

Sliding Mode Control of Half-car Active Suspension

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ABSTRACT

The purpose of the sliding mode controller for a vehicle suspension system is to reduce the discomfort sensed by passengers which arises from road roughness and to increase the ride handling associated with the pitching and rolling movements. This necessitates a very fast and accurate controller to meet as much control objectives, as possible. This paper deals with introducing a new technique such as Proportional Integral Sliding Mode controller (PISMC), sliding mode controller (SMC) and PID approaches to Half-Car Active Suspension to design a stability to meet the control objectives. The advantage of this controller is that it can handle the nonlinearities faster than other conventional controllers. The approach of the proposed controller is to minimize the vibrations on each corner of vehicle by supplying control forces to suspension system when travelling on rough road. Simulation results and a comparison with a classical PID controller are presented and discussed. We studied the stability of the three used controllers in the presence of disturbances. A good performance for the sliding mode controller (SMC) is achieved in simulation studies despite the disturbances.

General Terms

Vehicle, Sliding mode control

Keywords

Half-car, dynamic system, active suspension system, PISMC, SMC, PID controller, disturbances

1. INTRODUCTION

A car suspension system is the mechanism that physically separates the car body from the wheels of the car. The suspension system can be categorized into passive, semi-active and active suspension system according to external power input to the system and/or a control bandwidth [1]. Active suspensions differ from the conventional passive suspensions in their ability to inject energy into the system, as well as store and dissipate it. Various control strategies have been proposed by numerous researchers to improve the trade-off between ride comfort and road handling that occurred when passive car suspension is used. A number of researchers have suggested control methods for vehicle suspension systems. A linear controller

was designed for a quarter or half vehicle [2],[3],[4],[5],[6]. Due to its robustness, the author used a H1 controller for car active suspension with electric linear motor in order to provide comfort and safety for the passengers [7]. In reference [8], the authors presented the optimal semi-active preview control response of a half car vehicle model with magnetorheological damper in order to minimize a performance index. Others authors used a nonlinear controller, such as a nonlinear optimal control law based on quadratic cost function which is developed, and applied on a half-car model for the control of active suspension system [9]. In [10], a mathematical models of a seven-degree of freedom suspension system based on the whole vehicle are established and the fuzzy controller of vehicle semi-active suspension system is designed. A large class of fuzzy approaches for vehicle suspension system are developed [11],[12],[13],[14],[15],[16]. A lot of researches have suggested control methods for vehicle suspension systems which combined two intelligent controls, fuzzy logic and neural network control [17],[18]. In order to achieve the desired ride comfort and road handling and to solve the mismatched condition problem due to the nature of the road disturbances, a proportional, integral sliding mode control technique is presented to deal with the system and uncertainties [19]. In this paper, a SMC strategy is developed based on the proportional sliding surface which is very quickly to achieve the desired trajectory than the proportional integral sliding surface which was presented in the last reference. The arrangement of this paper is as follows. The dynamic model of the half car suspension is given in the second section. Strategies of control for the half car which are the Proportional Integral Sliding Mode controller (PISMC), sliding mode controller (SMC) and classical PID controller are presented in the third section. Simulation results are introduced in the fourth section. Finally, conclusion and future works are given in the last section.

2. ACTIVE SUSPENSION SYSTEM

Modeling of the active suspension systems in the early days considered that input to the active suspension is a linear force as [20]. Recently, due to the development of new control theory, the force input to the active suspension systems has been replaced by an input to control the actuator. Therefore, the dynamic of the active suspension systems now consists of the dynamic of suspension system plus the dynamic of the actuator system. Hydraulic actuators are widely used in the vehicle active suspension systems as considered

in [22], [23], [24]. The active suspension system of the half car model is shown in figure 1. Let and be the force inputs for the front and rear actuators, respectively.

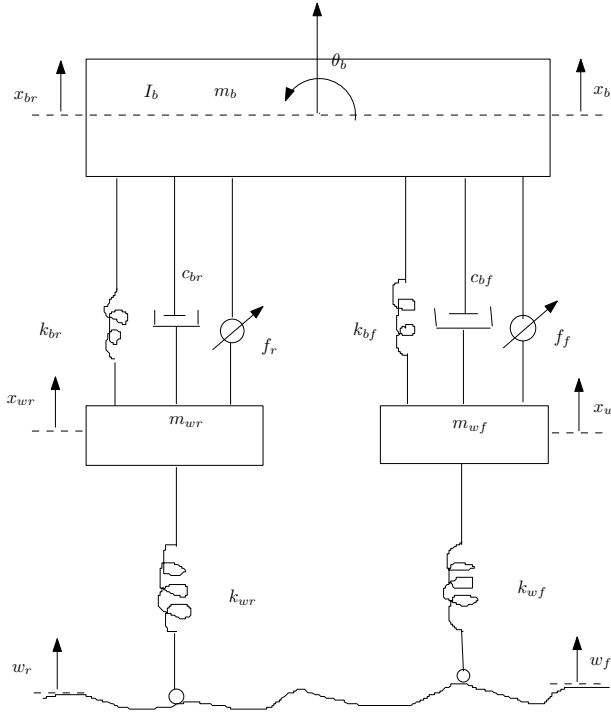


Fig. 1. Active suspension for the half car

Therefore, the motion equations of the active suspension for the half car model may be determined as follows [25]:

$$\frac{m_b}{L}(L_f \ddot{x}_{bf} + L_r \ddot{x}_{br}) + c_{bf}(\dot{x}_{bf} - \dot{x}_{wf}) + k_{bf}(x_{bf} - x_{wf}) + c_{br}(\dot{x}_{br} - \dot{x}_{wr}) + k_{br}(x_{br} - x_{wr}) - f_f - f_r = 0 \quad (1)$$

$$\frac{I_b}{L}(\ddot{x}_{bf} - \ddot{x}_{wf}) + L_f[c_{bf}(\dot{x}_{bf} - \dot{x}_{wf}) + k_{bf}(x_{bf} - x_{wf}) - f_f] - L_r[c_{br}(\dot{x}_{br} - \dot{x}_{wr}) + k_{br}(x_{br} - x_{wr}) - f_r] = 0 \quad (2)$$

$$m_{wf} \ddot{x} - c_{bf}(\dot{x}_{bf} - \dot{x}_{wf}) - k_{bf}(x_{bf} - x_{wf}) + k_{wf}(x_{wf} - w_f) + f_f = 0 \quad (3)$$

$$m_{wr} \ddot{x}_{wr} - c_{br}(\dot{x}_{br} - \dot{x}_{wr}) - k_{br}(x_{br} - x_{wr}) + k_{wr}(x_{wr} - w_r) + f_r = 0 \quad (4)$$

Equation (1)-(4) can be written in the following form:

$$M_h \ddot{X}_h(t) + S_h \dot{X}_h(t) + T_h X_h(t) = D_h F_h(t) + E_h W_h(t) \quad (5)$$

where,

$$X_h(t) = [\dot{x}_{bf}(t) \dot{x}_{wf}(t) x_{br}(t) x_{wr}(t)]^T; \quad (6)$$

$$W_h(t) = [w_f(t) w_r(t)]^T; \quad (7)$$

$$M_h = \begin{bmatrix} \frac{L_r m_b}{L} & 0 & \frac{L_f m_b}{L} & 0 \\ \frac{L_f}{L} & 0 & -\frac{L_r}{L} & 0 \\ 0 & m_{wf} & 0 & 0 \\ 0 & 0 & 0 & m_{wr} \end{bmatrix}; \quad (8)$$

$$S_h = \begin{bmatrix} c_{bf} & -c_{bf} & c_{br} & -c_{br} \\ L_f c_{bf} & -L_f c_{bf} & -L_r c_{br} & L_r c_{br} \\ -c_{bf} & c_{bf} & 0 & 0 \\ 0 & 0 & -c_{br} & c_{br} \end{bmatrix}; \quad (9)$$

$$T_h = \begin{bmatrix} k_{bf} & -k_{bf} & k_{br} & -k_{br} \\ L_f k_{bf} & -L_f k_{bf} & -L_r k_{br} & L_r k_{br} \\ -k_{bf} & k_{bf} + k_{wf} & 0 & 0 \\ 0 & 0 & -k_{br} & k_{br} + k_{wr} \end{bmatrix}; \quad (10)$$

$$E_h = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ k_{wf} & 0 \\ 0 & k_{wr} \end{bmatrix}, \quad (11)$$

$$D_h = \begin{bmatrix} 1 & 1 \\ L_f & -L_r \\ -1 & 0 \\ 0 & -1 \end{bmatrix}, \quad (12)$$

$$F_h = [f_f \ f_r]^T \quad (13)$$

To integrate the actuators dynamics to the half car suspension system, the following mathematical approach is proposed. The derivation starts with rewriting the motion equation of half car active suspension in equation (5) into the following form,

$$\dot{X}_h + M_h^{-1} S_h \dot{X}_h + M_h^{-1} T_h X_h = M_h^{-1} D_h F_h + M_h^{-1} E_h W_h \quad (14)$$

Defining the new state vector:

$$x_h(t) = [\dot{X}_h(t) \ X_h(t) \ f_h]^T \quad (15)$$

$$x_h(t) = \begin{bmatrix} \dot{x}_{bf}(t) \\ \dot{x}_{wf}(t) \\ \dot{x}_{br}(t) \\ x_{bf} \\ x_{wr} \\ x_{br} \\ x_{wr} \\ f_f \\ f_r \end{bmatrix}, \quad (16)$$

Let the rate of change of the control forces for the front and rear hydraulic actuators can be written as:

$$\dot{f}_h = F_{1h} f_h - F_{2h} \dot{x}_h + F_{3h} u \quad (17)$$

where,

$$f_h(t) = [f_f \ f_r(t)]^T; \quad (18)$$

$$x_h(t) = [x_{bf} \ x_{wf} \ x_{br} \ x_{wr}]^T; \quad (19)$$

$$x_h(t) = [u_f \ u_r]^T \quad (20)$$

$$F_{1h} = \begin{bmatrix} -\frac{1}{A_{ef}} & 0 \\ 0 & -\frac{1}{A_{er}} \end{bmatrix}, \quad (21)$$

$$F_{2h} = \begin{bmatrix} -\frac{A_{yf}}{A_{ef}} & \frac{A_{yf}}{A_{ef}} & 0 & 0 \\ 0 & 0 & -\frac{A_{yr}}{A_{er}} & \frac{A_{yr}}{A_{er}} \end{bmatrix}, \quad (22)$$

$$F_{3h} = \begin{bmatrix} \frac{1}{A_{ef}} & 0 \\ 0 & \frac{1}{A_{er}} \end{bmatrix}. \quad (23)$$

Therefore, by augmenting equation (7) and equation (9) the state space representation of the half car active suspension system with the hydraulic dynamics may be obtained as follows:

$$\begin{bmatrix} \ddot{X}_h \\ \dot{X}_h \\ \dot{f}_h \end{bmatrix} = \begin{bmatrix} -M_h^{-1}S_h & -M_h^{-1}S_h & -M_h^{-1}F_h \\ I & 0 & 0 \\ -F_{2h} & 0 & F_{1h} \end{bmatrix} \begin{bmatrix} X_h \\ X_h \\ f_h \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ F_{3h} \end{bmatrix} u + \begin{bmatrix} M_h^{-1}E_h \\ 0 \\ 0 \end{bmatrix} W_h \quad (24)$$

The dynamic equation for the hydraulically actuated active suspension system for the half car model in state space form as follows:

$$\dot{x}_h = A_h x_h + B_h u + f_h \quad (25)$$

where,

$$x_h(t) = \begin{bmatrix} \dot{x}_{bf}(t) \\ \dot{x}_{wf}(t) \\ \dot{x}_{br}(t) \\ \dot{x}_{wr}(t) \\ x_{bf} \\ x_{wr} \\ x_{br} \\ x_{wr} \\ f_f \\ f_r \end{bmatrix}, \quad (26)$$

;

$$B_h = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0.885 & 0 \\ 0 & 0.885 \end{bmatrix} \quad (27)$$

$$f_h = \begin{bmatrix} 0 & 0 \\ 1160 & 0 \\ 0 & 0 \\ 0 & 0.72 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} w_f \\ w_r \end{bmatrix} \quad (28)$$

A_h is a matrix (10 × 10); f_h represent the road disturbance applied to the half car. The road disturbances $\omega(t)$ representing a double bump may be given by the following equation:

$$\omega = a(1 - \cos(8\pi t)) \text{ if } 1.5 \leq t \leq 2.5 \quad (29)$$

$$\omega = b(1 - \cos(8\pi t)) \text{ if } 6.5 \leq t \leq 7.5$$

$$\omega = 0 \text{ otherwise}$$

where a and b denote the bump amplitude.

3. CONTROL STRATEGIES

The aim of this section is to make comparison between STFIS and Zigler-Nicholas PID controller. The next two subsections of this section present the PID controller and the STFIS control system.

3.1 PID controller

PID controller consists of proportional $P(e(t))$, integral $I(e(t))$ and derivative $D(e(t))$ parts. Assuming that each amplitude is completely decoupled and controlled independently from other amplitudes, the control input $u(t)$ is given by:

$$u(t) = K_p e(t) + K_I \int e(t) dt + K_D \frac{de(t)}{dt} \quad (30)$$

In equation, $e(t)$ is the control error:

$$e(t) = x_d(t) - x_a(t) \quad (31)$$

where $x_d(t)$ is the desired response and $x_a(t)$ is the actual response. K_p is called the proportional gain, K_I the integral gain and K_D the derivative gain. Zeigler-Nicholas methods are used to determine the optimum PID gain parameters.

3.2 Structure and proportional integral sliding mode controller(PISMC)

In sliding mode controlled systems, n^{th} order tracking problem is transformed into first order stability problem, which makes the problem easy to cope with. The control action has the state errors progress on stable and unstable trajectories and reach the sliding surfaces. Then, state errors quickly reach the zero value.

The proportional integral sliding surface for a half car suspension model defined as :

$$s = C_h x_h - \int (C_h A_h + C_h B_h K_h)(\tau) d(\tau) \quad (32)$$

where, $B_h \in R^{mn}$ is the input matrix for a half car model, $C_h \in R^{mn}$ and $K_h \in R^{mn}$ are the constant matrices, respectively, m is the number of inputs and n is the number of system states. Thus, it

can be seen that the active suspension system for the half car model has two sliding surfaces.

The sliding mode controller law is proposed as:

$$u(t) = u_{eq}(t) + u_s(t) \quad (33)$$

where, $u_s(t)$ is the switching control, $u_{eq}(t)$ such as $\dot{s}(t) = 0$;

$$u_s(t) = -(C_h B_h)^{-1} \rho \text{sgn}(s(t))$$

$$u_{eq}(t) = (C_h B_h)^{-1} (s(t) - C_h f_h(t) + C_h B_h K_h x_h(t))$$

where, ρ : positive constant, K_h is defined by using the poles placement method:

$$C_h = \begin{bmatrix} 2 & 4 & 3 & 5 & 6 & 8 & 9 & 2 & 1 & 4 \\ 7 & 1 & 3 & 5 & 4 & 2 & 7 & 5 & 4 & 3 \end{bmatrix}$$

Stability analysis

$$s(t) = C_h x_h(t) - \int (C_h A_h + C_h B_h K_h) x(\tau) d(\tau) \quad (34)$$

and

$$\dot{s}(t) = C_h \dot{x}(t) - C_h (A_h + B_h K_h) x(t) \quad (35)$$

u_{eq} such as $\dot{s}(t) = 0$

$$s(t) = C_h x_h(t) - \int (C_h A_h + C_h B_h K_h) x(\tau) d(\tau) \quad (36)$$

$$\begin{aligned} s\dot{s}(t) &= s(t)(C_h \dot{x}(t) - C_h (A_h + B_h K_h) x(t)) \\ &= s(t)(C_h A_h x(t) + C_h B_h (u_{eq}(t) + u_s(t)) \\ &\quad + C_h f_h - C_h (A_h + B_h K_h) x(t)) \\ &= s(t)(C_h A_h x(t) + C_h B_h ((C_h B_h)^{-1} (s(t) \\ &\quad - C_h f_h(t) + C_h B_h K_h x_h(t)) \\ &\quad - (C_h B_h)^{-1} \rho \text{sgn}(s(t)) + C_h f_h - C_h (A_h \\ &\quad + B_h K_h) x(t)) \end{aligned} \quad (37)$$

After simplification we have:

$$s\dot{s} = s(t)(s - \rho \text{sign}(s)) \quad (38)$$

$$= -s^2 \left(\rho \frac{\text{sign}(s)}{s} - 1 \right) \leq 0 \quad (39)$$

Thus, the half-car system is stable.

The following figure shows a control architecture which uses SMC controller.

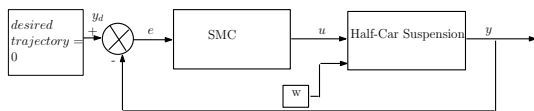


Fig. 2. Sliding mode controller architecture

3.3 Conventional sliding mode controller(SMC)

The state space form of a non-linear dynamic system can be written as,

$$\dot{x}_h = A_h x_h + B_h u + f_h \quad (40)$$

For a control system, the sliding surface can be selected as,

$$s(t) = C e(t) \quad (41)$$

Here $e(t)$ is the difference between the reference value and system response. C includes the sliding surface slopes and has positive elements. For stability, the following Lyapunov function candidate, which is proposed for a non-chattering action, has to be positive definite and its derivative has to be negative semi-definite.

$$V(s) = \frac{s^T s}{2} > 0 \quad (42)$$

$$\frac{dV(s)}{dt} = \dot{s}^T s \leq 0 \quad (43)$$

If the limit condition is applied to equation (44), and from equation (41) and equation (42) the controller force for the limit case is obtained:

$$u_{eq} = (C B_h)^{-1} (-C A_h x_h - C f_h) \quad (44)$$

Equivalent control is valid only on the sliding surface. So an additional term should be defined to pull the system to the surface. For this purpose the switching control $u_s(t)$ is selected as follows:

$$u_s(t) = (C B_h)^{-1} \rho \text{sgn}(s(t)) \quad (45)$$

Therefore, the proposed conventional sliding mode is given as follows:

$$\begin{aligned} u(t) &= (C B_h)^{-1} (-C A_h x_h - C f_h) \\ &\quad - (C B_h)^{-1} \rho \text{sgn}(s(t)). \end{aligned} \quad (46)$$

4. SIMULATIONS AND RESULTS

Simulation for controller of active half car suspension model is done by using MATLAB simulink. Three type of controllers are applied, they are PID controller which is tuned by Ziegler-Nicholas PISM and conventional SMC. We have considered a total mass of the body equal to $m_b = 1794.4 \text{ Kg}$. Fig.3 indicates that the suspension deflection controlled by SMC smaller than that of passive and PID and it achieved the desired trajectory very quickly than the PISM. From fig.4 and 6, it is observed that the amplitude of the wheel velocity (front and rear) for an active suspension based on PISM and SMC shows considerable improvement compared to the passive travels. Fig.5 and 8 illustrates how effectively the active suspension with PISM and SMC absorbs the vehicle vibration compared to the passive system. Fig.8 shows that the curve using SMC achieve the desired trajectory before the PISM and its amplitude is very small than the PID controller. Finally, fig.9 shows the conventional sliding surfaces (s_1 ; s_2) applied to the half car vehicle. Thus the active suspension with SMC scheme could greatly contribute to the improvement of the vehicle ride comfort and marginally contribute to the road holding ability and also it is very quickly to achieve the desired trajectory than the PISM and the PID controller.

5. CONCLUSIONS

A comparison between the PISM, SMC and the PID controller shows the validity of the proposed and the technique of the SMC. Thus, simulation results demonstrate the effectiveness of the proposed controller. SMC based active suspension provides higher

ride comfort and road handling qualities when compared to existing passive and other controllers such as PISMC and PID controller. Future works will essentially investigate the possibility to control the vehicle by the renewable energy.

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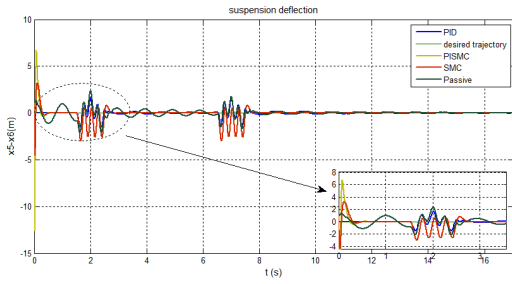


Fig. 3. The suspension deflection for the half car

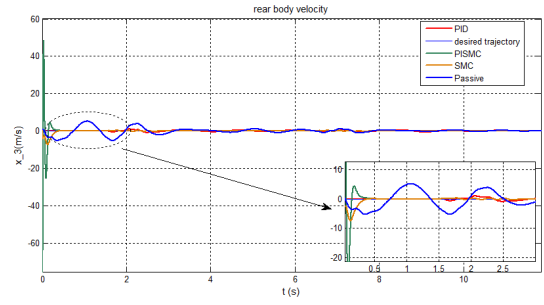


Fig. 6. Rear body velocity for the half car

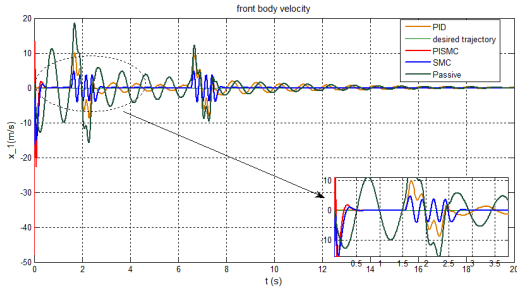


Fig. 4. Front body velocity for the half car

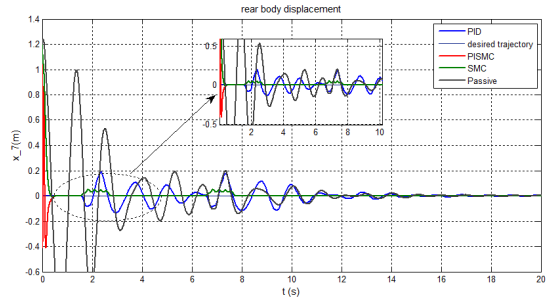


Fig. 7. Rear body displacement for the half car

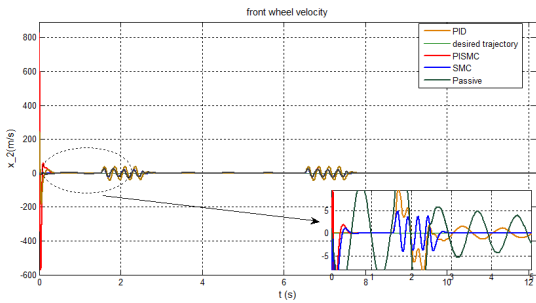


Fig. 5. Front wheel velocity for the half car

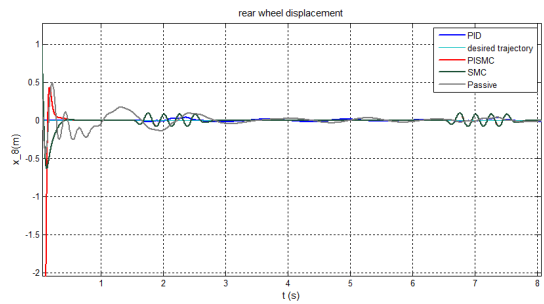


Fig. 8. Rear wheel displacement for the half car



Fig. 9. Conventional sliding surfaces