International Journal of Computer Applications (0975 – 8887)
Volume 155 – No 3, December 2016

Statistical Image Compression using Fast Fourier Coefficients

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ABSTRACT
Uncompressed images occupy more memory and it contains redundant data. For storage and transmission efficiency compression is required. The purpose of image compression is to reduce the number of bits to represent in image while maintaining visual quality of images. In this paper we implement the Fast Fourier co efficient Transform with non overlapping 3 x 3, 9 x 9 and 27x 27 block sizes of the images. The purpose of the study is to reduce the original image into asmall set of pixels by using the Fourier coefficients with this idea the compression is successfully implemented on various images, computed the Peak Signal Noise Ratio (PSNR) and

Compression Ratio (CR) for the various images.

General Terms
1. Read the image into the MATLAB environment and call it the original image. 2. Subdivide the original image into 3 x 3, 9 x 9, and 27 x 27 blocks called as Mega Pixel.3. Compute the 2D Discrete Fourier Transform (2D DFT) using FFT algorithm, for each mega pixel. It returns Fourier coefficients of the same size. 4. Obtain the Fourier spectrum using the function 'abs', computes the magnitudes of each element. 5. Using average of these magnitudes as threshold, remove those magnitudes lower than the threshold value. 6. For these magnitudes, which are higher than threshold value, obtain the Fourier Coefficients and replace the remaining coefficients with zero since a significant number of coefficients have small magnitudes in most images and can be discarded entirely with little image distortion. 7. Now each mega pixel containing only a few Fourier coefficients and remaining values are zero's. 8. Obtain the estimated image matrix by taking the inverse Fourier transform.

The process discussed above is illustrated step by step is as under for a 3 x 3 matrix in section 2.4

Keywords
Fast Fourier Transform (FFT), Root Mean Square Error (RMSE), Compression Ratio (CR), Peak Signal Noise Ratio (PSNR), Image.

1. INTRODUCTION
The term compression refers to the process of reducing the amount of data required to represent a given quantity of information. A most common characteristic of image is that the neighboring pixels are correlated and therefore contain redundant information. Now, the task is to find less correlated representation of the image.

Image compression addresses the problem of reducing the amount of data required to represent a digital image. The basis of the reduction process is the removal of redundant data. The process of reduction is transforming a 2D pixel array into a statistically uncorrelated dataset. The transformation is applied prior to storage or transmission of the image. The compression image is then decompressed to reconstruct the original image. Image compression plays an important role in "digital imaging technology".

The advancement of imaging technology has been producing superior quality image that require large storage space and higher band width for data transformation. By applying digital image compression techniques, the number of bits required to represent an image can be minimized primarily for achieving information transmission and storage efficiency [Ghafourian et.al (1995), Rafael c Gonzalez, Richard E Woods (2008), Chanda, Majumder Dutta B (2004), Padmaja.N (2012)]

Image compression is the natural technology for handling the increased spatial resolutionof today's imaging sensors and evolving broadcast television standards. And it play a major role in many important and diverse applications like televise conferencing , remote sensing, document and medical imaging, facsimile transmission (FAX), the control of remotely piloted vehicles in military, space and hazardous waste control applications. In short an ever expanding number of applications depend on the efficient manipulation, Storage and transmission of binary, grey scale or color images.

2. IMAGE COMPRESSION TECHNIQUES
Image compression techniques can be classified into two categories: Lossless and Lossy Schemes. In Lossless scheme, the exact original data can be recovered, while in Lossy Scheme, only a close approximation of the original data can be obtained.

$$\text{CompressionRatio(C.R)} = \frac{n_2}{n_1} = \frac{\text{number of bits of compressed image}}{\text{number of bits of original image}}$$

n_1 = number of bits of original image ,

n_2 = number of bits of compressed image

$$\text{RMSE} = \left[\frac{1}{M \times N} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left[g(x,y) - f(x,y) \right]^2 \right]^{1/2}$$

Where f(x,y), x=0,1,2, …M-1, y=0,1,2,…N-1 is the original image and g(x,y), x=0,1,2, …M-1, y=0,1,2,…N-1 is the reconstructed image of the original.

Peak Signal Noise Ratio (PSNR) refers the ratio between the maximum possible power ofa signal and the power of corrupting noise that affects the fidelity of its representation and is usually expressed in terms of the logarithmic decibel scale. The PSNR is most commonly used as a measure of quality of reconstructed image and is defined through mean square error.

$$\text{PSNR} = 20 \log_{10}\left(\frac{255}{\text{RMSE}}\right)$$

2.1 Block Based image compression using Fourier coefficients

Image compression techniques are generally classified into three categories, namely pixel coding, predictive coding, and transform coding. The Fourier transform is an important image processing tool based on transform coding. The output of the transformation represents the image in the Fourier or frequency domain, while the output image is the spatial domain equivalent. In the Fourier frequency domain image, each point represents a particular frequency contained in the spatial domain image. The Fourier transform is used in a wide range of applications, such as image analysis, image filtering, image reconstruction, and image compression.

In practice, images always have finite size and for an image array of size m x n, the Discrete Fourier Transform (DFT) decomposes the array of values into components of different frequencies. This operation is useful in many fields. A Fast Fourier Transform (FFT) is an efficient algorithm to compute the DFT and its inverse using two dimensional FFT, with few of the Fourier coefficients by considering different thresholds, image can be reconstructed and compressed.

The Fourier transform is a representation of an image as a sum of complex exponentials of varying magnitudes, frequencies. The Fourier transform plays a major role in image processing, including enhancement, restoration, and compression. Transform coding plays a major role in image compression. It uses a unitary matrix with a fast algorithm that represents an excellent compromise in terms of computational complexity versus coding performance. Thus, transforming coding outperforms more sophisticated schemes by a margin for given cost [Donoho .D .L. et al.(1998)].

A typical image’s intensity often varies significantly throughout the image. It makes the image compression in the spatial domain difficult. However, images tend to have a compact representation in the frequency domain packed around the low frequencies, which makes compression in the frequency domain more efficient and effective. In transform coding, first an image is transformed from spatial domain to the frequency domain, and then frequency domain coefficients are quantized and encoded to achieve the compression [Shapiro .J. (1993)]. The transform should be de- correlated to reduce redundancy and to have a maximum amount of information stored in the smaller number of coefficients. These coefficients are then coded as accurately as possible to retain the information.

2.2 Discrete Fourier Transform

Let f (x,y), for x = 0,1,2….M-1 and y = 0,1,2,…N-1, denote an M*N image .The 2D discrete Fourier transform (2D DFT) of an image f, denoted by F(u,v) is given by,

$$F(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

For u=0, 1, 2..,.M-1 and v =0,1,2…N-1

The exponential term can be divided into sines and cosines with the variables u and v. TheFrequency domain is the coordinate system spanned by F (u, v) with the u and v as Frequency variables where as the spatial domain is the coordinate system spanned by f(x, y) with x and y as spatial variables

$$f(x,y) = \frac{1}{MN}\sum\sum F(u,v)\, e^{j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

The values of F(u,v) in the above equations are called Fourier coefficients. for u=0, v=0, we have

$F(0,0) = \sum_{x}\sum_{y} f(x,y)$ is the total gray scale level of the image also called the discrete cosine component of the Fourier transform also represent zero frequency.

2.3 Fast Fourier Transform:

Let FFT is defined as:

$$FFT_N(k,f) = \sum_{n=0}^{N-1} f(n)\, e^{-\frac{j2\pi kn}{N}}$$

The process of dividing the frequency components into even and odd parts is what gives this algorithm and its name is “Decimation in frequency”. If N is a regular power of 2, we can apply this method recursively until we get to the trivial 1 point transform. By using FFTs we discuss in the next section the compression on various block sizes are explored.

2.4 Procedure for block based image compression using Fourier coefficients

Transform compression is based on a simple premise: when the signal passed through theFourier transform, the resulting data values will no longer be equal in their information carrying roles. In particular, the low frequency components of a signal are more important than the high frequency components. Image construction and compression have been carried out using two dimensional FFT by considering only a few of the Fourier coefficients. An illustration follows

Table 1.Digital data of the original image is given below

0.841	0.9134	0.2785
0.9058	0.6324	0.5469
0.127	0.975	0.9575

For the above digital data of the image we compute the Fourier coefficients.

Table 2. Fourier coefficients

5.2737+0.000i	0.1344-0.1208i	0.1344+0.1208i
0.3730-0.7821i	0.4482+1.5058i	0.9700-0.2645i
0.3730+0.7821i	0.9700-0.2645i	0.4482+1.5058i

For the above Fourier coefficients we compute the magnitudes and they are:

Table 3.Magnitudes

5.2737	0.1808	0.1808
0.8665	1.5711	1.0055
0.8665	1.0055	1.5711

For these magnitudes we find the magnitudes for more than mean are:

Table4. Magnitudes more than the threshold (Mean=1.3923)

5.2737	0	0
0	1.5711	0
0	0	1.5711

Now we compute the Fourier coefficients, of these more than mean values and they are:

Table 5. Fourier Coefficients more than threshold value

5.2737+0.000i	0.0000+0.0000i	0.0000+0.0000i
0.0000+0.0000i	0.4482-1.5058i	0.0000+0.0000i
0.0000+0.0000i	0.0000+0.0000i	-0.4482-1.5058i

With these coefficients we reconstruct the image and the root mean square error: 0.2105, PSNR: 61.6658

0.4864	0.9256	0.346
0.9256	0.346	0.4864
0.346	0.4864	0.9256

This process is experimented on the following 9 images and the results are tabulated for the 5 images are listed here

Experimental Results:

This procedure is on 9 randomly chosen gray scale images. These are of different sizes and varying image density and having different noise levels and these are:

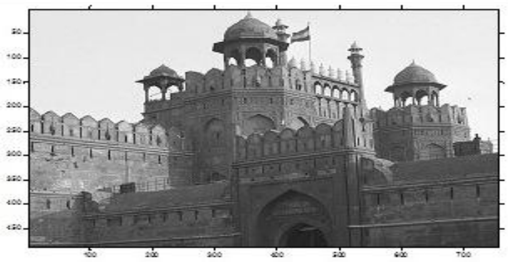
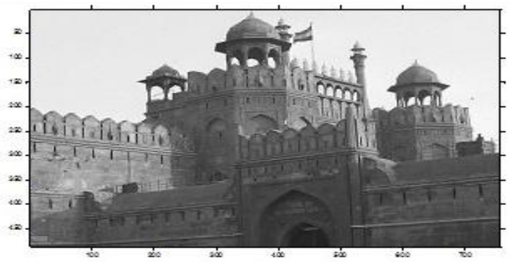
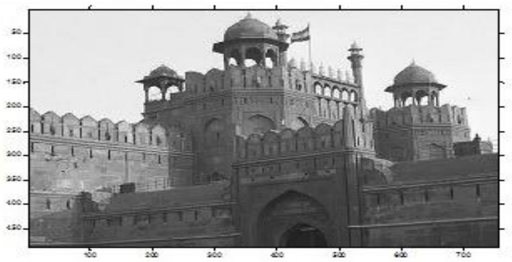
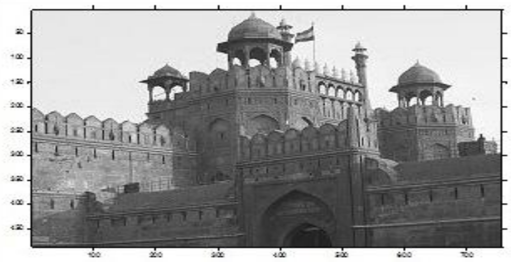
1) RED FORT 2) LOGO 3) ISLAND 4) BIRD

5) FLOWER 6) BABY 7) TREE 8) YEN NOTE 9) TIGER

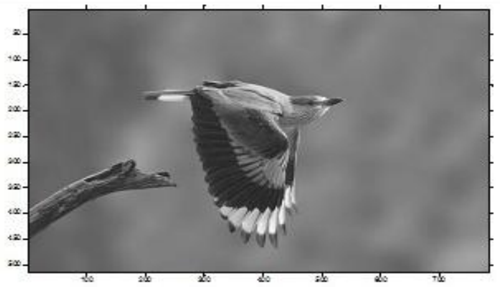
Table 5

S.NO	IMAGE NAME	IMAGE SIZE	PIXELS USED		COMPRESSION RATIO	RMSE	PSNR
1.	REDFORT	367416	46998	3 x 3	12.7915	12.3204	26.3217
			33066	9 x 9	8.9996	10.2736	27.8964
			60116	27 x 27	16.3618	8.1887	29.8666
2.	LOGO	210681	29009	3 x 3	13.7692	23.9779	23.9779
			20073	9 x 9	9.5277	16.1905	23.9456
			30885	27 x 27	14.6596	14.0998	25.1465
3.	ISLAND	65356	31146	3 x 3	11.7374	12.1843	26.4148
			21570	9 x 9	8.1257	9.4844	28.5906
			36456	27 x 27	13.7385	7.5066	30.6219
4.	BIRD	401679	45285	3 x 3	11.2739	4.6819	34.7224
			12735	9 x 9	3.1704	3.7109	36.7411
			21545	27 x 27	5.3637	2.8203	39.1249
5.	FLOWER	553311	71943	3 x 3	13.0023	11.0115	27.2939
			72323	9 x 9	13.0709	9.2736	28.7858
			88929	27 x 27	16.0722	7.4888	30.5856
6.	BABY	309825	35375	3 x 3	11.4177	7.1449	31.0509
			16229	9 x 9	5.2381	6.5901	31.7530
			28123	27 x 27	9.0771	4.9539	34.0584
7.	TREE	7002045		3 x 3	11.1383	5.4650	33.3790
			382789	9 x 9	5.4668	5.1417	33.9087
			746657	27 x 27	10.6634	3.5264	37.1847
8.	YEN NOTE	338256	49908	3 x 3	14.7545	23.2611	20.7982
			55128	9 x 9	16.2977	18.4640	22.8043
			91740	27 x 27	27.1215	15.2331	24.4750
9.	TIGER	2277396	262634	3 x 3	11.5322	6.7512	31.5432
			188136	9 x 9	8.2610	5.6465	33.0953
			269660	27 x 27	11.8407	4.0715	35.9358

Note: The Images Tree & Tiger are too big to fit on screen of size 3 x 3.

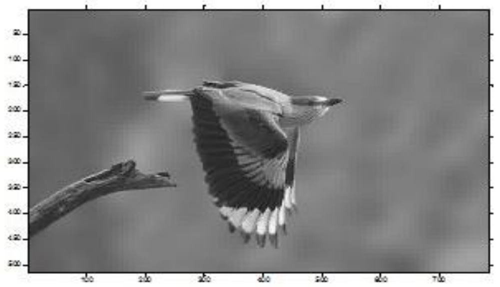
	
Original Image	**Reconstructed Image 3 x 3** **CR (12.7915) PSNR (26.3217)**
	
Reconstructed Image 9 x 9 **CR (8.9996) PSNR (27.8964)**	**Reconstructed Image 27 x 27** **CR (16.3618) PSNR (29.8666)**

	
Original Image	**Reconstructed Image 3 x 3** **CR(11.4177) PSNR (31.0509)**
	
Reconstructed Image 9 x 9 **CR(5.2381) PSNR(31.7530)**	**Reconstructed Image 27 x 27** **CR(9.0771) PSNR(34.0584)**

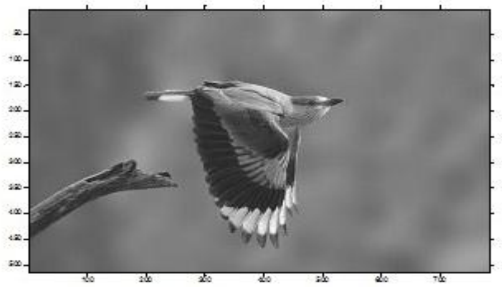


Original Image

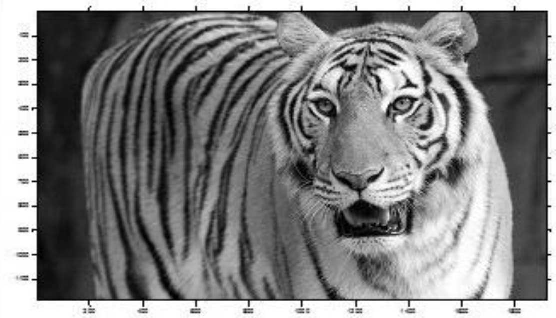
Reconstructed Image 3 x 3
CR(11.2739) PSNR(34.7224)



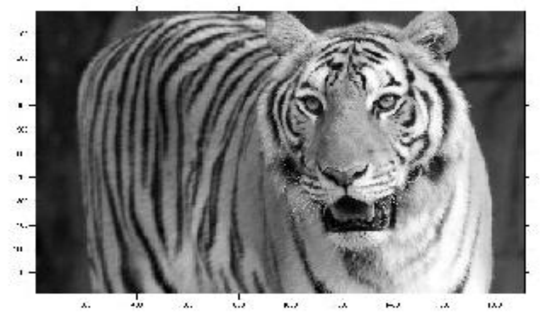
Reconstructed Image 9 x 9
CR(3.1704) PSNR(36.7411)



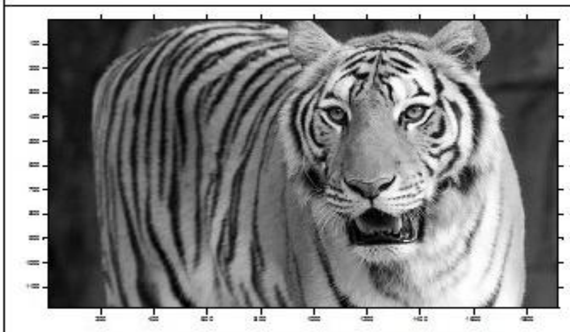
Reconstructed Image 27 x 27
CR(5.3637) PSNR(39.1249)



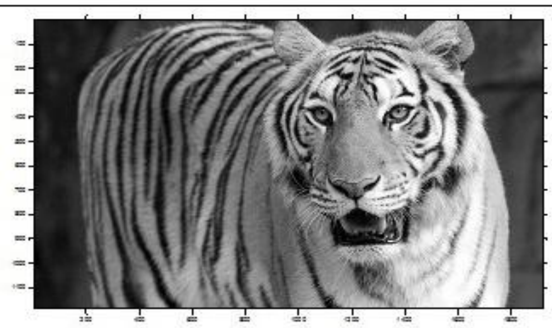
Original Image



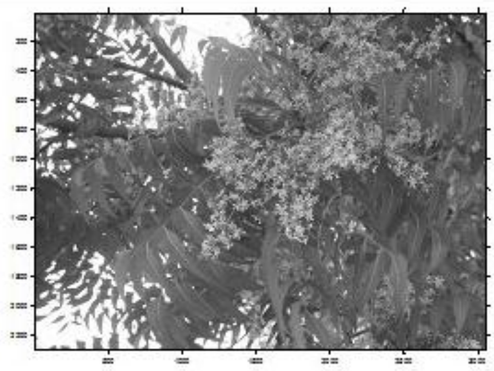
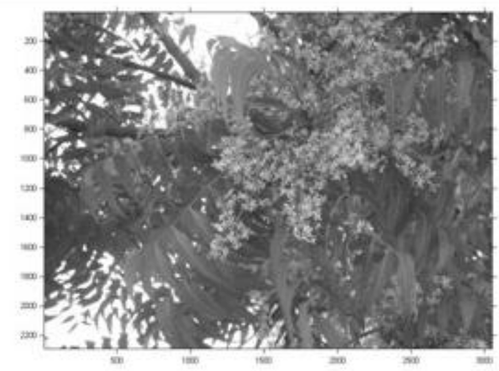
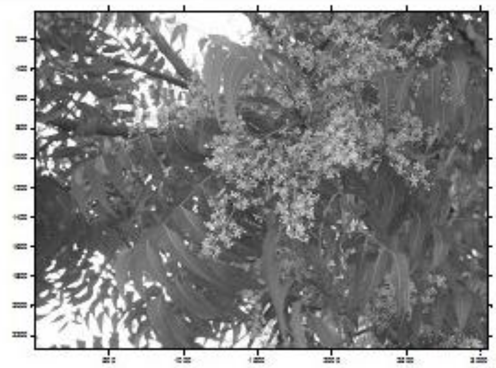
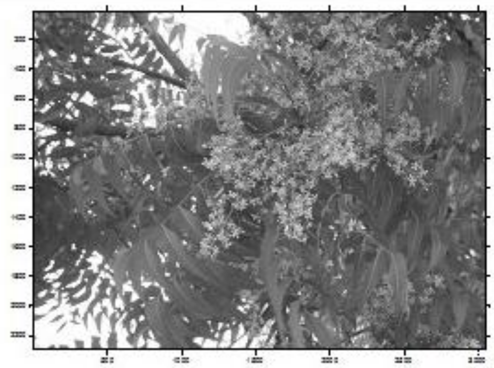
Reconstructed Image 3 x 3
CR (11.5322) PSNR (31.5432)



Reconstructed Image 9 x 9
CR(8.2610) PSNR(33.0953)



Reconstructed Image 27 x 27
CR(11.8407) PSNR(35.9358)

	
Original Image	**Reconstructed Image 3 x 3 CR(11.1383) PSNR(33.3790)**
	
Reconstructed Image 9 x 9 CR(5.4668) PSNR(33.9087)	**Reconstructed Image 27 x 27 CR(10.6634) PSNR(37.1847)**

3. CONCLUSIONS

From the above table 5, it is seen that a good compression is achieved for all sizes of images. However the compression is the best when it is sub divided into 9 * 9 blocks. The compression ratio and PSNR of the image for instance"REDFORT"for a 3*3 block size are 12.7915 and 26.3217 respectively. While the compression ratio and PSNR for a block size 9 * 9 of the same image (REDFORT) are 8.9996 and 27.8964 respectively. This indicates that when the block size is increased from 3 x 3 to 9x9, higher compression and also better quality of the image is achieved. While for block size 27 x 27 for the "REDFORT" images are the CR & PSNR 16.3618 and 29.8666 thus we can comment that as block size increases from 9 x 9 to 27 x 27, further quality is achieved but lower compression as compared to 9 x 9 block size also the percentage of increase in the quality of the image (REDFORT), the PSNR for the 9 x 9 block and for the same image 27 x 27 blocks size is approximately 7% while the percentage of increase in compression ratio is 82% and thus we conclude that a good compression is achieved for a 9 x 9 blocks size and it is seen that the reproduced image is also visually clear. Similar compression pattern is observed across all the images of different sizes considered with this compression approach..

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