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TOPSIS for Multi Criteria Decision Making in Intuitionistic Fuzzy Environment

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ABSTRACT
This paper is based on intuitionistic fuzzy sets, we introduce an extension of fuzzy TOPSIS for multi criteria decision making problem in intuitionistic fuzzy environment. Intuitionistic fuzzy sets are more suitable to deal with uncertainty than other generalized forms of fuzzy sets. The rating of each alternative and the weight of each criterion are expressed in intuitionistic fuzzy number. The normalized intuitionistic fuzzy number is calculated by using the concept of $\alpha - cuts$. Ranking function is used for determining the positive ideal solution and negative ideal solution. For application and verification, a numerical example is discussed at the end of this paper and compare with existing method.

Keywords
Intuitionistic fuzzy number, ranking of intuitionistic fuzzy number, positive ideal solution, negative ideal solution, multicriteria decision making

1. INTRODUCTION
Multi Criteria Decision Making (MCDM) is concerned with structuring and solving decision and planning problems involving multiple criteria. The purpose is to support decision makers facing such problems. Typically, there does not exist an unique optimal solution for such problems and it is necessary to use decision maker's performance to differentiate between solutions. MCDM has been an active area of research since the 1970's. Different approaches have been proposed by many researchers, including the Analytic Hierarchy Process (AHP), Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) and MCDM.

The TOPSIS (Technique for Order of Preference by Similarity to ideal solution) is a multi-criteria decision analysis method developed by Hwang and Yoon (1981) with further developments by Yoon (1987) and Hwang, Lai & Liu in 1993. TOPSIS is based on the concept that the chosen alternative should have the shortest distance from the positive ideal solution (PIS) and the farthest distance from the negative ideal solution (NIS).

Atanassov and Gargov (1989) also proposed the concept of Atanassov's interval-valued intuitionistic fuzzy sets as a further generalization of fuzzy set theory. Boran et al.[4] introduced a TOPSIS method with intuitionistic number in supplier selection problem for a multi-criteria group decision making (MCGDM) problem. Izadikhan [12] extended the concept of TOPSIS to develop group decision making process for supplier selection under interval-valued intuitionistic fuzzy environment. Recently many researchers [6,19] extended the concept of TOPSIS to develop methodology for multi-criteria decision making (MCDM) and multi attribute decision making (MADM) for interval valued intuitionistic fuzzy environment.

In real-world situation, because of incomplete or non-obtainable information, the data (attributes) are often not so deterministic, and therefore the usually are fuzzy/imprecise. Therefore, some researches try to use TOPSIS method for fuzzy/imprecise data. For example, Tsaur et al. [18] first convert a fuzzy MCDM problem into a crisp problem via centroid defuzzification and then solve the non fuzzy MCDM problem using the TOPSIS method. Chu [9] proposed a fuzzy TOPSIS approach for selecting plant location. Chen and Tzeng [8] transform a fuzzy MCDM problem into a nonfuzzy MCDM using fuzzy integral. Byun and Lee [5] provide a decision support system for the selection of a rapid prototyping process using the modified TOPSIS method. Recently, in some researches, TOPSIS method is considered for extension. For example, Chen [7] extends the concept of TOPSIS to develop a methodology for solving multi person multi criteria decision making problems in fuzzy environment. Mahdavi et al.[16] designed a model of TOPSIS for the fuzzy environment with the introduction of appropriate negations for obtaining ideal solutions. Abo-Sinha et al. [1] extend the TOPSIS method to solve multi objective nonlinear programming problems. Also, Jahanshahloo et al. [14,15] and Izadikhan [13] extended the TOPSIS method for decision making problems with interval and fuzzy data.

In this paper we propose the multicriteria TOPSIS method in the intuitionistic fuzzy environment by using ranking method given by [17] and distance measure. The rest of the paper organized as follows:

In section 2 we firstly introduced some basic definitions of the intuitionistic fuzzy set, intuitionistic fuzzy number, distance measure and ranking method given by [17]. In section 3 the different steps in the proposed intuitionistic fuzzy TOPSIS method are presented. A numerical example of the proposed model is presented in section 4. The paper is concluded in section 5.

2. PRELIMINARIES
In this section some basic definitions of intuitionistic fuzzy set, intuitionistic fuzzy number, α-cut trapezoidal intuitionistic fuzzy number, distance measure between intuitionistic fuzzy number are presented and ranking method given by [17] is also presented in this section.

Definition 1. Intuitionistic fuzzy set (IFS).

An IFS $\tilde{A}$ on a universe X is defined is an object of the following form:

$$\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)\rangle : \forall x \in X\} \quad (1)$$

Where the functions $\mu_{\tilde{A}}(x): \to [0,1]$ and $\nu_{\tilde{A}}(x): \to [0,1]$ represent the degree of membership and the degree of non-membership of an element $x \in \tilde{A} \subset X$ respectively.

Definition 2. Intuitionistic fuzzy Number(IFN).

An IFS $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)\rangle : \forall x \in X\}$ of the real line is called an IFN if

(a) $\tilde{A}$ is IF-normal,
(b) $\tilde{A}$ is IF-convex
(c) $\mu_{\tilde{A}}$ is upper semi continuous and is lower semi continuous.
(d) $\tilde{A} = \{x \in X | \nu_{\tilde{A}}(x) < 1\}$ is bounded.

Definition 3. $(\alpha - Cut)$.

The α-cuts of a IFN are a non fuzzy sets defined as

$$(\tilde{A}^{+})_{\alpha} = \{x \in R | \mu_{\tilde{A}}(x) \geq \alpha\} = \tilde{A}_{\alpha} \quad (2)$$

$$(\tilde{A}^{-})_{\alpha} = \{x \in R | 1 - \nu_{\tilde{A}}(x) \geq \alpha\} = \tilde{A}_{1-\alpha} \quad (3)$$

If the sides of the fuzzy numbers are strictly monotone then the convention that

$f_{\tilde{A}}^{-1}(\alpha) = \tilde{A}_L^{+}(\alpha)$, $g_{\tilde{A}}^{-1}(\alpha) = \tilde{A}_U^{+}(\alpha)$, $h_{\tilde{A}}^{-1}(\alpha) = \tilde{A}_L^{-}(\alpha)$, $k_{\tilde{A}}^{-1}(\alpha) = \tilde{A}_U^{-}(\alpha)$

In particular if the decreasing functions $g_{\tilde{A}}$ and $h_{\tilde{A}}$ and increasing functions $f_{\tilde{A}}$ and $k_{\tilde{A}}$ be linear then we will have the TIF numbers.

Definition 4. (Trapezoidal Intuitionistic Fuzzy Number).

Let $\tilde{A}$ be an TIFN Intuitionistic fuzzy number with parameters $b_1 \leq a_1 \leq b_2 \leq a_2 \leq a_3 \leq b_3 \leq a_4 \leq b_4$ and denoted as $\tilde{A} = \langle a_1, a_2, a_3, a_4; b_1, b_2, b_3, b_4\rangle$ on a real number set R, then its membership and non-membership are defined as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a_1 \\ \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ 1, & a_2 \leq x \leq a_3 \\ \frac{a_1-x}{a_4-a_3}, & a_3 \leq x \leq a_4 \\ 0, & x > a_4 \end{cases} \quad (4)$$

$$\nu_{\tilde{A}}(x) = \begin{cases} 0, & x < b_1 \\ \frac{b_2-x}{b_2-b_1}, & b_1 \leq x \leq b_2 \\ 1, & b_2 \leq x \leq b_3 \\ \frac{x-b_3}{b_4-b_3}, & b_3 \leq x \leq b_4 \\ 0, & x > b_4 \end{cases} \quad (5)$$

If in a TIFN $\tilde{A}$, we let $b_2 = b_3$(and hence $a_2 = a_3$) then we will give a Triangular Intuitionistic Fuzzy Number (TrIFN) with parameters $b_1 \leq a_1 \leq b_2 \leq (a_2 = a_3 = b_3) \leq a_4 \leq b_4$ and denoted by $\langle a_1, a_2, a_3; b_1, b_2, b_3\rangle$.

Definition 5.

Let $\tilde{A}_1 = \langle a_{11}, a_{12}, a_{13}, a_{14}; b_{11}, b_{12}, b_{13}, b_{14}\rangle$ and $\tilde{A}_2 = \langle a_{21}, a_{22}, a_{23}, a_{24}; b_{21}, b_{22}, b_{23}, b_{24}\rangle$ be two trapezoidal Intuitionistic fuzzy numbers then we have

$$\tilde{A}_1 \oplus \tilde{A}_2 = \langle a_{11} + a_{21}, a_{12} + a_{22}, a_{13} + a_{23}, a_{14} + a_{24}; b_{11} + b_{21}, b_{12} + b_{22}, b_{13} + b_{23}, b_{14} + b_{24}\rangle,$$

$$\lambda\tilde{A}_1 = \langle \lambda a_{11}, \lambda a_{12}, \lambda a_{13}, \lambda a_{14}; \lambda b_{11}, \lambda b_{12}, \lambda b_{13}, \lambda b_{14}\rangle, \quad \lambda > 0$$

$$\lambda\tilde{A}_1 = \langle \lambda a_{14}, \lambda a_{13}, \lambda a_{12}, \lambda a_{11}; \lambda b_{14}, \lambda b_{13}, \lambda b_{12}, \lambda b_{11}\rangle, \quad \lambda < 0$$

$$\tilde{A}_1 \otimes \tilde{A}_2 = \langle a_{11}a_{21}, a_{12}a_{22}, a_{13}a_{23}, a_{14}a_{24}; b_{11}b_{21}, b_{12}b_{22}, b_{13}b_{23}, b_{14}b_{24}\rangle \quad (6)$$

Definition 6. Let $\tilde{A}$ and $\tilde{B}$ be two trapezoidal Intuitionistic fuzzy numbers with α-cuts representations, then the distance between $\tilde{A}$ and $\tilde{B}$ is defined as follows:

$$d(\tilde{A}, \tilde{B}) = \sqrt{\frac{1}{4}\int_0^1 \left[\left(\tilde{A}_L^{+}(\alpha) - \tilde{B}_L^{+}(\alpha)\right)^2 + \left(\tilde{A}_U^{+}(\alpha) - \tilde{B}_U^{+}(\alpha)\right)^2 + (\tilde{A}_L^{-}(\alpha) - \tilde{B}_L^{-}(\alpha))^2 + (\tilde{A}_L^{+}(\alpha) - \tilde{B}_L^{+}(\alpha))^2\right] d\alpha} \quad (7)$$

Definition 7.

Let $\tilde{A}_1$ and $\tilde{A}_2$ be two trapezoidal Intuitionistic fuzzy number, $\tilde{A}_1 = \langle a_{11}, a_{12}, a_{13}, a_{14}; b_{11}, b_{12}, b_{13}, b_{14}\rangle$ and $\tilde{A}_2 = \langle a_{21}, a_{22}, a_{23}, a_{24}; b_{21}, b_{22}, b_{23}, b_{24}\rangle$. Thus, the distance between $\tilde{A}_1$ and $\tilde{A}_2$ is obtained by

$$d(\tilde{A}_1, \tilde{A}_2) = \sqrt{\frac{1}{12}\left\{\begin{array}{l} \sum_{i=1}^{4}(a_{2i} - a_{1i})^2 + \sum_{i=1}^{4}(b_{2i} - b_{1i})^2 + \\ \sum_{i\in\{1,3\}}(a_{2i} - a_{1i})(a_{2i'} - a_{1i'}) + \\ \sum_{i\in\{1,3\}}(b_{2i} - b_{1i})(b_{2i'} - b_{1i'}) \end{array}\right\}} \quad (8)$$

Where $i' = i + 1$ for all $i \in \{1,3\}$.

Definition 8: Let $\tilde{A}_1$ and $\tilde{A}_2$ be two triangular intuitionistic fuzzy number, $\tilde{A}_1 = \langle a_{11}, a_{12}, a_{13}; b_{11}, b_{12}, b_{13}\rangle$ and $\tilde{A}_2 = \langle a_{21}, a_{22}, a_{23}; b_{21}, b_{22}, b_{23}\rangle$. Thus, the distance between $\tilde{A}_1$ and $\tilde{A}_2$ is obtained by

$$d(\tilde{A}_1, \tilde{A}_2) = \sqrt{\frac{1}{12}\left\{\begin{array}{l} \sum_{i=1}^{3}(a_{2i} - a_{1i})^2 + \sum_{i=1}^{3}(b_{2i} - b_{1i})^2 + \\ \sum_{i\in\{1,2\}}(a_{2i} - a_{1i})(a_{2i'} - a_{1i'}) + \\ \sum_{i\in\{1,2\}}(b_{2i} - b_{1i})(b_{2i'} - b_{1i'}) + \\ (a_{22} - a_{12})^2 + (b_{22} - b_{12})^2 \end{array}\right\}} \quad (9)$$

Where $i' = i + 1$ for all $i \in \{1,2\}$.

Definition 9. Let $\tilde{A}_1$ and $\tilde{A}_2$ be two triangular intuitionistic fuzzy numbers. The fuzzy number $\tilde{A}_1$ is closer to intuitionistic fuzzy number $\tilde{A}_2$ as $d(\tilde{A}_1, \tilde{A}_2)$ approaches to 0.

Property 1. If $\tilde{A}_1$ and $\tilde{A}_2$ are real numbers, then the distance measurement $d(\tilde{A}_1, \tilde{A}_2)$ is identical to the Euclidean distance.

Proof. Let $\tilde{A}_1 = \langle a_{11}, a_{12}, a_{13}; b_{11}, b_{12}, b_{13}\rangle$ and $\tilde{A}_2 = \langle a_{21}, a_{22}, a_{23}; b_{21}, b_{22}, b_{23}\rangle$ are two real numbers, then let $a_{11} = a_{12} = a_{13} = b_{11} = b_{12} = b_{13} = A_1$ and $a_{21} = a_{22} = a_{23} = b_{21} = b_{22} = b_{23} = A_2$. Using equation (9) distance measurement $d(\tilde{A}_1, \tilde{A}_2)$ can be calculated as

$$d(\tilde{A}_1, \tilde{A}_2) = \sqrt{\frac{1}{12}\left\{\begin{array}{l} 3(a_{21} - a_{11})^2 + 3(b_{21} - b_{11})^2 + \\ 2(a_{21} - a_{11})(a_{22} - a_{12}) + \\ 2(b_{21} - b_{11})(b_{22} - b_{12}) + \\ (a_{22} - a_{12})^2 + (b_{22} - b_{12})^2 \end{array}\right\}}$$

$$= \sqrt{\frac{1}{12}\begin{Bmatrix} 3(A_2-A_1)^2+3(A_2-A_1)^2+ \\ 2(A_2-A_1)(A_2-A_1)+ \\ 2(A_2-A_1)(A_2-A_1)+ \\ (A_2-A_1)^2+(A_2-A_1)^2 \end{Bmatrix}}$$

$$= \sqrt{\frac{1}{12}\begin{Bmatrix} 3(A_2-A_1)^2+3(A_2-A_1)^2+ \\ 2(A_2-A_1)^2+2(A_2-A_1)^2+ \\ (A_2-A_1)^2+(A_2-A_1)^2 \end{Bmatrix}}$$

$$= \sqrt{(A_2-A_1)^2}$$

$$= |A_1-A_2|.$$

Property 2. Two Triangular intuitionistic fuzzy numbers $\tilde{A}_1$ and $\tilde{A}_2$ are identical if and only if $d(\tilde{A}_1,\tilde{A}_2)=0$.

Proof. Let $\tilde{A}_1=\langle a_{11},a_{12},a_{13};b_{11},b_{12},b_{13}\rangle$ and $\tilde{A}_2=\langle a_{21},a_{22},a_{23};b_{21},b_{22},b_{23}\rangle$ be two triangular intuitionistic fuzzy numbers.

If $\tilde{A}_1$ and $\tilde{A}_2$ are identical, then $a_{11}=a_{21}$, $a_{12}=a_{22}$, $a_{13}=a_{23}$, $b_{11}=b_{21}$, $b_{12}=b_{22}$ and $b_{13}=b_{23}$. Using equation (9) the distance between $\tilde{A}_1$ and $\tilde{A}_2$ is

$$d(\tilde{A}_1,\tilde{A}_2) = \sqrt{\frac{1}{12}\begin{Bmatrix} 3(a_{21}-a_{11})^2+3(b_{21}-b_{11})^2+ \\ 2(a_{21}-a_{11})(a_{22}-a_{12})+ \\ 2(b_{21}-b_{11})(b_{22}-b_{12})+ \\ (a_{22}-a_{12})^2+(b_{22}-b_{12})^2 \end{Bmatrix}}$$

$$= \sqrt{\frac{1}{12}\begin{Bmatrix} 3(a_{21}-a_{21})^2+3(b_{21}-b_{21})^2+ \\ 2(a_{21}-a_{21})(a_{22}-a_{22})+ \\ 2(b_{21}-b_{21})(b_{22}-b_{22})+ \\ (a_{22}-a_{22})^2+(b_{22}-b_{22})^2 \end{Bmatrix}}$$

$$= \sqrt{\frac{1}{12}\{3(0)^2+3(0)^2+2(0)+2(0)+(0)^2+(0)^2\}}$$

$$= 0.$$

Conversely, If $d(\tilde{A}_1,\tilde{A}_2)=0$, then from equation (9) we have

$$d(\tilde{A}_1,\tilde{A}_2) = \sqrt{\frac{1}{12}\begin{Bmatrix} (a_{21}-a_{11})^2+(a_{22}-a_{12})^2+(a_{23}-a_{13})^2+(b_{21}-b_{11})^2 \\ +(b_{22}-b_{12})^2+(b_{23}-b_{13})^2+(a_{21}-a_{11})(a_{22}-a_{12}) \\ +(a_{22}-a_{12})(a_{23}-a_{13})+(b_{21}-b_{11})(b_{22}-b_{12}) \\ +(b_{22}-b_{12})(b_{23}-b_{13})+(a_{22}-a_{12})^2+(b_{22}-b_{12})^2 \end{Bmatrix}}$$

$$= 0$$

Implies that $a_{11}=a_{21}$, $a_{12}=a_{22}$, $a_{13}=a_{23}$, $b_{11}=b_{21}$, $b_{12}=b_{22}$ and $b_{13}=b_{23}$. Therefore, two triangular intuitionistic fuzzy numbers $\tilde{A}_1$ and $\tilde{A}_2$ are identical and the property has been proved.

Property 3. If $\tilde{A}_1$, $\tilde{A}_2$ and $\tilde{A}_3$ are three triangular intuitionistic fuzzy numbers. The intuitionistic fuzzy number $\tilde{A}_2$ is closer to intuitionistic fuzzy number $\tilde{A}_1$ than the other intuitionistic fuzzy number $\tilde{A}_3$ if and only if $d(\tilde{A}_1,\tilde{A}_2)<d(\tilde{A}_1,\tilde{A}_3)$.

This property is trivial. For example, Three intuitionistic fuzzy numbers $\tilde{A}_1=\langle 2,3,4;1,3,5\rangle$, $\tilde{A}_2=\langle 3,4,5;2,4,6\rangle$, $\tilde{A}_3=\langle 4,5,6;3,5,7\rangle$. From Fig. 1 & 2, we can easily see that the intuitionistic fuzzy number $\tilde{A}_2$ is closer to intuitionistic fuzzy number $\tilde{A}_1$ than the intuitionistic fuzzy number $\tilde{A}_3$.

$$d(\tilde{A}_1,\tilde{A}_2) = \sqrt{\frac{1}{12}\begin{Bmatrix} (3-2)^2+(4-3)^2+(5-4)^2+(2-1)^2+ \\ (4-3)^2+(6-5)^2+(3-2)(4-3)+ \\ (4-3)(5-4)+(2-1)(4-3)+ \\ (4-3)(6-5)+(4-3)^2+(4-3)^2 \end{Bmatrix}} = 1$$

$$d(\tilde{A}_1,\tilde{A}_3) = \sqrt{\frac{1}{12}\begin{Bmatrix} (4-2)^2+(5-3)^2+(6-4)^2+(3-1)^2+ \\ (5-3)^2+(7-5)^2+(4-2)(5-3)+ \\ (5-3)(6-4)+(3-1)(5-3)+ \\ (5-3)(7-5)+(5-3)^2+(5-3)^2 \end{Bmatrix}}$$

$$= \sqrt{\frac{1}{12}\{48\}}$$

$$= 2$$

According to the distance measurement and definition 9, we conclude that the intuitionistic fuzzy number $\tilde{A}_2$ is closer to intuitionistic fuzzy number $\tilde{A}_1$ than the other intuitionistic fuzzy number $\tilde{A}_3$.

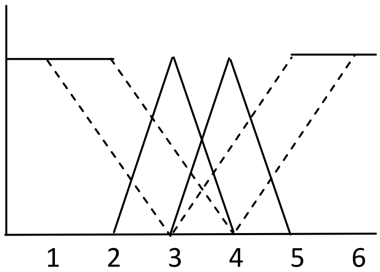

Figure-1

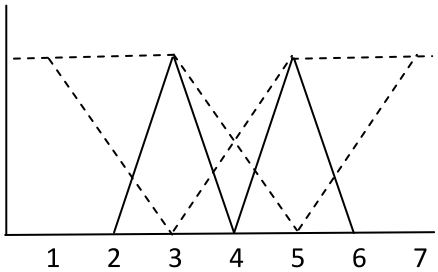

Figure-2

2.1 Ranking method for TIFS-

For an arbitrary Intuitionistic fuzzy number $\tilde{A}=\{\langle x,\mu_{\tilde{A}}(x),\nu_{\tilde{A}}(x)\rangle : \forall x\in X\}$. S. Sagaya Roseline & E.C. Henry amirtha raj [16] define the magnitude of membership and non-membership function for intuitionistic fuzzy number denoted by $Mag(\tilde{A}_\mu)$, $Mag(\tilde{A}_\nu)$ respectively as

$$Mag(\tilde{A}_\mu)=\frac{1}{2}\left(\int_0^1 (f_{\tilde{A}}^{-1}(r)+g_{\tilde{A}}^{-1}(r)+a_2+a_3)f(r)dr\right) \tag{10}$$

$$Mag(\tilde{A}_\nu)=\frac{1}{2}\left(\int_0^1 (h_{\tilde{A}}^{-1}(r)+k_{\tilde{A}}^{-1}(r)+b_2+b_3)f(r)dr\right) \tag{11}$$

Where the function $f(r)$ is a non negative and increasing function on [0,1] with $f(0)=0, f(1)=1$ and $\int_0^1 f(r)dr=$

$\frac{1}{2}$. Obviously function $f(r)$ can be considered as a weighting function. In actual applications, function $f(r)$ can be chosen according to the actual situation. Here we use $f(r) = r$. In particular, let $\tilde{A} = \langle a_1, a_2, a_3, a_4; b_1, b_2, b_3, b_4 \rangle$ be a TIFN with membership and non-membership functions, defined as in equation (4). In this case we have

$$Mag(\tilde{A}_\mu) = \frac{1}{12}(a_1 + 5a_2 + 5a_3 + a_4) \quad (12)$$

$$Mag(\tilde{A}_\nu) = \frac{1}{12}(b_1 + 5b_2 + 5b_3 + b_4) \quad (13)$$

Now, an ordering could be given on TIFNs as shown in the following algorithm

2.2 Algorithm

As a ranking method we compare two membership and non-membership of $\tilde{A}$ and $\tilde{B}$ using the following steps:

Step-1 Compute $Mag(\tilde{A}_\mu)$ and $Mag(\tilde{B}_\mu)$ using (12) then compare them as

(1) $Mag(\tilde{A}_\mu) > Mag(\tilde{B}_\mu)$ if and only if $\tilde{A}_\mu > \tilde{B}_\mu$.

(2) $Mag(\tilde{A}_\mu) < Mag(\tilde{B}_\mu)$ if and only if $\tilde{A}_\mu < \tilde{B}_\mu$.

(3) $Mag(\tilde{A}_\mu) = Mag(\tilde{B}_\mu)$ if and only if $\tilde{A}_\mu = \tilde{B}_\mu$.

Step-2 Compute $Mag(\tilde{A}_\nu)$ and $Mag(\tilde{B}_\nu)$ using (13) then compare as

(1) $Mag(\tilde{A}_\nu) > Mag(\tilde{B}_\nu)$ if and only if $\tilde{A}_\nu > \tilde{B}_\nu$.

(2) $Mag(\tilde{A}_\nu) < Mag(\tilde{B}_\nu)$ if and only if $\tilde{A}_\nu < \tilde{B}_\nu$.

(3) $Mag(\tilde{A}_\nu) = Mag(\tilde{B}_\nu)$ if and only if $\tilde{A}_\nu = \tilde{B}$.

Note- For Triangular intuitionistic fuzzy number

$$Mag(\tilde{A}_\mu) = \frac{1}{12}(a_1 + 10a_2 + a_3) \quad (14)$$

$$Mag(\tilde{A}_\nu) = \frac{1}{12}(b_1 + 10b_2 + b_3) \quad (15)$$

3 TOPSIS METHOD-

Topsis proposed by Hwang and Yoon (1981) and Yoon and Hwang (1985), is a kind of method to solve multi-attribute decision-making problem and based on the concept that the chosen alternative should have the shortest distance from the positive ideal solution and farthest distance from the negative ideal solution.

3.1 TOPSIS method is presented as follows:

Step 1: Create an evaluation matrix consisting of m alternatives and n criteria, with the intersection of each alternative and criteria given as x_{ij} , we therefore have a matrix $(x_{ij})_{m\times n}$.

Step 2: The matrix $(x_{ij})_{m\times n}$ is then normalized to form the matrix $(n_{ij})_{m\times n}$, using the normalization method

$$n_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} {x_{ij}}^2}, \quad i = 1,2, \dots, m, \; j = 1,2, \dots, n. \quad (16)$$

Step 3: Calculate the weighted normalized decision matrix

$$V = (v_{ij})_{m\times n} = (w_j v_{ij})_{m\times n}, \quad i = 1,2, \dots, m \quad (17)$$

Where $w_j = \frac{W_j}{\sum_{j=1}^{n} W_j}$, $j = 1,2, \dots, n$ so that $\sum_{j=1}^{n} w_j = 1$, and W_j is the original weight given to the indicator w_j, $j = 1,2, \dots, n$.

Step 4: Determine the positive ideal solution and negative ideal solution

$$A^+ = \{v_1^+, v_2^+, \dots, v_n^+\}$$
$$= \{(\max_j v_{ij} \mid i \in I), (\min_j v_{ij} \mid i \in J)\}, \quad (18)$$

$$A^- = \{v_1^-, v_2^-, \dots, v_n^-\}$$
$$= \{(\min_j v_{ij} \mid i \in I), (\max_j v_{ij} \mid i \in J)\}, \quad (19)$$

Where I is associated with benefit criteria and J is associated with cost criteria.

Step 5: Calculate the separation measure between the target alternative and the positive ideal solution

$$d_i^+ = \{\textstyle\sum_{j=1}^{n} (v_{ij} - v_j^+)^2\}^{\frac{1}{2}}, \; i = 1,2, \dots, m. \quad (20)$$

And the distance between alternative and negative ideal solution is

$$d_i^- = \{\textstyle\sum_{j=1}^{n} (v_{ij} - v_j^-)^2\}^{\frac{1}{2}}, \; i = 1,2, \dots, m. \quad (21)$$

Step 6: Calculate the closeness coefficient to the ideal solution

$$CC_i = \frac{d_i^-}{(d_i^+ + d_i^-)} \quad i = 1,2, \dots, m. \quad (22)$$

Table 1: Linguistic variables for the importance weight of each criteria

Very Low(VL)	$\langle 0,0,0.1; 0,0,0.2 \rangle$
Low(L)	$\langle 0,0.1,0.3; 0,0.1,0.4 \rangle$
Medium Low(ML)	$\langle 0.1,0.3,0.5; 0.05,0.3,0.55 \rangle$
Medium(M)	$\langle 0.3,0.5,0.7; 0.2,0.5,0.8 \rangle$
Medium High(MH)	$\langle 0.5,0.7,0.9; 0.45,0.7,0.95 \rangle$
High(H)	$\langle 0.7,0.9,1.0; 0.6,0.9,1.0 \rangle$
Very High(VH)	$\langle 0.9,1.0,1.0; 0.8,1.0,1.0 \rangle$

Table 2 Linguistic variable for the rating

Very Poor(VL)	$\langle 0,0,1; 0,0,2 \rangle$
Poor(P)	$\langle 0,1,3; 0,1,4 \rangle$
Medium Poor(MP)	$\langle 1,3,5; 0.5,3,5.5 \rangle$
Fair(F)	$\langle 3,5,7; 2,5,8 \rangle$
Medium Good(MG)	$\langle 5,7,9; 4.5,7,9.5 \rangle$
Good(G)	$\langle 7,9,10; 6,9,10 \rangle$
Very Good(VG)	$\langle 9,10,10; 8,10,10 \rangle$

3.2 Topsis method with Intuitionistic fuzzy data

A systematic approach to extend the TOPSIS to the Intuitionistic fuzzy environment is proposed in this section.

Let $A = \{A_1, \dots, A_m\}$ be a set of m alternatives and decision maker will choose the best one from A according to a criterion set $C = \{C_1, \dots, C_n\}$ which include n criteria.

The importance weights of various criteria and the ratings of qualitative criteria are considered as linguistic variables. These linguistic variables can be expressed in positive triangular intuitionistic fuzzy numbers as Table 1 and Table 2.

The importance weight of each criteria can be obtained by either directly assign or indirectly using pairwise comparisons. In here, it is suggested that the decision makers use the linguistic variables (shown as Table 1 and 2) to evaluate the importance of the criteria and the ratings of alternatives with respect to various criteria.

Assume that a decision group has K person, then the importance of the criteria and rating of alternatives with respect to each criteria can be calculated as

$$\tilde{x}_{ij} = \frac{1}{K}[\tilde{x}_{ij}^1 + \tilde{x}_{ij}^2 + \cdots + \tilde{x}_{ij}^k],$$

$$\tilde{w}_j = \frac{1}{K}[\tilde{w}_j^1 + \tilde{w}_j^2 + \cdots + \tilde{w}_j^k]$$

Where $\tilde{x}_{ij}^k$ and $\tilde{w}_j^k$ are the rating and the importance weight of the k^{th} decision maker.

For alternative A_i, the rating of the j^{th} aspect is denoted by $\tilde{x}_{ij}$. Various steps in the proposed Intuitionistic fuzzy Topsis are as follows:

Step 1: *Construct an Intuitionistic fuzzy decision matrix.*

In Intuitionistic fuzzy decision matrix, we suppose that, each $\tilde{x}_{ij}$ is triangular Intuitionistic fuzzy number, i.e., $\tilde{x}_{ij} = \left(a_{ij}, b_{ij}, c_{ij}; a_{ij}^{'}, b_{ij}, c_{ij}^{'}\right)$. A MCDM problem with Intuitionistic fuzzy data can be concisely expressed in matrix format as

$$\begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \ldots & C_n \end{array} \\ \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{bmatrix} \end{array}$$

$i = 1,2, \ldots, m; j = 1,2, \ldots, n.$

$\widetilde{W} = [\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n],$

Where $\tilde{w}_j$ is the weight of criterion C_j and is a normalized Intuitionistic fuzzy number.

Step 2: We calculate the normalized Intuitionistic fuzzy decision matrix as follows:

First, for each Intuitionistic fuzzy number $\tilde{x}_{ij} = \langle a_{ij}, b_{ij}, c_{ij}; a_{ij}^{'}, b_{ij}, c_{ij}^{'}\rangle$, we calculate the set of α-cut as

$$(\tilde{x}_{ij})_\alpha = \{[a_{ij} + \alpha(b_{ij} - a_{ij}), c_{ij} - \alpha(c_{ij} - b_{ij})], [a_{ij}^{'} + \alpha(b_{ij} - a_{ij}^{'}), c_{ij}^{'} - \alpha(c_{ij}^{'} - b_{ij})]\}, \quad (23)$$

$\alpha \in [0,1]$.

Therefore, each Intuitionistic fuzzy $\tilde{x}_{ij}$ is transform to an interval, now by an approach proposed in Jahanshahloo et al. [10].we can transform this interval into normalized interval as follows:

$$[\tilde{n}_{ij}]_\alpha^L = \frac{a_{ij} + \alpha(b_{ij} - a_{ij})}{\sqrt{\sum_{i=1}^m \{(a_{ij} + \alpha(b_{ij} - a_{ij}))^2 + (c_{ij} - \alpha(c_{ij} - b_{ij}))^2\}}}$$

$$[\tilde{n}_{ij}]_\alpha^U = \frac{c_{ij} - \alpha(c_{ij} - b_{ij})}{\sqrt{\sum_{i=1}^m \{(a_{ij} + \alpha(b_{ij} - a_{ij}))^2 + (c_{ij} - \alpha(c_{ij} - b_{ij}))^2\}}}$$

$$[\tilde{m}_{ij}]_\alpha^L = \frac{a_{ij}^{'} + \alpha(b_{ij} - a_{ij}^{'})}{\sqrt{\sum_{i=1}^m \{(a_{ij}^{'} + \alpha(b_{ij} - a_{ij}^{'}))^2 + (c_{ij}^{'} - \alpha(c_{ij}^{'} - b_{ij}))^2\}}}$$

$$[\tilde{m}_{ij}]_\alpha^U = \frac{c_{ij}^{'} - \alpha(c_{ij}^{'} - b_{ij})}{\sqrt{\sum_{i=1}^m \{(a_{ij}^{'} + \alpha(b_{ij} - a_{ij}^{'}))^2 + (c_{ij}^{'} - \alpha(c_{ij}^{'} - b_{ij}))^2\}}}$$

Now, interval $([(\tilde{n}_{ij})_\alpha^L, (\tilde{n}_{ij})_\alpha^U], [(\tilde{m}_{ij})_\alpha^L, (\tilde{m}_{ij})_\alpha^U])$ is normalized of interval (23). Now we can transform this normalized interval in to a Intuitionistic fuzzy number such as $\widetilde{N}_{ij} = \langle s_{ij}, n_{ij}, t_{ij}; s_{ij}^{'}, m_{ij}, t_{ij}^{'}\rangle$ such that n_{ij} and m_{ij} is obtained when $\alpha = 1$ i.e.,

$$n_{ij} = [\tilde{n}_{ij}]_{\alpha=1}^L = [\tilde{n}_{ij}]_{\alpha=1}^U$$

$$m_{ij} = [\tilde{m}_{ij}]_{\alpha=1}^L = [\tilde{m}_{ij}]_{\alpha=1}^U$$

Also by setting $\alpha = 0$ we have

$$s_{ij} = n_{ij} - [\tilde{n}_{ij}]_{\alpha=0}^L$$

$$t_{ij} = [\tilde{n}_{ij}]_{\alpha=0}^U - n_{ij}$$

$$s_{ij}^{'} = m_{ij} - [\tilde{m}_{ij}]_{\alpha=0}^L$$

$$t_{ij}^{'} = [\tilde{m}_{ij}]_{\alpha=0}^U - m_{ij}$$

And $\widetilde{N}_{ij}$ is a normalized positive triangular Intuitionistic fuzzy number i.e., $\widetilde{N}_{ij}$ is normalized of Intuitionistic fuzzy number $\tilde{x}_{ij}$.

Step 3: *Construct the weighted normalized intuitionistic fuzzy decision matrix*.

The weighted intuitionistic fuzzy normalized value v_{ij} can be obtained by aggregating the weight vector $\tilde{w}_j$ and the intuitionistic fuzzy number $\widetilde{N}_{ij}$ as:

$$\tilde{v}_{ij} = \widetilde{N}_{ij} \cdot \tilde{w}_j, \quad i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n.$$

Where $\tilde{w}_j$ is the weight of j^{th} attribute or criterion and $\sum_{i=1}^n w_j = 1$.

Step 4: *Intuitionistic fuzzy positive ideal solution (IFPIS) and intuitionistic fuzzy negative ideal solution (IFPIS).*

Let P_1 and P_2 be benefit criteria & cost criteria respectively. $\tilde{A}^+$ is IFPIS and $\tilde{A}^-$ is IFNIS. Then by using the equation (13) we have obtained $\tilde{A}^+$ and $\tilde{A}^-$ as follows;

$$\tilde{A}^+ = (\tilde{v}_1^+, \ldots, \tilde{v}_n^+)$$

Where $\tilde{v}_j^+ = \max \tilde{v}_{ij}$

$$= \langle(\max_i \mu_{ij} | j \in P_1); (\min_i \nu_{ij} | j \in P_2)\rangle$$

$$\tilde{A}^- = (\tilde{v}_1^-, \ldots, \tilde{v}_n^+)$$

Where $\tilde{v}_j^- = \min \tilde{v}_{ij}$

$$= \langle(\min_i \mu_{ij} | j \in P_1); (\max_i \nu_{ij} | j \in P_2)\rangle$$

Step 5: *Calculate the distance measure of each alternative A_i from IFPIS and IFNIS.*

The separation of each alternative from the Intuitionistic fuzzy positive ideal solution, by using the equation (9) we have

$\tilde{d}_i^+ = \sum_{j=1}^n d(\tilde{v}_{ij}, \tilde{v}_{ij}^+),\ i = 1, \ldots, m.$

Similarly, the separation from the fuzzy negative ideal solution can be calculated as

$\tilde{d}_i^- = \sum_{j=1}^n d(\tilde{v}_{ij}, \tilde{v}_{ij}^-),\ i = 1, \ldots, m.$

Step 6: *Calculate the relative closeness coefficient (CC_i) to the ideal solution*

The relative closeness coefficient (CC_i) of the alternative A_i with respect to the intuitionistic fuzzy ideal solutions is defined as:

$$CC_i = \frac{\tilde{d}_i^-}{(\tilde{d}_i^+ + \tilde{d}_i^-)}\ , \quad i = 1, \ldots, m.$$

Obviously, an alternative A_i is closer to the $\tilde{A}^+$and farther from $\tilde{A}^-$as CC_i approaches to 1. Therefore, according to the closeness coefficient, we can determine the ranking order of all alternatives and select the best one from among a set of feasible alternatives.

4 NUMERICAL EXAMPLE

In this section, we still use the problem discussed in [7] to illustrate the proposed MCDM method in intuitionistic fuzzy environment.

Now suppose that a software company wants to hire a software engineer. After Preliminary screening three candidate $\{A_1,\ A_2,\ A_3\}$ have remained as alternative for further evaluation. A committee of three decision maker, D_1, D_2 and D_3 has been formed to conduct the interview and to select the most suitable candidate. The three possible alternatives can be evaluated under five criteria:

(1) Emotional steadiness (C_1),

(2) Oral communication skill (C_2),

(3) Personality (C_3),

(4) Past experience (C_4),

(5) Self confidence (C_5).

The proposed method is currently applied to solve this problem and the computational procedure is summarized as follows:

Step 1: The decision-makers gives the importance weights (shown in table 1) and rating (shown in table 2) of criteria with linguistic terms.

Step 2: Converting the linguistic evaluation (shown in table 3 and 4) into triangular intuitionistic fuzzy number to construct the intuitionistic fuzzy decision matrix and determine the intuitionistic fuzzy weight of each criterion as table 5.

Step 3: Determine the normalized intuitionistic fuzzy decision matrix by using Step 2 of proposed method shown in table 6.

Step 4: By using the formula (18) constructing the weighted normalized intuitionistic fuzzy decision matrix as

table 7.

Step 5: Determine FPIS and FNIS as

$A^+ = \{\langle 0.19,0.35,0.55; 0.17,0.37,0.45\rangle,$
$\langle 0.39,0.47,0.48; 0.15,0.33,0.48\rangle,$

$\langle 0.30,0.42,0.48; 0.18,0.33,0.45\rangle,$

$\langle 0.36,0.41,0.45; 0.22,0.37,0.49\rangle,$

$\langle 0.16,0.30,0.45; 0.04,0.17,0.38\rangle\}$

$A^- = \{\langle 0.22,0.37,0.44; 0.15,0.35,0.57\rangle,$

$\langle 0.22,0.33,0.43; 0.30,0.47,0.48\rangle,$

$\langle 0.21,0.33,0.43; 0.27,0.42,0.49\rangle,$

$\langle 0.28,0.37,0.45; 0.29,0.41,0.45\rangle,$

$\langle 0.07,0.17,0.32; 0.13,0.30,0.52\rangle\}$

Step 6: By using formula (9) calculate the distance measure of each alternative from FPIS and FNIS, respectively, as table 8.

Step 8: The closeness coefficient of each alternative A_i are obtained as shown in table 8.

Step 9: According to the closeness coefficient the ranking order of the three candidates are $A_3 > A_1 > A_2$. Obviously, the best selection is candidate A_3.

Table 3: The importance weight of the criteria

	D_1	D_2	D_3
C_1	H	VH	MH
C_2	VH	VH	VH
C_3	VH	H	H
C_4	VH	VH	VH
C_5	M	MH	MH

Table 4: The ratings of the three candidates by decision makers under all criteria

Criteria	Candidates		Decision-makers	
		D_1	D_2	D_3
C_1	A_1	MG	G	MG
	A_2	G	G	MG
	A_3	VG	G	F
C_2	A_1	G	MG	F
	A_2	VG	VG	VG
	A_3	MG	G	VG
C_3	A_1	F	G	G
	A_2	VG	VG	G
	A_3	G	MG	VG
C_4	A_1	VG	G	VG
	A_2	VG	VG	VG
	A_3	G	VG	MG
C_5	A_1	F	F	F
	A_2	VG	MG	G
	A_3	G	G	MG

Step 10: Finally compare with the results obtained using other method[16].The results are listed in table 9.

Table 5: The intuitionistic fuzzy decision matrix and intuitionistic fuzzy weights of each three alternatives

	C_1	C_2	C_3	C_4	C_5
A_1	⟨5.7,7.7,9.3; 5,7.7,9.7 ⟩	⟨5, 7, 8.7; 4.2,7,9.2⟩	⟨5.7,7.7,9; 4.7,7.7,9.3⟩	⟨8.3,9.7,10; 7.3,9.7,10⟩	⟨3,5,7; 2,5,8⟩
A_2	⟨6.3,8.3,9.7; 5.5,8.3,9.8⟩	⟨9,10,10; 8,10,10⟩	⟨8.3,9.7,10; 7.3,9.7,10⟩	⟨9,10,10; 8,10,10⟩	⟨7,8.7,9.7; 6.2,8.7,9.8⟩
A_3	⟨6.3,8,9; 5.3,8,9.3⟩	⟨7,8.7,9.7; 6.2,8.7,9.8⟩	⟨7,8.7,9.7; 6.2,8.7,9.8⟩	⟨7,8.7,9.7; 6.2,8.7,9.8⟩	⟨6.3,8.3,9.7; 5.5,8.3,9.8⟩
Weight	⟨0.7,0.87,0.97; 0.62,0.87,0.98⟩	⟨0.9,1,1; 0.8,1,1⟩	⟨0.77,0.93,1; 0.67,0.93,1⟩	⟨0.9,1,1; 0.8,1,1⟩	⟨0.43,0.63,0.83; 0.36,0.63,0.9⟩

Table 6 The Normalized Intuitionistic fuzzy decision making

	C_1	C_2	C_3	C_4	C_5
A_1	⟨0.29,0.39,0.48; 0.26,0.39,0.51⟩	⟨0.24,0.33,0.42; 0.21,0.33,0.46⟩	⟨0.28,0.36,0.44; 0.24,0.36,0.47⟩	⟨0.37,0.42,0.45; 0.34,0.42,0.47⟩	⟨0.16,0.27,0.38; 0.11,0.27,0.44⟩
A_2	⟨0.33,0.42,0.50; 0.29,0.42,0.52⟩	⟨0.44,0.47,0.48; 0.39,0.47,0.49⟩	⟨0.40,0.45,0.48; 0.37,0.45,0.47⟩	⟨0.41,0.43,0.45; 0.38,0.43,0.47⟩	⟨0.38,0.47,0.53; 0.34,0.47,0.54⟩
A_3	⟨0.31,0.41,0.47; 0.28,0.41,0.49⟩	⟨0.34,0.41,0.47; 0.31,0.41,0.49⟩	⟨0.34,0.41,0.47; 0.31,0.41,0.49⟩	⟨0.32,0.37,0.44; 0.29,0.37,0.46⟩	⟨0.34,0.45,0.53; 0.30,0.45,0.54⟩

Table 7 The Weighted normalized Intuitionistic fuzzy decision matrix

	C_1	C_2	C_3	C_4	C_5
A_1	⟨0.20,0.34,0.47; 0.16,0.34,0.49⟩	⟨0.22,0.33,0.42; 0.17,0.33,0.46⟩	⟨0.22,0.33,0.44; 0.16,0.33,0.47⟩	⟨0.33,0.42,0.45; 0.27,0.42,0.47⟩	⟨0.07,0.17,0.32; 0.04,0.17,0.39⟩
A_2	⟨0.23,0.36,0.48; 0.18,0.36,0.51⟩	⟨0.39,0.47,0.48; 0.32,0.47,0.49⟩	⟨0.31,0.42,0.48; 0.25,0.42,0.50⟩	⟨0.37,0.43,0.45; 0.30,0.43,0.47⟩	⟨0.16,0.29,0.44; 0.12,0.29,0.49⟩
A_3	⟨0.22,0.36,0.46; 0.17,0.36,0.48⟩	⟨0.31,0.41,0.47; 0.25,0.41,0.49⟩	⟨0.26,0.38,0.47; 0.21,0.38,0.49⟩	⟨0.29,0.37,0.44; 0.23,0.37,0.46⟩	⟨0.15,0.28,0.44; 0.12,0.28,0.49⟩

Table 8: Separation measures and the relative closeness coefficient of each alternative

	$\tilde{d}_i^+$	$\tilde{d}_i^-$	Closeness coefficient	Rank
A_1	0.2553	0.234	0.4788	2
A_2	0.2415	0.2068	0.4613	3
A_3	0.1624	0.5406	0.5406	1

5 CONCLUSION

In this paper we have presented the TOPSIS method with intuitionistic fuzzy data. In the evaluation process the rating of each alternative with respect to each criteria are taken as intuitionistic fuzzy number. The normalized fuzzy decision matrix is calculated by using the concept of $\alpha - cuts$. In this approach the distance of an alternative from the intuitionistic fuzzy positive ideal solution and intuitionistic fuzzy negative ideal solution is also considered. The closeness coefficients of alternatives are obtained and alternatives has ranked. The comparison of results with Mahdavi et al.[16] are shown on table 9.

Table 9. Comparison with other methods

		$\tilde{d}_i^+$	$\tilde{d}_i^-$	Closeness coefficient	Rank	Ordering
Proposed Method	A_1	0.2553	0.234	0.4788	2	$A_3 > A_1 > A_2$
	A_2	0.2415	0.2068	0.4613	3	
	A_3	0.1624	0.5406	0.5406	1	
Mahdavi et al.	A_1	4.3673	4.9326	0.5304	1	$A_1 > A_3 > A_2$
	A_2	5.0000	4.3044	0.4626	3	

A_3	4.6722	4.5614	0.4940	2

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