ABSTRACT
In this paper, we presented a new test statistic for testing exponentiality against new better than renewal used in the RP order (NBRU$_{rp}$) based on moment inequality. Pitman's asymptotic efficiency, the Pitman asymptotic relative efficiency (PARE) are studied for other tests. Critical values are tabulated for sample size $n = 5(1)30(5)50$, the power of the test are calculate. Also we proposed a test for testing exponentiality versus (NBRU$_{rp}$) for right censored data and the power estimates of this test are also simulated for some commonly used distributions in reliability. Finally, real data are given to elucidate the use of the proposed test statistic in the reliability analysis.

Keywords
Life distributions, (NBRU$_{rp}$) aging class, moment inequalities, exponentiality U-statistic, asymptotic normality, efficiency, Monte Carlo method, power and censored data.

1. INTRODUCTION
In reliability, various aging classes of life distributions have been introduced to describe several types of improvement that accompany aging. The residual probability (RP) function is a well-known reliability measure which has applications in many disciplines such as reliability theory, survival analysis, and actuarial studies. The RP function uniquely determines the distribution function of $F$ (and hence the distribution function of $G$), under the condition that the ratio of the hazard rates of $X$ and $Y$ is known. In addition, when the ratio of the hazard rates of $X$ and $Y$ is a monotone function of time, then RP function is also a monotone function of time. The study of the properties of RP function might be important for engineers and system designers to compare the lifetime of the products and, hence, to design better products. For example, consider a series system with two independent components. If $X$ and $Y$ denote the lifetime of the components, then clearly the lifetime of the system is $T = min(X,Y)$. It is easily seen that $R(t) = P(Y = T|T > t)$, that is, the probability that the component with lifetime $Y$ causes the system failure given that the system has survived up to time $t$ (cf. Zardasht and Asadi [27] for several reliability properties, Tan and Lü [26] for some biological background, and Lü and Chen [20], Chen et al. [9] and Zhou et al. [28] for some real world applications). Formally, in view of the RP function, the lifetime random variable $X$ is said to be smaller than $Y$ in the RP order (denoted by $X \leq rp Y$) if and only if

$$R(t) \leq 0.5, \quad \forall t > 0.$$
moments inequalities for NRBUD and RNBU classes of life distributions.

In this paper, we derive the moment inequalities for the NBRU_{rp} class in section 2, we present attest statistic based on a U-statistic for testing H_0 : is exponential against H_1 : is NBRU_{rp} and not exponential, the Pitman asymptotic efficiencies are calculated for some commonly used distributions in reliability, in Section 3. In Section 4 Monte Carlo null distribution critical points are simulated for sample sizes n=5,10,30,50, the power estimates of this test are calculated at the significant level α = 0.05 for some common alternatives distribution and some application are given. In Section 5, we dealing with right-censored data and selected critical values are tabulated; the power estimates for censor data of this test are tabulated. Finally, we discuss some applications to elucidate the usefulness of the proposed test in reliability analysis.

2. MOMENT INEQUALITY

The next result provides moments inequality for the NBRU_{rp} distributions.

In this, as well as subsequent results all moments are assumed to exist and are finite.

**Theorem 2.1** If F is NBRU_{rp}, then for all integer t ≥ 0,
\[ \int_0^{\infty} x^{t+2} F(x) dF(x) \geq \frac{r + 2}{r + 4} \int_0^{\infty} x^{t+1} \left( \int_0^x tdF(t) \right) dF(x). \] (2.1)

**Proof:** since F is NBRU_{rp} then
\[ \int_t^{\infty} F^2(x) dx \geq \int_t^{\infty} f(x) \left( \int_x^{\infty} F(u) du \right) dx \]
\[ = \int_t^{\infty} F(x) \left( \int_x^{\infty} f(u) du \right) dx \]
\[ = \int_t^{\infty} [F(t) F(x) - F^2(x)] dx. \]

This can be written in the form
\[ 2 \int_0^{\infty} F^2(x) dx \geq \int_0^{\infty} F(t) \int_0^{\infty} F(x) dx dt. \]

Multiplying both sides by \( t^r \) for r ≥ 0 and integrating over (0, ∞) w.r.t. t, we get,
\[ 2 \int_0^{\infty} t^r \int_t^{\infty} F^2(x) dx dt \geq \int_0^{\infty} t^r F(t) \int_t^{\infty} F(x) dx dt. \] (2.2)

It is easy to show that,
L.H.S
\[ = 2 \int_0^{\infty} t^r \int_t^{\infty} F^2(x) dx dt, \]
\[ = 2 \int_0^{\infty} F^2(x) \left( \int_0^x t^r dt \right) dx, \]
\[ = \frac{4}{(r + 1)(r + 2)} \int_0^{\infty} x^{r+2} F(x) dF(x). \] (2.3)

And,
R.H.S
\[ = \int_0^{\infty} t^r F(t) \int_t^{\infty} F(x) dx dt, \]
\[ = \int_0^{\infty} F(x) \left( \int_0^x t^r F(t) dt \right) dx, \]
\[ = \frac{1}{(r + 1)(r + 2)} \int_0^{\infty} x^{r+2} F(x) dF(x) \]
\[ + \int_0^{\infty} x^{r+1} \left( \int_0^x tdF(t) \right) dF(x). \] (2.4)

Making use of (2.3), (2.4) in (2.2) the result follows.

3. TESTING AGAINST NBRU_{rp} CLASS FOR NON-CENSORED DATA

Here we present a test statistic based on the moment inequality, the test presented depends on a sample X_1, X_2, ..., X_n from a population with distribution function F. We test the null hypothesis for testing H_0 : F is exponential against an alternative that H_1 : F is belongs to NBRU_{rp} class and not exponential. We propose the following measure of departure using theorem (2.1) as
\[ \delta_{rp} = \int_0^{\infty} x^{r+2} F(x) dF(x) \]
\[ - \frac{r + 2}{r + 4} \int_0^{\infty} x^{r+1} \left( \int_0^t (t-x) dF(t) \right) dF(x). \] (3.1)

Where,
\[ I(t > x) = \begin{cases} \frac{1}{0}, & t > x \\ 0, & O.W. \end{cases} \]

Note that under H_0 : \( \delta_{rp} = 0 \), while under H_1 : \( \delta_{rp} > 0 \)

3.1 Empirical Test Statistic NBRU_{rp}

**Alternative**

To estimate \( \delta_{rp} \), let X_1, X_2, ..., X_n be a random sample from F. Let \( F_n(x) \) denote the empirical distribution of the survival function \( F(x) \) where
\[ F_n(x) = \frac{1}{n} \sum_{i=1}^{n} I(X_i > x), \quad dF_n(x) = \frac{1}{n} \] (3.2)

and let \( \delta_{rp} \) be the empirical estimate of \( \delta_{rp} \) where can be written as
\[ \delta_{rp} = \frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \left( \frac{X_i^{r+2} - \frac{r + 2}{r + 4} X_i^{r+1} X_j^{r+1}}{X_i^{r+2}} \right) I(X_i > X_j) \] (3.3)

To make the test \( \delta_{rp} \) scale invariant, we let
\[ \bar{\delta}_{rp} = \frac{\delta_{rp}}{X^{r+2}} \] (3.4)

Set,
\[ \phi(X_1, X_2) = \frac{X_1^{r+2} - \frac{r + 2}{r + 4} X_1^{r+1} X_2^{r+1}}{X_1^{r+2}} I(X_2 > X_1). \] (3.5)

And define the symmetric kernel as
\[ \psi(X, X) = \frac{1}{2l} \sum_{i} \phi(X_i, X'_i). \]

Where the sum is over all arrangements of \( X_i \) and \( X'_i \), this leads to \( \hat{\Delta}_{\text{rp}} \) is equivalent to \( U_n \), statistic given by

\[ U_n = \frac{1}{\binom{n}{r}} \sum_{i=0}^{n-r} \phi(X_i, X'_i). \]

The following theorem summarizes the asymptotic normality of \( \hat{\Delta}_{\text{rp}} \).

**Theorem 3.1** As \( n \to \infty, \sqrt{n}(\hat{\Delta}_{\text{rp}} - \delta_{\text{rp}}) \) is asymptotically normal with mean 0 and variance,

\[ \sigma^2 = \text{Var} \left[ \frac{1}{r + 4} \int_{X_1}^{X_{r+2}} e^{-x} \, dx - \frac{r + 2}{r + 4} X_1 \int_{X_1}^{X_1} e^{-x} \, dx \right] \]

Under \( H_0 \) and \( r = 0 \), the variance \( \sigma^2 \) reduces to

\[ \sigma^2 = \text{Var} \left[ -2X e^{-X} - 2 e^{-X} - \frac{1}{2} X + 2 \right] \]  

(3.6)

**Proof:** Let \( \eta_1(X_1) = E[\psi(X_1, X_2)] \),

\[ \eta_1(X_1) = \int_{X_1}^{X_{r+2}} e^{-x} \, dx - \frac{r + 2}{r + 4} X_1 \int_{X_1}^{X_1} e^{-x} \, dx \]

And,

\[ \eta_2(X_1) = E[\psi(X_2, X_3)] \]

\[ \eta_2(X_1) = \int_{X_1}^{X_{r+2}} e^{-x} \, dx - \frac{r + 2}{r + 4} X_1 \int_{X_1}^{X_1} e^{-x} \, dx \]

Set,

\[ \zeta(X_1) = \eta_1(X_1) + \eta_2(X_1) \]

\[ \zeta(X_1) = \frac{1}{r + 4} (2X_1^{r+2} - (r+2)X_1^{r+1})e^{-X_1} + \int_{X_1}^{X_{r+2}} e^{-x} \, dx - \frac{r + 2}{r + 4} X_1 \int_{X_1}^{X_1} e^{-x} \, dx. \]

Then,

\[ \sigma^2 = \text{Var}[\zeta(X_1)]. \]

Under \( H_0 \), the variance reduces to eq. (3.6), after calculation \( \sigma^2 = \frac{1}{25} \).

### 3.2 The Pitman Asymptotic Efficiency

To judge on the quality of this procedure, Pitman asymptotic efficiencies (PAE) are computed. We use the concept of Pitman’s asymptotic efficiency (PAE) which is defined as

\[ \text{PAE}(\delta_{\text{rp}}(\theta)) = \frac{1}{\alpha_0} \left[ \int_{\theta}^{\theta_0} \frac{d}{d\theta} \delta_{\text{rp}}(\theta) \right]_{\theta=\theta_0}. \]

And compared with some other tests for the following alternative distributions:

(i) Linear failure rate family (LFR),

\[ F_1(x) = e^{-\theta x^2}, x \geq 0, \theta \geq 0, \]

(ii) Makeham family,

\[ F_2(x) = e^{-\theta(x + e^{-x} - 1)}, x \geq 0, \theta \geq 0, \]

(iii) Gamma family,

\[ F_3(x) = \int_{x}^{\infty} e^{-\theta x} \, dx, x > 0, \theta \geq 0, \]

(iv) Weibull family,

\[ F_4(x) = e^{-x^\theta}, x > 0, \theta \geq 0. \]

Note that \( H_0 \) (the exponential distribution) is attained at \( \theta_0 = 0 \) in (i), (ii) and at \( \theta_0 = 1 \) in (iii), (iv).

Since,

\[ \delta_{\text{rp}}(\theta) = \int_{0}^{\infty} x^{r+2} F_0(x) dF_0(x) - \frac{r + 2}{r + 4} \int_{0}^{\infty} x^{r+1} \left( \int_{x}^{\infty} t dF_0(t) \right) dF_0(x). \]

The \( \text{PAE}(\delta_{\text{rp}}(\theta)) \) can be written as,

\[ \text{PAE}(\delta_{\text{rp}}(\theta)) = \frac{1}{\alpha_0} \left[ \int_{0}^{\infty} x^{r+2} F_0(x) dF_0(x) - \frac{r + 2}{r + 4} \int_{0}^{\infty} x^{r+1} \left( \int_{x}^{\infty} t dF_0(t) \right) dF_0(x) \right]. \]

Using MATHEMATICA 9 program to calculate the Pitman asymptotic efficiency for \( \text{NBRU}_{\text{rp}} \) test statistic in case of Weibull, Gamma family and direct calculations for linear failure rate family (LFR) and Makeham. In the above cases we get the following \( \text{PAE} \) values:

(i) Linear failure rate family \( F_1 \):

\[ \text{PAE}(\delta_{\text{rp}}(\theta)) = \frac{1}{\alpha_0} \left[ \frac{r + 2}{r + 4} 2^{-(r+2)} \Gamma(r + 2) \right]. \]

(ii) Makeham family \( F_2 \):

\[ \text{PAE}(\delta_{\text{rp}}(\theta)) = \frac{1}{\alpha_0} \left[ \frac{r + 2}{2(r + 4)} 3^{-(r+2)} \Gamma(r + 2) \right]. \]

(iv) Weibull family \( F_3 \):

\[ \text{PAE}(\delta_{\text{rp}}(\theta)) = 0.709668, \]

(iii) Gamma family \( F_4 \):

\[ \text{PAE}(\delta_{\text{rp}}(\theta)) = 0.291887. \]

Direct calculations of the asymptotic efficiencies of \( \text{NBRU}_{\text{rp}} \) test are given in Table 1. at \( r = 0 \).

**Table 1: Comparison between the PAF of our test and some other tests:**

<table>
<thead>
<tr>
<th>Test</th>
<th>( F_1 )</th>
<th>( F_2 )</th>
<th>( F_3 )</th>
<th>( F_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mugaridi et al  ( \delta_1 )([23])</td>
<td>0.408 0.039</td>
<td>0.170 -</td>
<td>\</td>
<td></td>
</tr>
<tr>
<td>Abdal-Azir ( \delta_2 )([1])</td>
<td>0.535 0.184</td>
<td>0.223 -</td>
<td>\</td>
<td></td>
</tr>
</tbody>
</table>
It is clear from Table 1, that the new test statistic \( \delta_{rp} \) for NBRU\(_p\) is more efficient than \( \delta_1, \delta_2, \delta_3 \) and \( \delta_4 \). Also the Pittman asymptotic relative efficiency (PARE) of our test \( \delta_{rp} \), comparing to \( \delta_1, \delta_2, \delta_3, \delta_4 \) is calculated where

\[
PARE(T_1, T_2) = \frac{P_{AE}(T_1)}{P_{AE}(T_2)}
\]

Table 2: show that the asymptotic relative efficiencies for our test:

<table>
<thead>
<tr>
<th>Test</th>
<th>LFR</th>
<th>Makeham</th>
<th>Weibull</th>
</tr>
</thead>
<tbody>
<tr>
<td>( PARE(\delta_{rp}, \delta_1) )</td>
<td>2.252</td>
<td>5.231</td>
<td>4.176</td>
</tr>
<tr>
<td>( PARE(\delta_{rp}, \delta_2) )</td>
<td>1.718</td>
<td>1.109</td>
<td>3.184</td>
</tr>
<tr>
<td>( PARE(\delta_{rp}, \delta_3) )</td>
<td>4.235</td>
<td>1.417</td>
<td>14.20</td>
</tr>
<tr>
<td>( PARE(\delta_{rp}, \delta_4) )</td>
<td>1.609</td>
<td>0.761</td>
<td>0.498</td>
</tr>
</tbody>
</table>

It is clear from Table 2, that our test statistic \( \delta_{rp} \) for NBRU\(_p\) is more efficiently than the other four cases.

4. MONTE CARLO NULL DISTRIBUTION CRITICAL POINTS

In this section the Monte Carlo null distribution critical points of \( \hat{\delta}_{rp} \) are simulated based on 5000 generated samples of size \( n = 5(1)30(5)50 \) from the standard exponential distribution by using Mathematica 9 program, simulated percentiles for small samples are commonly used by applied statisticians and reliability analyst. We have simulated the upper percentile values for 90%; 95%; 98% and 99%. Table 3, presented these percentile values of the statistics \( \hat{\delta}_{rp} \) in (3.4).

![Fig. 1. Relation between critical values, sample size and confidence levels.](image)

Table 3. Critical values of statistic \( \hat{\delta}_{rp} \)

<table>
<thead>
<tr>
<th>n</th>
<th>90%</th>
<th>95%</th>
<th>98%</th>
<th>99%</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.06131</td>
<td>0.07848</td>
<td>0.09722</td>
<td>0.10896</td>
</tr>
<tr>
<td>6</td>
<td>0.05981</td>
<td>0.07624</td>
<td>0.09403</td>
<td>0.10448</td>
</tr>
<tr>
<td>7</td>
<td>0.05612</td>
<td>0.07179</td>
<td>0.08898</td>
<td>0.09925</td>
</tr>
<tr>
<td>8</td>
<td>0.05206</td>
<td>0.06746</td>
<td>0.08243</td>
<td>0.09295</td>
</tr>
<tr>
<td>9</td>
<td>0.05046</td>
<td>0.06559</td>
<td>0.08152</td>
<td>0.09662</td>
</tr>
<tr>
<td>10</td>
<td>0.05067</td>
<td>0.06355</td>
<td>0.07557</td>
<td>0.08427</td>
</tr>
<tr>
<td>11</td>
<td>0.04704</td>
<td>0.06069</td>
<td>0.07652</td>
<td>0.08511</td>
</tr>
<tr>
<td>12</td>
<td>0.04479</td>
<td>0.05711</td>
<td>0.07185</td>
<td>0.08453</td>
</tr>
<tr>
<td>13</td>
<td>0.04436</td>
<td>0.05506</td>
<td>0.06692</td>
<td>0.07687</td>
</tr>
<tr>
<td>14</td>
<td>0.04246</td>
<td>0.05368</td>
<td>0.06761</td>
<td>0.07633</td>
</tr>
<tr>
<td>15</td>
<td>0.04085</td>
<td>0.05116</td>
<td>0.06260</td>
<td>0.06914</td>
</tr>
<tr>
<td>16</td>
<td>0.04045</td>
<td>0.05193</td>
<td>0.06533</td>
<td>0.07158</td>
</tr>
<tr>
<td>17</td>
<td>0.03932</td>
<td>0.04935</td>
<td>0.06146</td>
<td>0.06852</td>
</tr>
<tr>
<td>18</td>
<td>0.03784</td>
<td>0.04875</td>
<td>0.06083</td>
<td>0.06935</td>
</tr>
<tr>
<td>19</td>
<td>0.03766</td>
<td>0.04917</td>
<td>0.05873</td>
<td>0.06745</td>
</tr>
<tr>
<td>20</td>
<td>0.03619</td>
<td>0.04718</td>
<td>0.05759</td>
<td>0.06457</td>
</tr>
<tr>
<td>21</td>
<td>0.03581</td>
<td>0.04476</td>
<td>0.05676</td>
<td>0.06257</td>
</tr>
<tr>
<td>22</td>
<td>0.03426</td>
<td>0.04348</td>
<td>0.05467</td>
<td>0.06286</td>
</tr>
<tr>
<td>23</td>
<td>0.03346</td>
<td>0.04320</td>
<td>0.05413</td>
<td>0.06144</td>
</tr>
<tr>
<td>24</td>
<td>0.03235</td>
<td>0.04239</td>
<td>0.05465</td>
<td>0.06213</td>
</tr>
<tr>
<td>25</td>
<td>0.03279</td>
<td>0.04188</td>
<td>0.05183</td>
<td>0.05827</td>
</tr>
<tr>
<td>26</td>
<td>0.03251</td>
<td>0.04065</td>
<td>0.04995</td>
<td>0.05676</td>
</tr>
<tr>
<td>27</td>
<td>0.03094</td>
<td>0.03868</td>
<td>0.04950</td>
<td>0.05486</td>
</tr>
<tr>
<td>28</td>
<td>0.03099</td>
<td>0.03966</td>
<td>0.04886</td>
<td>0.05615</td>
</tr>
<tr>
<td>29</td>
<td>0.03115</td>
<td>0.03988</td>
<td>0.04853</td>
<td>0.05566</td>
</tr>
<tr>
<td>30</td>
<td>0.02957</td>
<td>0.03704</td>
<td>0.04616</td>
<td>0.05215</td>
</tr>
<tr>
<td>31</td>
<td>0.02857</td>
<td>0.03657</td>
<td>0.04483</td>
<td>0.05143</td>
</tr>
<tr>
<td>40</td>
<td>0.02680</td>
<td>0.03451</td>
<td>0.04228</td>
<td>0.04749</td>
</tr>
<tr>
<td>45</td>
<td>0.02526</td>
<td>0.03224</td>
<td>0.04002</td>
<td>0.04315</td>
</tr>
<tr>
<td>50</td>
<td>0.02291</td>
<td>0.02956</td>
<td>0.03733</td>
<td>0.04294</td>
</tr>
</tbody>
</table>

It is clear from Table 3, and Fig.1, that the critical values are increasing as the confidence level increasing and are almost decreasing as the sample size increasing.

4.1 Power Estimates of the Test \( \hat{\delta}_{rp} \)

In this section, the power of our test \( \hat{\delta}_{rp} \) is estimated at (1 - \( \alpha \))% confidence level, \( \alpha = 0.05 \) with suitable parameters values of at \( n = 10, 20 \) and 30 with respect to three alternatives linear failure rate (LFR), Weibull, and Gamma distributions based on 5000 simulated samples.

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Parameter</th>
<th>Sample Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>LFR</td>
<td>( \theta )</td>
<td>( n = 10 ) ( n = 20 ) ( n = 30 )</td>
</tr>
<tr>
<td>2</td>
<td>0.9982</td>
<td>0.9996</td>
</tr>
<tr>
<td>3</td>
<td>0.9990</td>
<td>1.0000</td>
</tr>
<tr>
<td>4</td>
<td>0.9986</td>
<td>0.9998</td>
</tr>
<tr>
<td>2</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
</tbody>
</table>
The power estimates of our test $\Delta_{rp}$ increases as the value of the parameter $\theta$ and sample size $n$ increases, and it is clear that our test has good powers.

### 4.2 Applications Using Complete (Uncensored) Data

In this section, we apply our test to some real data-sets at 95% confidence level.

#### Data-set #1.

Consider the data that given in Abouammoh et al. [2]. These data represent set of 40 patients suffering from blood cancer (leukemia) from one of ministry of health hospitals in Saudi Arabia. In this case, we get $\Delta_{rp} = 0.107634$ and this value exceeds the tabulated critical value in Table 3. It is evident at the significant level %95, that the data set has NBRU$_{rp}$ property.

#### Data-set #2.

Consider the data set given in Grubbs [13]. This data gives the times between arrivals of 25 customers at a facility. It is easily to show that $\Delta_{rp} = 0.154287$ which is greater than the critical value of Table 3. Then we accept $H_1$ which shows that the data set have NBRU$_{rp}$ property but not exponential.

#### Data-set #3.

Consider the data, which represent failure times in hours, for a specific type of electrical insulation in an experiment where the insulation was subjected to a continuously increasing voltage stress (Lawless [18], p.138). The value of test statistic for the data set by formula (3.4) is given by $\Delta_{rp} = 0.028851$ which is less than the critical value of table 3. Then we accept the null hypothesis of exponentially property. This means that this kind of data doesn't fit with NBRU$_{rp}$ property.

### 5. TESTING AGAINST NBRU$_{rp}$ CLASS FOR CENSORED DATA

In this section, a test statistic is proposed to test $H_0$ versus $H_1$ with randomly right-censored data. Such a censored data is usually the only information available in a life-testing model or in a clinical study where patients may be lost (censored) before the completion of a study. This experimental situation can formally be modeled as follows.

Suppose $n$ objects are put on test, and $X_1, X_2, ..., X_n$ denote their true life time. We assume that $X_1, X_2, ..., X_n$ be independent, identically distributed (i.i.d) according to a continuous life distribution $F$. Let $Y_1, Y_2, ..., Y_n$ be (i.i.d) according to a continuous life distribution $G$. Also we assume that $X$s and $Y$s are independent. In the randomly right-censored model, we observe the pairs $(Z_i, \delta_i), i = 1, ..., n$ where $\delta_i = \min(X_i, Y_i)$ and $\delta_i = \begin{cases} 1 & \text{if } Z_i = X_i (j-th \ observation \ is \ uncensored), \\ 0 & \text{if } Z_i = Y_i (j-th \ observation \ is \ censored) \end{cases}.$

Let $Z(0) = 0 < Z(1) < Z(2) < \cdots < Z(n)$ denote the ordered $Z$s and $\delta_i$ is the $\delta_i$ corresponding to $Z_i$ respectively.

Using the censored data $(Z_i, \delta_i), j = 1, ..., n$ Kaplan and Meier [14] proposed the product limit estimator.

$$F_\ell(X) = 1 - F_\ell(X) = \prod_{j=1}^{\delta_i} \left( \frac{n-j}{n-j+1} \right), X \in [0, Z_n]$$

Now, for testing $H_0 : \Delta_{rp} = 0$, against $H_1 : \Delta_{rp} > 0$, using the randomly right censored data, we propose the following test statistic:

$$\tilde{\Delta}_{rp}^{c} = \frac{1}{\mu^{\frac{n}{2}}} \prod_{i=1}^{n} \left( \frac{n}{n-j+1} \right)^{\delta_i}, X \in [0, Z_n]$$

Since

$$\int_{0}^{\infty} x^{r+1} F^2(x) dx = \frac{2}{r+2} \int_{0}^{\infty} x^{r+2} F(x) dx$$

For computational purposes and for $r = 0$, $\tilde{\Delta}_{rp}$ can be rewritten as

$$\tilde{\Delta}_{rp}^{c} = \frac{1}{\mu^{\frac{n}{2}}} (\eta - \beta).$$

Where,

$$\mu = \sum_{i=1}^{n} \prod_{m=1}^{i-1} \left( \frac{\delta_{m}}{C_{m}^{\delta_{m}}} \right) (Z(i) - Z(i-1)),$$

$$\eta = \sum_{i=1}^{n} \prod_{m=1}^{i-1} \left( \frac{\delta_{m}}{C_{m}^{\delta_{m}}} \right)^{2} (Z(i) - Z(i-1)),$$

$$\beta = \frac{1}{2} \sum_{i=1}^{n} \prod_{m=1}^{i-1} \left[ \prod_{q=1}^{j-2} C_{q}^{\delta_{q}} - \prod_{q=1}^{j-1} C_{q}^{\delta_{q}} \right] \left( \prod_{p=1}^{i-2} C_{p}^{\delta_{p}} - \prod_{p=1}^{i-1} C_{p}^{\delta_{p}} \right)$$

And

$$dF_n(Z_i) = F(Z_{i-1}) - F(Z_i), \quad C_k = \frac{n-k}{n-k+1}$$

Table 5 gives the critical values percentiles of $\tilde{\Delta}_{rp}$ test for sample sizes $n = 5(5)30(10)70,81,86$, based on 5000 replications.

<table>
<thead>
<tr>
<th>n</th>
<th>90%</th>
<th>95%</th>
<th>98%</th>
<th>99%</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1.91182</td>
<td>2.26190</td>
<td>2.65184</td>
<td>2.97898</td>
</tr>
<tr>
<td>10</td>
<td>0.47058</td>
<td>0.53694</td>
<td>0.60793</td>
<td>0.67943</td>
</tr>
<tr>
<td>15</td>
<td>0.31263</td>
<td>0.36263</td>
<td>0.41703</td>
<td>0.45568</td>
</tr>
<tr>
<td>20</td>
<td>0.21942</td>
<td>0.26526</td>
<td>0.31678</td>
<td>0.35392</td>
</tr>
</tbody>
</table>
5.1 Power Estimates of the Test $\Delta_{r\mu}$

In this section, we present an estimation of the power for testing exponentiality versus $NBRU_{r\mu}$. Using significance level $\alpha = 0.05$ with suitable parameter values of $\theta$ at $n = 10, 20$ and $30$, and for commonly used distributions in reliability such as LFR family, Weibull family and Gamma family alternatives as shown in Table 6.

![Critical values vs sample size](Fig2.png)

We notice from Table 5. That our test has a good power, and the power increases as the sample size increases.

5.2 Applications for Censored Data

We present two good real examples to illustrate the use of our test statistics $\Delta_{r\mu}$ in the case of censored data at 95% confidence level.

**Data-set #4.**

Consider the data from Susarla and Vanryzin [25], which represent 81 survival times (in months) of patients melanoma. Out of these 46 represents non-censored data. Now, taking into account the whole set of survival data (both censored and uncensored). It was found that the value of test statistic for the data set using formula (5.2) is given by $\Delta_{r\mu}^c = 1.49772 \times 10^{-9}$ and this value is less than the tabulated critical value in Table 5. This means that the data set have the exponential property.

**Data-set #5.**

On the basis of right censored data for lung cancer patients from Pena [24]. These data consists of 86 survival times (in month) with 22 right censored. Now account the whole set of survival data (both censored and uncensored), and computing the test statistic given by formula (4.2). It was found that $\Delta_{r\mu}^c = 1.42503 \times 10^{-9}$. This is less than the tabulated value in Table 5. so we accept the null hypotheses.

6. CONCLUSION

Moments inequalities of $NBRU_{r\mu}$ class of life distributions are deduced. Based on these inequalities a new test for exponentiality versus $NBRU_{r\mu}$ class is constructed. The critical values of this test are calculated. Based on $PAEs$ comparison between our test and tests of Mugdadi et al $\delta_1$ [23], Abdal-Aziz $\delta_2$ [1], Mahmoud et al $\delta_3$ [21] and Kayid $\delta_4$ [16] are given. Our study showed that our test performs higher $PAE$ with respect to $r$. Based on right censored data a test for exponentiality versus $NBRU_{r\mu}$ class is also given. The power estimates of this test are simulated for uncensored and censored data. Finally sets of real data are used to elucidate the proposed test for practical problems.

7. REFERENCES


[27] Zardasht, V. and Asadi, M. (2010).Evaluation of $P \left( X_i > Y_j \right)$ when both $X_i$ and $Y_j$ are residual lifetimes of two systems. Statistics Neer landica, vol. 64, no. 4, pp. 460.481.