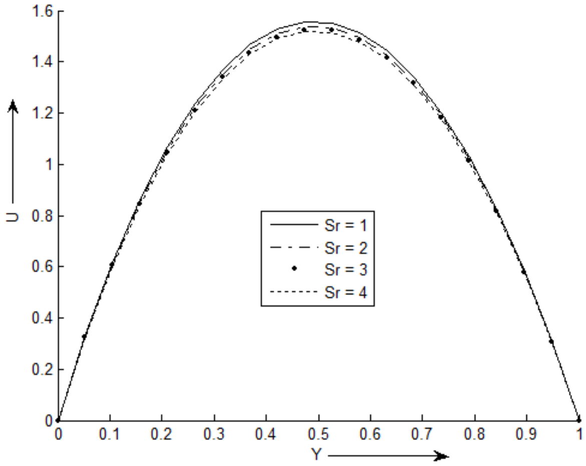
Figure 2: Effects of Dufour number D_f taking $D_f = (0.5, 1, 1.5, 2)$, $Re = 1$, $Da = 1$, $G_T = 5$, $G_C = 5$, $Pr = 0.71$, $Sr = 0.5$, $M = 1$, $Sc = 0.6$, $Ec = 0.01$, $\gamma = 0.5$, $\delta = 0.2$, $n = 2$, $a = 2$, $b = 2$ on (a) velocity profiles (b) temperature profiles (c) concentration profiles



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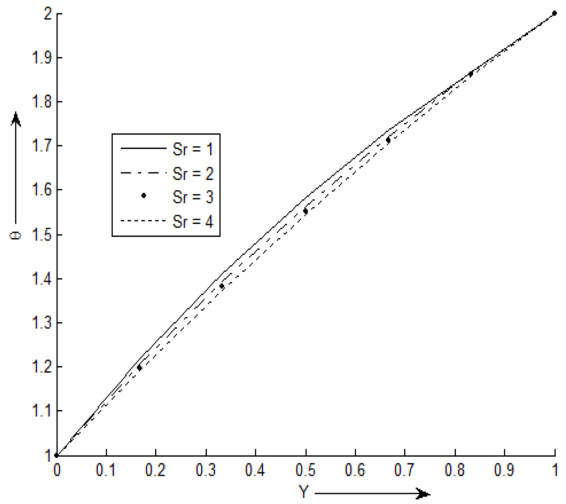

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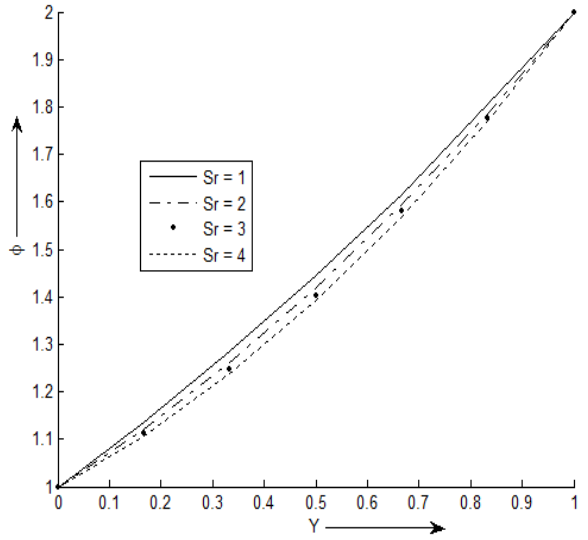

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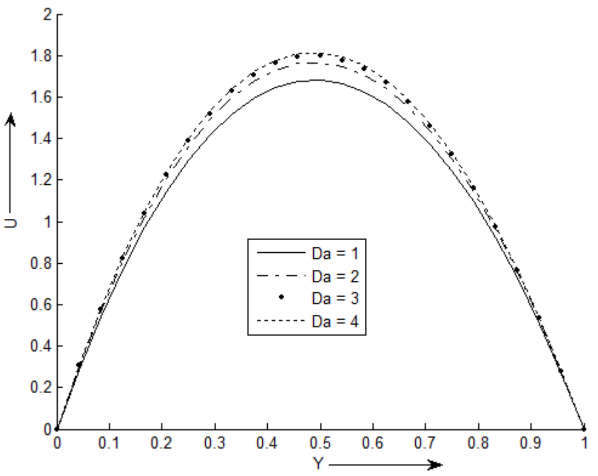
Figure 3: Effects of Soret number Sr taking $D_f = 1, Re = 1, Da = 1, G_T = 5, G_C = 5, Pr = 0.71, Sr = (1, 2, 3, 4), M = 1, Sc = 0.6, Ec = 0.01, \gamma = 0.5, \delta = 0.2, n = 2, a = 2, b = 2$ on (a) velocity profiles (b) temperature profiles (c) concentration profiles



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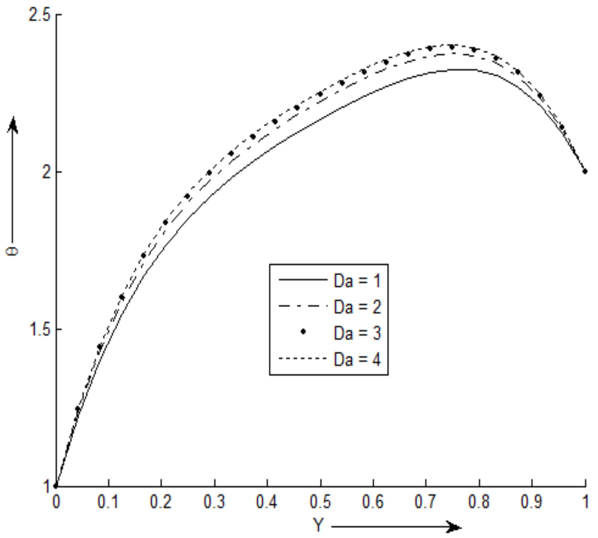

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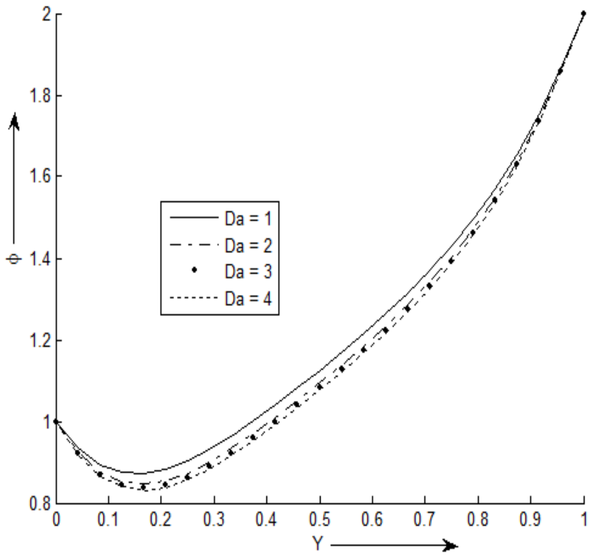

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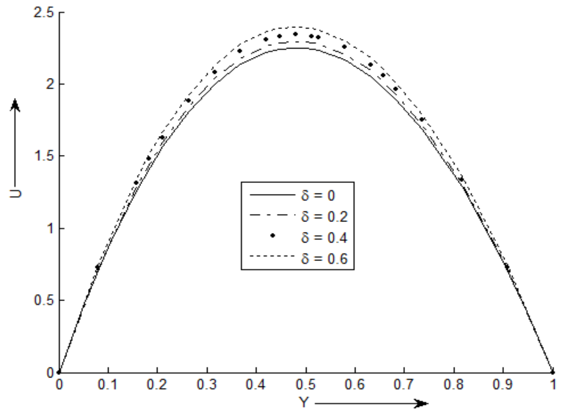
Figure 4: Effects of porosity parameter taking $D_f = 1, Re = 1, Da = (1, 2, 3, 4), G_T = 5, G_C = 5, Pr = 0.71, Sr = 0.5, M = 1, Sc = 0.6, Ec = 0.01, \gamma = 0.5, \delta = 0.2, n = 2, a = 2, b = 2$ on (a) velocity profiles (b) temperature profiles (c) concentration profiles



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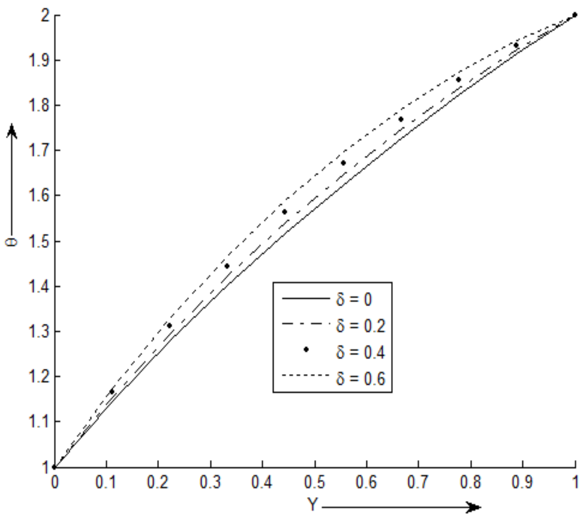

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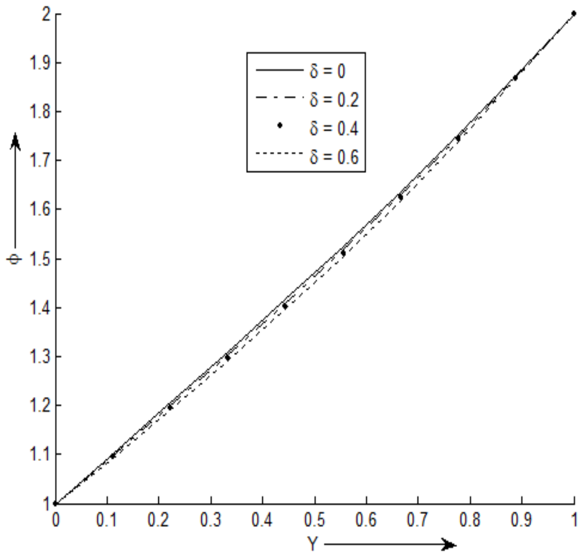

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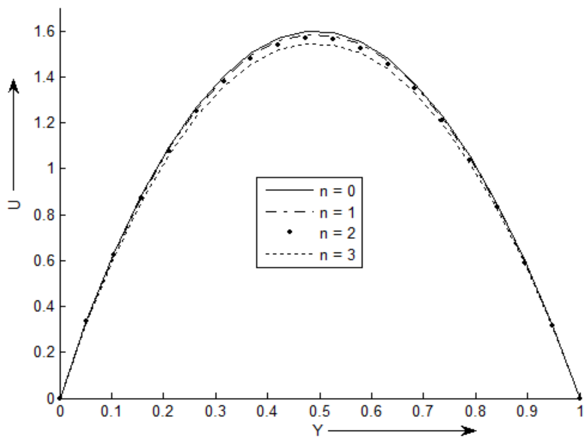
Figure 5: Effects of dimensionless heat source parameter δ taking $D_f = 1$, $Re = 1$, $Da = 1$, $G_T = 5$, $G_C = 5$, $Pr = 0.71$, $Sr = 0.5$, $M = 1$, $Sc = 0.6$, $Ec = 0.01$, $\gamma = 0.5$, $\delta = (0, 0.2, 0.4, 0.6)$, $n = 2$, $a = 2$, $b = 2$ on (a) velocity profiles (b) temperature profiles (c) concentration profiles



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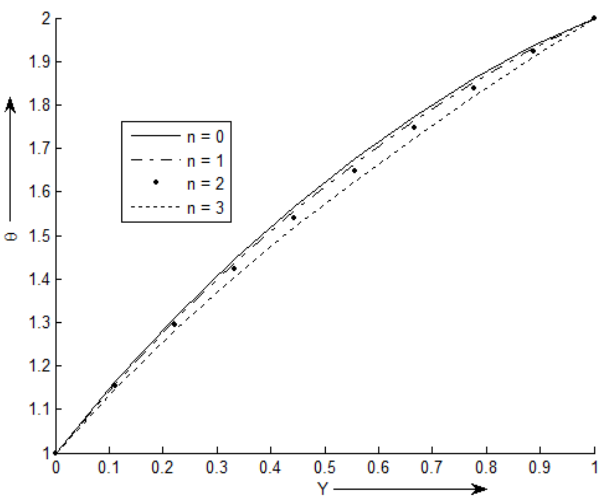

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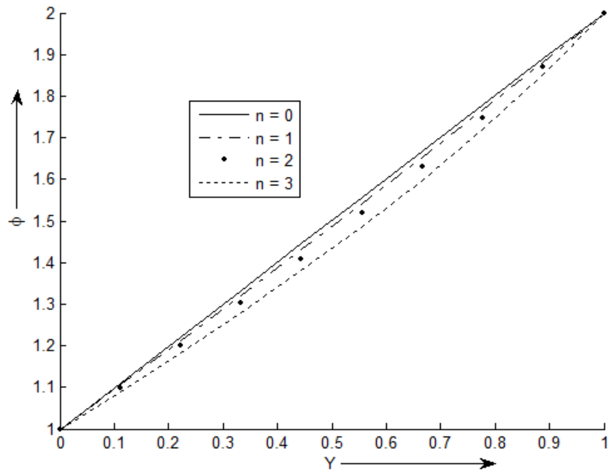

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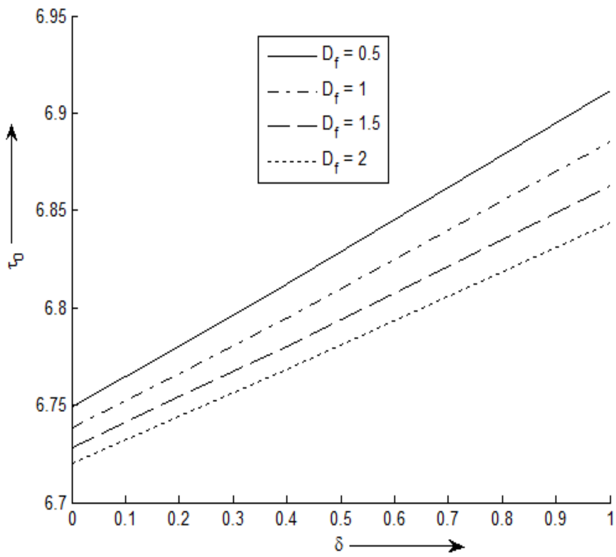
Figure 6: Effects of order of chemical reaction n taking $D_f = 1$, $Re = 1$, $Da = 1$, $G_T = 5$, $G_C = 5$, $Pr = 0.71$, $Sr = 0.5$, $M = 1$, $Sc = 0.6$, $Ec = 0.01$, $\gamma = 0.5$, $\delta = 0.2$, $n = (0, 1, 2, 3)$, $a = 2$, $b = 2$ on (a) velocity profiles (b) temperature profiles (c) concentration profiles

Finally, with the help of the figures 7 and 8, the behaviour of the coefficient of skin friction (τ_0, τ_1), Nusselt number (Nu_0, Nu_1) and Sherwood number (Sh_0, Sh_1) near the plates are observed.

Figure 6 show the effects of Dufour number D_f and dimensionless heat source parameter δ on the coefficient of skin friction, Nusselt number and Sherwood number near the plates Y = 0 and Y = 1. It is noticed that the skin friction $(\tau_0 = U'(0))$ and the Nusselt number $(Nu_0 = \theta'(0))$ near the plate Y = 0 and the Sherwood number $(Sh_1 = \phi'(1))$ near the plate Y = 1 increase with increasing values of heat source parameter and decreasing Dufour number but the skin friction $\left(\tau_1 = U'(1)\right)$ and Nusselt number near the plate Y = 1 $(Nu_1 = \theta'(1))$ and the Sherwood number $(Sh_0 = \phi'(0))$ near the plate Y = 0 seem to decrease with the increasing values of the heat source parameter δ and decreasing Dufour number D_f.



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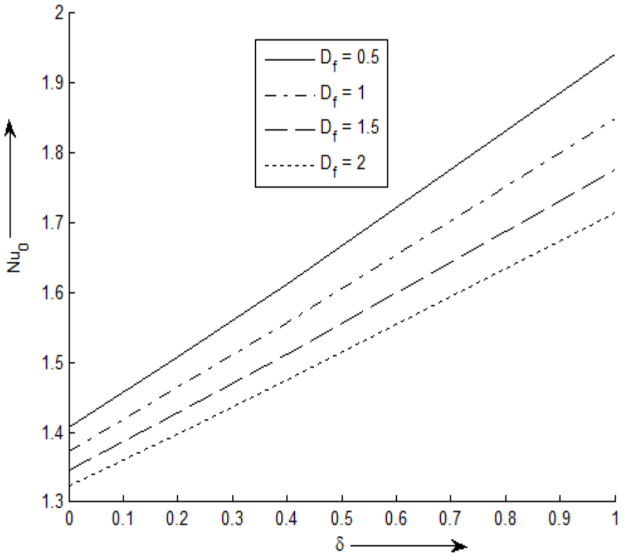

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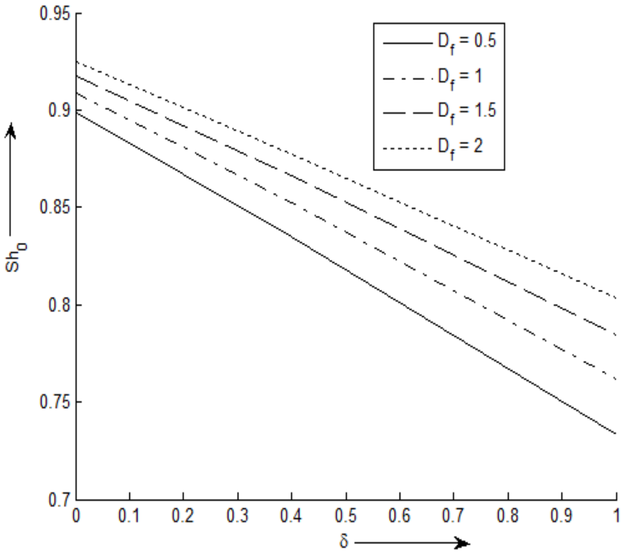

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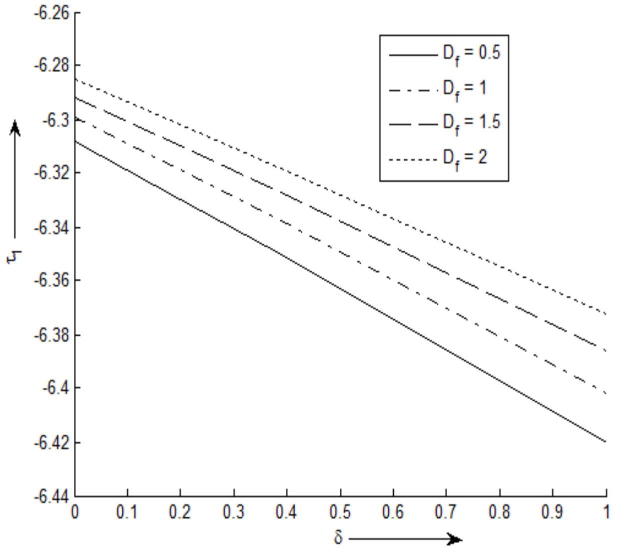

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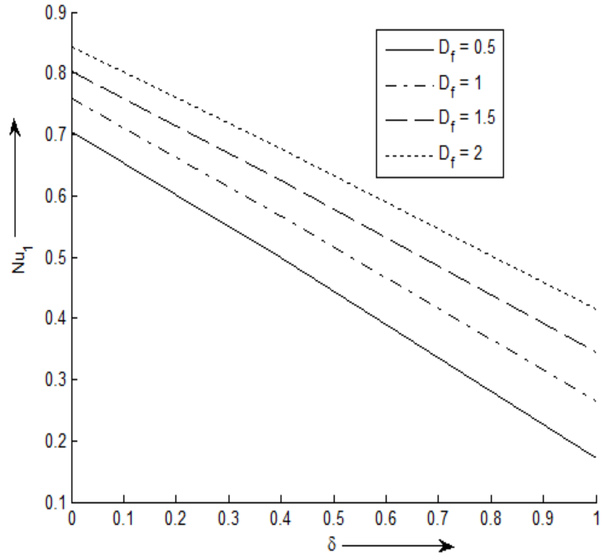

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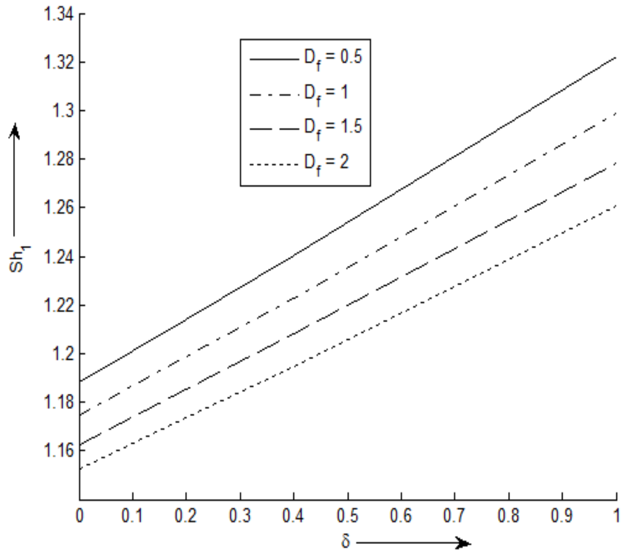

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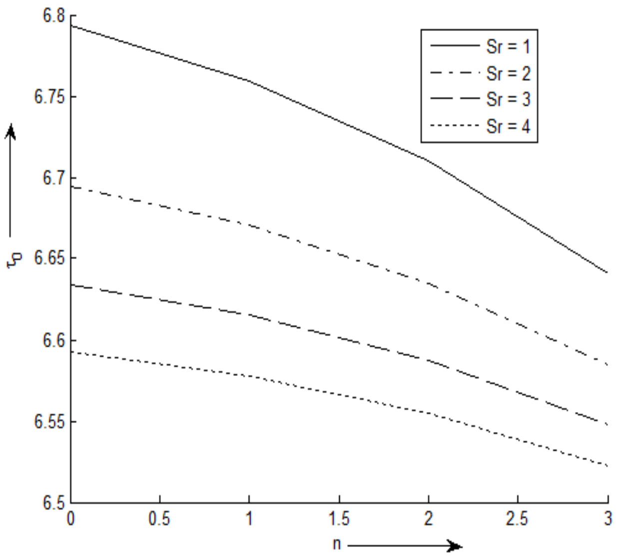
Figure 7: Effects of Dufour number D_f and dimensionless heat source parameter δ on (a) coefficient of skin friction, (b) Nusselt number (c) Sherwood number near Y = 0 and (d) coefficient of skin friction, (e) Nusselt number (f) Sherwood number near Y = 1

Figure 8 show the effects of Soret number Sr and order of chemical reaction n on the coefficient of skin frction, Nusselt number and Sherwood number near the plates Y = 0 and Y = 1. It is observed that the skin friction $(\tau_0 = U'(0))$, the Nusselt number $(Nu_0 = \theta'(0))$ and the Sherwood number $(Sh_0 = \phi'(0))$ near the plate Y = 0 decrease with increasing values of either the order of the chemical reaction or Soret number but near the plate Y = 1 these profiles $(\tau_1 = U'(1), Nu_1 = \theta'(1), Sh_1 = \phi'(1))$ seem to increase with the increasing values of either the Soret number Sr or the order of chemical reaction n.



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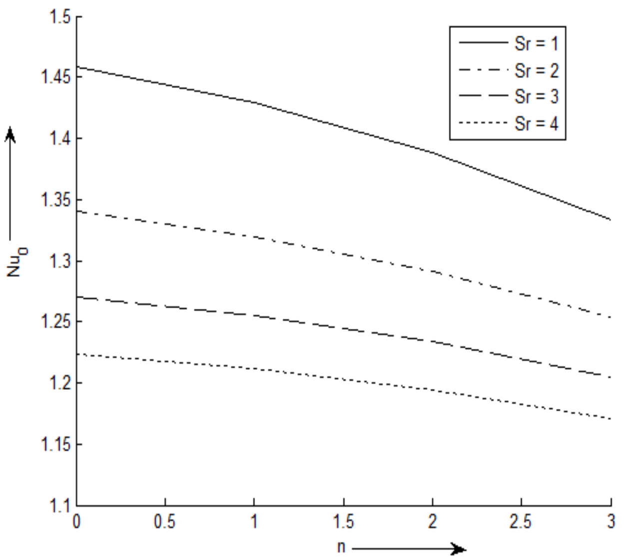

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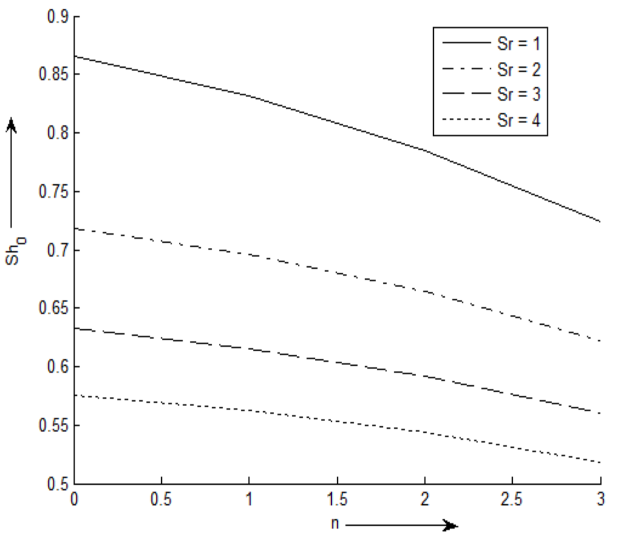

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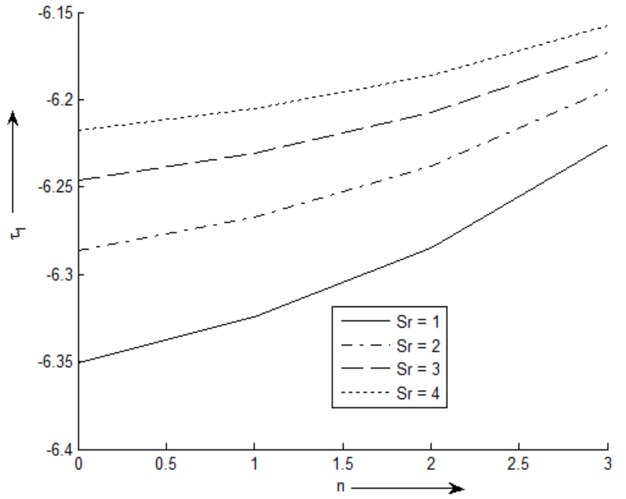

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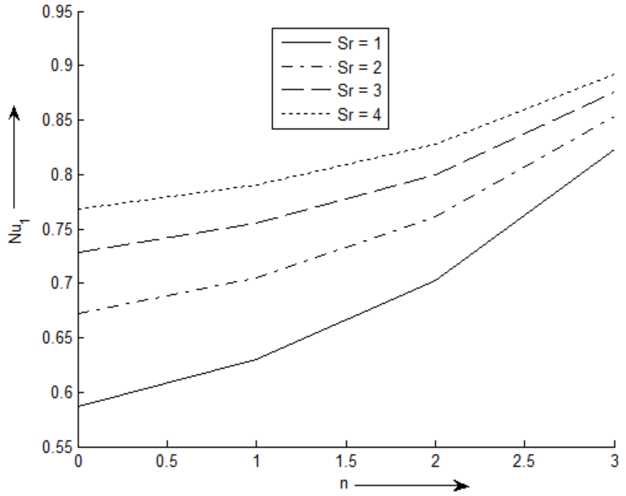

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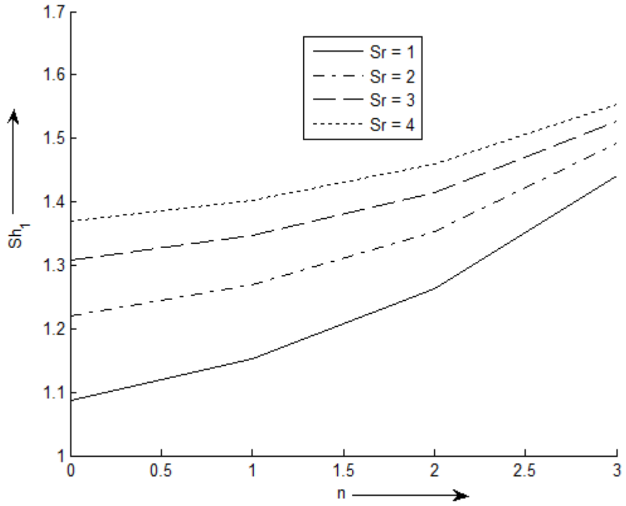

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Figure 8: Effects of Soret number Sr and the order of chemical reaction n on (a) coefficient of skin friction, (b) Nusselt number (c) Sherwood number near Y = 0 and (d) coefficient of skin friction, (e) Nusselt number (f) Sherwood number near Y = 1

5. CONCLUSION

It can be concluded that the velocity, temperature, coefficient of skin friction and Nusselt number near the plate Y = 0 of the binary fluid mixture increase while coefficient of skin friction and Nusselt number near the plate Y = 1 decrease with the decrease in the values of Dufour number, Soret number and order of chemical reaction but the effect gets reversed with the decrease of heat source parameter.

The concentration, Sherwood number near the plate Y = 0 increase while Sherwood number near the plate Y = 1 decrease with the decrease of Soret number, heat source parameter and order of chemical reaction but reverse effect is observed with decreasing values of Dufour number.

The results of this paper depict that high permeability of the porous medium will favour the temperature and the separation process of the components in the binary fluid mixture flowing through the vertical parallel plates embedded in the porous medium. Also the distance between the plates is inversely related to the permeability of the medium. Hence broaden vertical parallel plates results in low temperature of the fluid mixture.

On the basis of the conclusions of the paper, gas separating instruments can be installed in big cities as an engineering application so that harmful pollutants can be removed which are present in small quantities mixed with air. It also has its application in hyper velocity equipments such as missiles, aircrafts and so on and heat exchangers. The results obtained in this paper can be used by the researchers working in the field of chemical industries.

It is hoped that the present work will serve as a motivation for future experimental work which seems to be lacking at the present time.

6. REFERENCES

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