Approximations of Rough Sets via Filter by using *g*-increasing and *g*-decreasing Sets

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ABSTRACT
In this paper, we introduce a way of constructing a rough set via grill ordered topological spaces. Increasing and decreasing sets are defined based on grill and comparisons between current approximations and previous approximations by Shafei and Kandil are carried out. Also it is shown that the chances of getting better approximation by our method of approximations are greater than any of the available methods.

Keywords
G-increasing, *G*-decreasing.

1. INTRODUCTION
A rough set, first described by Polish computer scientist Zdzisaw I. Pawlak in 1991 [13], is a formal approximation of a crisp set (i.e., conventional set) interms of a pair of sets which give the lower and the upper approximation of the original set. In the standard version of rough set theory, the lower and upper approximation sets are crisp sets, but in other variations, the approximating sets may be fuzzy sets. Rough set methods can be applied as a component of hybrid solutions in machine learning and data mining. They have been found to be particularly useful for rule induction and feature selection. Rough set based data analysis methods have been successfully applied in bioinformatics, economics and finance, medicine, multimedia, web and text mining, signal and image processing, software engineering, robotics, and engineering (e.g. power systems and control engineering). Recently the three regions of rough sets are interpreted as regions of acceptance, rejection and deferment. This leads to three-way decision making approach with the model, which can potentially lead to interesting future applications. The set with the same lower and upper approximation is the crisp set, otherwise a rough set. The boundary region is the difference between the upper and lower approximations. The accuracy of the set or ambiguous depending on the boundary region is empty or not respectively. If the boundary region is empty we have succeeded in our attempt to define the set. The standard rough set theory starts from an equivalence relation R on a finite universe X.

This paper investigates a new notion of generalized closed rough sets using grills, filters and ordered relations. The increasing and decreasing sets with respect to grill were defined and hence their corresponding lower and upper approximations. Examples are given to illustrate the new approximations and comparative study was carried out with previous approximations of [5, 6].

In the approximation space (X, R), Pawlak [13] considered two operators, the lower and upper approximations of subsets.

Let A $\subseteq$X.

$\underline{R}(A) = \{x \in X : [x]_R \subseteq A\}$.

$\overline{R}(A) = \{x \in X : [x]_R \cap A \neq \phi\}$.

The Boundary, positive and negative regions are also defined:

$BN_R(A) = \underline{R}(A) - \overline{R}(A)$.

$POS_R(A) = \underline{R}(A)$.

$NEG_R(A) = X - \overline{R}(A)$.

2. PRELIMINARIES
In this section, the needed definitions and results that are necessary to understand this paper were given.

Definition 2.1.*[1] If R is a binary relation on X and A $\subseteq$X, then A is called "`after composed"(respectively after-c composed) set if A contains all the after (respectively fore) sets for all its elements, i.e., for all a $\in$A, aR$\subseteq$A*

(respectively Ra $\subseteq$A), where aR = {b : (a, b) $\in$R} and Ra = {b : (b, a) $\in$R}.

Definition 2.2.*[12] Let (X,R) be a poset. A set A$\subseteq$X is said to be*

(1) Decreasing if for every a $\in$A and x $\in$X such that xRa, then x $\in$A.

(2) Increasing if for every a $\in$A and x $\in$X such that aRx, then x $\in$A.

Theorem 2.3.*[12] Let (X, R) be a poset and A $\subseteq$X. Then, the class of all increasing (decreasing) sets forms a topology on X which is denoted by τ_{inc} (τ_{dec}).*

Definition 2.4.*[11] A subfamily* F *of P(X) is called a filter on X if*

(1) $\phi \notin$ F.

(2) If $A_1, A_2 \in$ F, *then* $A_1 \cap A_2 \in$ F.

(3) If $A \in$ F *and* $A \subseteq B \subseteq X$, *then* $B \in$F.

Definition 2.5.*[11] A subset* B *of P(X) is called a filter base if*

(1) $\phi \notin$ B.

(2) If $B_1, B_2 \in$ B, *then there exist* $B_3 \subseteq B_1 \cap B_2$.

A filter base B *can be turned into a filter by including all sets of P(X) which contains a set of* B, *i.e.,* F_B=*{A $\in$P(X) : A$\supseteq$B, B $\in$*B}

Definition 2.6.*[11] Let $\xi \subseteq$P(X). Then $\xi$is called a filter-subbases on X if it satisfies the finite intersection property, i.e., any finite sub collection of ξhas anon-empty intersection.*

Definition 2.7.*[3] A collection G of nonempty subsets of a space X is called a grill on X if*

(i) A ∈G and A ⊆B ⊆X ⇒ B ∈G.

(ii) A,B ∈X and A ∪B ∈G ⇒A ∈G or B ∈G.

Definition 2.8.*[4] A triple (X, τ_R, ρ) where τ_R is the topology generated by any relation R and ρ is a partially order relation, is called an order topological approximation space "OTAS".*

Definition 2.9.*[4] Let triple (X, τ_R, ρ) be an OTAS, A ⊆X. Then the lower, upper approximations, boundary regions and accuracy levels respectively are given by:*

$\underline{R}_{inc}(A)= \cup\{G\in \tau_R$: G is an increasing and $G\subseteq A\}$.

$\underline{R}_{dec}(A)= \cup\{G\in \tau_R$: G is a decreasing and $G \subseteq A\}$.

$\overline{R}^{inc}(A)= \cap \{F \in \tau_R'$: F is an increasing and $A \subseteq F\}$.

$\overline{R}^{dec}(A)= \cap \{F \in \tau_R'$: F is a decreasing and $A \subseteq F\}$.

$BN_{inc}(A)= \overline{R}^{inc(A)} - \underline{R}_{inc(A)}$, $BN_{dec}(A)= \overline{R}^{dec(A)} - \underline{R}_{dec(A)}$.

$\alpha^{inc}(A) = \left|\frac{\underline{R}_{inc}(A)}{\overline{R}^{inc}(A)}\right|$, $\alpha^{dec}(A) = \left|\frac{\underline{R}_{dec}(A)}{\overline{R}^{dec}(A)}\right|$,

$\alpha^{inc}(A)$ *is an increasing accuracy and* $\alpha^{dec}(A)$ *is a decreasing accuracy.*

Definition 2.10.*[5] A triple (X, F_R, ρ) is said to be generalized order topological approximation space (GOTAS), where F_R is a filter generated by any relation R and ρis a partial ordered relation.*

Here, to construct the filter F_R, let $\xi= \{xR : x \in X\}$ be a subbase of a filter F_R.

Definition 2.11.*[5]A (X, F_R, ρ) be an GOTAS and A ⊆X. Then the lower, upper approximations, boundary regions and accuracy levels respectively are given by:*

$R_{*inc}(A)= \cup\{G\in F_R$: G is an increasing and $G \subseteq A\}$.

$R_{*dec}(A)= \cup\{G\in F_R$: G is a decreasing and $G \subseteq A\}$.

$$R^{*inc}(A)= \begin{cases} \cap \{H \in F_R': H \text{ is an increasing and } A \subseteq H\}. \\ X \text{ if not exists } H \in F_R' : H \text{ is an increasing and } A \subseteq H. \end{cases}$$

$$R^{*dec}(A)= \begin{cases} \cap \{H \in F_R': H \text{ is a decreasing and } A \subseteq H\}. \\ X \text{ if not exists } H \in F_R' : H \text{ is a decreasing and } A \subseteq H. \end{cases}$$

$BN_{*inc}(A)= R^{*inc(A)} - R_{*inc(A)}$,

$BN_{*dec}(A)= R^{*dec(A)} - R_{*dec(A)}$.

$\alpha^{*inc}(A) = \left|\frac{R_{*inc}(A)}{R^{*inc}(A)}\right|$, $\alpha^{*G\text{-}dec}(A) = \left|\frac{R_{*dec}(A)}{R^{*dec}(A)}\right|$,

$\alpha^{*inc}(A)$ *is an increasing accuracy and* $\alpha^{*dec}(A)$ *is a decreasing accuracy.*

Lemma 2.12. *[5] Let R be a binary relation on X. Then* $\tau_R - \phi \subseteq F_R$, *where* τ_R *is the topology generated by the subbase* $\xi= \{xR{:}x \in X\}$ *and* F_R *is a filter generated by the same subbase.*

Lemma 2.13.*[5] In any GOTAS (X, F_R, ρ),* $\tau_R - \phi \subseteq F_R$.

In the next definition, the generalized order topological approximation space is also defined by using the subbase $\xi^*= \{RxR : x \in X\}$ to generate filter F_R^*.

Definition 2.14.*[6] Let (X, F_R^*, ρ) is said to be generalized order topological approximation space (GOTAS), where F_R^* is a filter generated by any relation R and ρis a partially ordered relation.*

Definition 2.15.*[6] A (X, F_R^*, ρ) be an GOTAS and A ⊆X. Then the lower, upper approximations, boundary regions and accuracy respectively are given by:*

$R_{**inc}(A)= \cup\{G\in F_R^*$: G is an increasing and $G \subseteq A\}$.

$R_{**dec}(A)= \cup\{G\in F_R^*$: G is a decreasing and $G\subseteq A\}$.

$R^{**inc}(A)= \cap \{H\in F_R^{*\prime}$: H is an increasing and $A \subseteq H\}$.

$R^{**dec}(A)= \cap \{H\in F_R^{*\prime}$: H is a decreasing set and $A\subseteq H\}$.

$BN_{**inc}(A)= R^{**inc(A)} - R_{**inc(A)}$,

$BN_{**dec}(A)= R^{**dec(A)} - R_{**dec(A)}$.

$\alpha^{**inc}(A) = \left|\frac{R_{**inc}(A)}{R^{**inc}(A)}\right|$, $\alpha^{**G\text{-}dec}(A) = \left|\frac{R_{**dec}(A)}{R^{**dec}(A)}\right|$,

$\alpha^{**inc}(A)$ *is an increasing accuracy and* $\alpha^{**dec}(A)$ *is a decreasing accuracy.*

3. ROUGH SETS BY USING GRILL IN ORDERED TOPOLOGICAL SPACES

In this section, we introduced a new rough set via grill in topological ordered spaces depending on a general binary relation and a partially order relation.

Using the grill, we defined the two sets namely $\mathcal{G}$ -increasing, $\mathcal{G}$ -decreasing and with the general binary relation, a topology τ_R was generated with $\xi= \{xR{:}\ x\epsilon X\}$as a subbase. The lower and upper approximations were defined with respect to the sets $\mathcal{G}$-increasing and $\mathcal{G}$-decreasing.

Definition 3.1. Let (X, R) be a poset and $\mathcal{G} \subseteq P(X)$ be a grill on X. Then a set A ⊆X is called

(1) $\mathcal{G}$-decreasing set iff $Ra \cap A' \in G$ for all a ∈A, where Ra = {b : (b, a) ∈R}.

(2) $\mathcal{G}$-increasing set iff $aR \cap A' \in G$ for all a ∈A, where aR = {b: (a, b)∈R}.

Definition 3.2.A quadruple (X, τ_R, ρ, $\mathcal{G}$) is called a general ordered topological approximation space along with a grill " $\mathcal{G}$ *-GOTAS* ", where τ_R is a topology generated by any relation R and ρ is a partial order relation.

Definition 3.3. *A (X, τ_R, ρ, $\mathcal{G}$) be a $\mathcal{G}$-GOTAS, A ⊆X. Then the lower, upper approximations, boundary regions and the accuracy levels respectively are given by:*

$\underline{R}_{G\text{-}inc}(A)= \cup\{H \in \tau_R$: H is a G-increasing set and $H\subseteq A\}$.

$\underline{R}_{G\text{-}dec}(A)= \cup\{H \in \tau_R$: H is a G-decreasing set and $H\subseteq A\}$.

$$\overline{R}^{G\text{-}inc}(A)= \begin{cases} \cap \{F \in \tau_R': F \text{ is a } G-\text{increasing set and } A \subseteq F\}. \\ X \text{ if not exists } F \in \tau_R': F \text{ is a } G-\text{increasing set and } A \subseteq F. \end{cases}$$

$$\overline{R}^{G\text{-}dec}(A)= \begin{cases} \cap \{F \in \tau_R': F \text{ is a } G-\text{decreasing set and } A \subseteq F\}. \\ X \text{ if not exists } F \in \tau_R': F \text{ is a } G-\text{decreasing set and } A \subseteq F. \end{cases}$$

$BN_{G\text{-}inc}(A) = \overline{R}^{G\text{-}inc(A)} - \underline{R}_{G\text{-}inc(A)}$.

$BN_{G\text{-}dec}(A) = \overline{R}^{G\text{-}dec(A)} - \underline{R}_{G\text{-}dec(A)}$.

$\alpha^{G\text{-}inc}(A) = \left|\frac{\underline{R}_{G-inc\ (A)}}{\overline{R}^{G-inc\ (A)}}\right|$, $\alpha^{G\text{-}dec}(A) = \left|\frac{\underline{R}_{G-dec\ (A)}}{\overline{R}^{G-dec\ (A)}}\right|$,

$\alpha^{G\text{-}inc}(A)$ is an increasing accuracy and $\alpha^{G\text{-}dec}(A)$ is a decreasing accuracy.

Example 3.4. *Let X = {a, b, c, d}, R = {(b, b),(c, c),(d, d),(a, b),(a, d),(b, c),(b, d),(c, a),(c, b),(c, d),(d, a),(d, b),(d, c)}, τ_R= {ϕ, X, {b, d},{b, c, d}}, τ_R'= {ϕ, X, {a}, {a, c}}, ρ= Δ∪{(a, c),(a, d),(b, c),(d, c)} and G = {X, {a}, {c},*

{a,b}, {a,c}, {a,d}, {b,c}, {c,d}, {a,b,d}, {a,c,d}, {b,c,d}}.

Table 1, gives the values of approximations, boundary values and the accuracy levels calculated using G - increasing sets for the same set of Shafei [4], using Definition 3.3, along with the above values using increasing sets of Shafei and it is seen that Definition3.3, minimized the boundary region in more number of cases than Shafei's method. Totally the probability of getting better accuracy is higher than Shafei's method as far the G-increasing sets are concerned.

Table 2, shows the values of approximations, boundary values and the accuracy levels which are calculated using G - decreasing sets. When a comparison is made with Shafei's values, where the above values are calculated using decreasing sets. It is found that there is no change in accuracy level. When considering the boundary values, it is obvious that Shafei's definition gives better result than our approximations.

Proposition 3.5. *Let (X, τ_R, ρ, G) be a G-GOTAS and A, B ⊆X. .Then*

(1) $\underline{R}_{G\text{-}inc}(A) \subseteq A \subseteq \overline{R}^{G\text{-}inc}(A)$ ($\underline{R}_{G\text{-}dec}(A) \subseteq A \subseteq \overline{R}^{G\text{-}dec}(A)$), equalityholds if A = ϕ or X.

(2) A ⊆B ⇒ $\overline{R}^{G\text{-}inc}(A) \subseteq \overline{R}^{G\text{-}inc}(B)$ ($\overline{R}^{G\text{-}dec}(A) \subseteq \overline{R}^{G\text{-}dec}(B)$).

(3) A ⊆B ⇒ $\underline{R}_{G\text{-}inc}(A) \subseteq \underline{R}_{G\text{-}inc}(B)$($\underline{R}_{G\text{-}dec}(A) \subseteq \underline{R}_{G\text{-}dec}(B)$).

(4) $\overline{R}^{G\text{-}inc}(A \cap B) \subseteq \overline{R}^{G\text{-}inc}(A) \cup \overline{R}^{G\text{-}inc}(B)$

($\overline{R}^{G\text{-}dec}(A \cap B) \subseteq \overline{R}^{G\text{-}dec}(A) \cup \overline{R}^{G\text{-}dec}(B)$).

(5) $\underline{R}_{G\text{-}inc}(A \cap B) = \underline{R}_{G\text{-}inc}(A) \cap \underline{R}_{G\text{-}inc}(B)$

($\underline{R}_{G\text{-}dec}(A \cap B) = \underline{R}_{G\text{-}dec}(A) \cap \underline{R}_{G\text{-}dec}(B)$)

(6) $\overline{R}^{G\text{-}inc}(A \cup B) = \overline{R}^{G\text{-}inc}(A) \cup \overline{R}^{G\text{-}inc}(B)$

($\overline{R}^{G\text{-}dec}(A \cup B) = \overline{R}^{G\text{-}dec}(A) \cup \overline{R}^{G\text{-}dec}(B)$).

(7) $\underline{R}_{G\text{-}inc}(A \cup B) \supseteq \underline{R}_{G\text{-}inc}(A) \cap \underline{R}_{G\text{-}inc}(B)$

($\underline{R}_{G\text{-}dec}(A \cup B) \supseteq \underline{R}_{G\text{-}dec}(A) \cap \underline{R}_{G\text{-}dec}(B)$)

(8) $\overline{R}^{G\text{-}inc}(\overline{R}^{G\text{-}inc}(A)) \supseteq \overline{R}^{G\text{-}inc}(A)$

($\overline{R}^{Gdec}(\overline{R}^{G\text{-}dec}(A)) \supseteq \overline{R}^{G\text{-}dec}(A)$).

(9) $\underline{\boldsymbol{R}}_{G\text{-}inc}(A)$ ($\underline{\boldsymbol{R}}_{G\text{-}inc}(A)$) ⊆ $\underline{\boldsymbol{R}}_{G\text{-}inc}(A)$

($\underline{\boldsymbol{R}}_{G\text{-}dec}(A)$ ($\underline{\boldsymbol{R}}_{G\text{-}dec}(A)$) ⊆ $\underline{\boldsymbol{R}}_{G\text{-}dec}(A)$).

4. GENERALIZED ROUGH SETS VIA FILTER BY USING G -INCREASING AND G -DECREASING SETS

In this section, we introduced a new rough set approximation by using the notion G -increasing and G -decreasing sets along with a filter space defined by a general binary relation together with a partially ordered relation.

Definition 4.1. *A quadruple (X, F_R, ρ, G) is called G_{F_R}-general ordered topological approximation space along with a grill G and is denoted as "G_{F_R}-GOTAS", where F_R is a filter generated by any relation R using the subbase ξ = {xR: x ∈X} and $\rho$is a partially order relation and G a given grill on X.*

Definition 4.2. *Let (X, F_R, ρ, G) be a G_{F_R}-GOTAS, A ⊆X. Then the lower, upper approximations, boundary regions and accuracy levels respectively are given by:*

$R_{*G\text{-}inc}(A)$= ∪{H ∈ F_R: H is a G-increasing set and H⊆A}.

$R_{*G\text{-}dec}(A)$= ∪{H ∈ F_R: H is a G-decreasing set and H⊆A}.

$R^{*G\text{-}inc}(A)$=

$$\begin{cases} \cap\ \{F\ \in F_R' : F\ is\ a\ G - increasing\ set\ and\ A\ \subseteq F\}. \\ X\ if\ not\ exists\ F\ \in F_R' : F\ is\ a\ G - increasing\ set\ and\ A\ \subseteq F. \end{cases}$$

$R^{*G\text{-}dec}(A)$=

$$\begin{cases} \cap\ \{F\ \in F_R' : F\ is\ a\ G - decreasing\ set\ and\ A\ \subseteq F\}. \\ X\ if\ not\ exists\ F\ \in F_R' : F\ is\ a\ G - decreasing\ set\ and\ A \subseteq F. \end{cases}$$

$BN_{*G\text{-}inc}(A) = R^{*G\text{-}inc(A)} - R_{*G\text{-}inc(A)}$,

$BN_{*G\text{-}dec}(A) = R^{*G\text{-}dec(A)} - R_{*G\text{-}dec(A)}$.

$\alpha^{*G\text{-}inc}(A) = \left|\frac{R_{*G-inc\ (A)}}{R^{*G-inc\ (A)}}\right|$, $\alpha^{*G\text{-}dec}(A) = \left|\frac{R_{*G-dec\ (A)}}{R^{*G-dec\ (A)}}\right|$,

$\alpha^{*G\text{-}inc}(A)$ is an increasing accuracy and $\alpha^{*G\text{-}dec}(A)$ is a decreasing accuracy.

In Table 3, calculated approximations, boundary values and accuracy levels via filters using G -increasing sets by Definition4.2, together with Kandil's [5] calculated values are given. It is seen that there is no significant difference between the values of both methods.

In Table 4, we compare the upper and lower approximations of sets via filters by Kandil and Definition 4.2, based on decreasing and G -decreasing sets respectively. Though the probability of accuracy is more in our attempt, Kandil's method reduces the boundary values in a better way.

Proposition 4.3. *Let (X, F_R, ρ, G) be a G_{F_R}-GOTAS, A ⊆X. (1) $R_{*G\text{-}inc}(A) \subseteq A \subseteq R^{*G\text{-}inc}(A)$ ($R_{*G\text{-}dec}(A) \subseteq A \subseteq R^{*G\text{-}dec}(A)$), equality holds if A = ϕ or X.*

(2) A ⊆B ⇒ $R^{*G\text{-}inc}(A) \subseteq R^{*G\text{-}inc}(B)$ ($R^{*G\text{-}dec}(A) \subseteq R^{*G\text{-}dec}(B)$).

(3) A ⊆B ⇒ $R_{*G\text{-}inc}(A) \subseteq R_{*G\text{-}inc}(B)$ ($R_{*G\text{-}dec}(A) \subseteq R_{*G\text{-}dec}(B)$).

(4) $R^{*G\text{-}inc}(A \cap B) \subseteq R^{*G\text{-}inc}(A) \cup R^{*G\text{-}inc}(B)$,

$R^{*G\text{-}dec}(A \cap B) \subseteq R^{*G\text{-}dec}(A) \cup R^{*G\text{-}dec}(B)$.

(5) $R_{*G\text{-}inc}(A \cap B) = R_{*G\text{-}inc}(A) \cap R_{*G\text{-}inc}(B,$

$R_{*G\text{-}dec}(A\cap B) = R_{*G\text{-}dec}(A) \cap R_{*G\text{-}dec}(B))$

(6) $R^{*G\text{-}inc}(A \cup B) = R^{*G\text{-}inc}(A) \cup R^{*G\text{-}inc}(B)$,

$R^{*G\text{-}dec}(A \cup B) = R^{*G\text{-}dec}(A) \cup R^{*G\text{-}dec}(B))$.

(7) $R_{*G\text{-}inc}(A\cup B) \supseteq R_{*G\text{-}inc}(A) \cap R_{*G\text{-}inc}(B)$

$R_{*G\text{-}dec}(A \cup B) \supseteq R_{*G\text{-}dec}(A) \cap R_{*G\text{-}dec}(B)$

(8) $R^{*G\text{-}inc}(R^{*G\text{-}inc}(A)) \supseteq R^{*G\text{-}inc}(A)$

$R^{*G\text{-}dec}(R^{*G\text{-}dec}(A)) \supseteq R^{*G\text{-}dec}(A)$.

(9) $R_{*G\text{-}inc}(R_{*G\text{-}inc}(A)) \subseteq R_{*G\text{-}inc}(A)$

$R_{*G\text{-}dec}(R_{*G\text{-}dec}(A)) \subseteq R_{*G\text{-}dec}(A)$.

5. GENERALIZED ROUGH SETS VIA FILTER BY AN AFTER-FORE SETS OF A RELATION

In this section, we calculate rough set approximation via filter by using G-increasing and G-decreasing sets which depends on a general binary relation together with a partially order relation. Here, the filter is generated by after-fore set that has a nonempty finite intersection.

The filter F_R^* was constructed by $\xi^* = \{RxR : x \in X\}$ as the subbase of a filter F_R^*.

Definition 5.1. *A quadruple (X, F_R^*, ρ, G) is called $G_{F_R^*}$ - grill ordered topological approximation space "$G_{F_R^*}$-GOTAS", where F_R^* is a filter generated by the after-fore sets of any relation R that has a non-empty finite intersection, ρ is a partial order relation and G is a grill on X.*

Definition 5.2. *Let (X, F_R^*, ρ, G) be a $G_{F_R^*}$-GOTAS , A ⊆X. Then the lower, upper approximations, boundary regions and accuracy levels respectively are given by:*

$R_{**G\text{-}inc}(A) = \cup\{H \in F_R^* : H \text{ is a G-increasing set and } H \subseteq A\}$.

$R_{**G\text{-}dec}(A) = \cup\{H \in F_R^* : H \text{ is a G-decreasing set and } H \subseteq A\}$.

$R^{**G\text{-}inc}(A) =$

$$\begin{cases} \cap \{F \in F_R^{*'} : F \text{ is a } G-\text{increasing set and } A \subseteq F\}. \\ X \text{ if not exists } F \in F_R^{*'} : F \text{ is a } G-\text{increasing set and } A \subseteq F. \end{cases}$$

$R^{**G\text{-}dec}(A) =$

$$\begin{cases} \cap \{F \in F_R^{*'} : F \text{ is a } G-\text{decreasing set and } A \subseteq F\}. \\ X \text{ if not exists } F \in F_R^{*'} : F \text{ is a } G-\text{decreasing set and } A \subseteq F. \end{cases}$$

$BN_{**G\text{-}inc}(A) = R^{**G\text{-}inc(A)} - R_{**G\text{-}inc(A)}$,

$BN_{**G\text{-}dec}(A) = R^{**G\text{-}dec(A)} - R_{**G\text{-}dec(A)}$.

$\alpha^{**G\text{-}inc}(A) = \left|\frac{R_{**G-inc\ (A)}}{R^{**G-inc\ (A)}}\right|$, $\alpha^{**G\text{-}dec}(A) = \left|\frac{R_{**G-dec\ (A)}}{R^{**G-dec\ (A)}}\right|$,

$\alpha^{**G\text{-}inc}(A)$ is an increasing accuracy and $\alpha^{**G\text{-}dec}(A)$ is a decreasing accuracy.

Note 5.3. All the inequalities hold good in G_{F_R}*-GOTAS in(X,* F_R*, ρ, G) are also true in* $G_{F_R^*}$*-GOTAS.*

Table 5, shows the comparison between the boundaries and accuracy levels of Kandil's Definition 3.2 [6] and the present Definition 5.2. (in case of increasing sets). It is seen that the present definition reduces the boundary region by increasing the lower approximations and decreasing the upper approximations.

Table 6, gives comparison between the boundaries and accuracy levels of Kandil's Definition 3.2 [6] and the present Definition 7.3.2. (in case of decreasing sets). The probability of getting better approximation is more by the present definition than Kandil's method. The lower approximation is increased but the upper approximation is not decreased in most of the cases in our method.

6. CONCLUSION

This paper, defined approximations by grill in three different ways, namely using, a topology, a filter generated by any relation and a filter generated by an after-fore sets of any relation. In all the cases, the probability of getting a better accuracy using G-increasing and G-decreasing sets is more than the method of Shafei and Kandil.

Among all the three cases, the approximation via filter generated by after-fore sets gives a better lower approximation in both the cases of G –increasing and G-decreasing sets.

Table 1. Comparison between the boundary and accuracy by using El-Shafei et al.'s method 2.10 [4] and the present Definition 3.2 in case of increasing (*G*-increasing) sets.

A	El-Shafei et al.'s method 2.10 [4]				The current method in Definition 3.2			
	$\underline{R}_{inc}(A)$	$\overline{R}^{inc}(A)$	$BN_{inc}(A)$	$\alpha^{inc}(A)$	$\underline{R}_{G\text{-}inc}(A)$	$\overline{R}^{G\text{-}inc}(A)$	$BN_{G\text{-}inc}(A)$	$\alpha^{G\text{-}inc}(A)$
ϕ	ϕ	ϕ	ϕ	0	ϕ	$\{a\}$	$\{a\}$	0
$\{a\}$	ϕ	X	X	0	ϕ	$\{a\}$	$\{a\}$	0
$\{b\}$	ϕ	X	X	0	ϕ	X	X	0
$\{c\}$	ϕ	X	X	0	ϕ	X	X	0
$\{d\}$	ϕ	X	X	0	ϕ	X	X	0
$\{a, b\}$	ϕ	X	X	0	ϕ	X	X	0
$\{a, c\}$	ϕ	X	X	0	ϕ	X	X	0
$\{a, d\}$	ϕ	X	X	0	ϕ	X	X	0
$\{b, c\}$	ϕ	X	X	0	ϕ	X	X	0

{b, d}	ϕ	X	X	0	{b, d}	X	{a, c}	0.5
{c, d}	ϕ	X	X	0	ϕ	X	X	0
{a, b, c}	ϕ	X	X	0	ϕ	X	X	0
{a, b, d}	ϕ	X	X	0	ϕ	X	X	0
{a, c, d}	ϕ	X	X	0	ϕ	X	X	0
{b, c, d}	{b, c, d}	X	{a}	0.75	{b, d}	X	{a, c}	0.5
X	X	X	ϕ	1	{b, d}	X	{a, c}	0.5

Table 2. Comparison between the boundary and accuracy by using Shafei et al.'s method 2.10 [4] and the present Definition 3.2 in case of decreasing (*G*-decreasing) sets.

A	Shafei et al.'s method				The current method in Definition 3.2			
	$\underline{R}_{dec}(A)$	$\overline{R}^{dec}(A)$	$BN_{dec}(A)$	$\alpha^{dec}(A)$	$\underline{R}_{G\text{-}dec}(A)$	$\overline{R}^{G\text{-}dec}(A)$	$BN_{G\text{-}dec}(A)$	$\alpha^{G\text{-}dec}(A)$
ϕ	ϕ	ϕ	ϕ	0	ϕ	X	X	0
{a}	ϕ	{a}	{a}	0	ϕ	X	X	0
{b}	ϕ	X	X	0	ϕ	X	X	0
{c}	ϕ	X	X	0	ϕ	X	X	0
{d}	ϕ	X	X	0	ϕ	X	X	0
{a, b}	ϕ	X	X	0	ϕ	X	X	0
{a, c}	ϕ	X	X	0	ϕ	X	X	0
{a, d}	ϕ	X	X	0	ϕ	X	X	0
{b, c}	ϕ	X	X	0	ϕ	X	X	0
{b, d}	ϕ	X	X	0	ϕ	X	X	0
{c, d}	ϕ	X	X	0	ϕ	X	X	0
{a, b, c}	ϕ	X	X	0	ϕ	X	X	0
{a, b, d}	ϕ	X	X	0	ϕ	X	X	0
{a, c, d}	ϕ	X	X	0	ϕ	X	X	0
{b, c, d}	ϕ	X	X	0	ϕ	X	X	0
X	X	X	ϕ	1	ϕ	X	X	0

Table 3. Comparison between the boundary and accuracy by using Kandil et al. Definition 5.2 [5] and the present Definition 4.2 in case of increasing sets

A	Kandil ‘s method				The current method in Definition 4.2			
	$R_{*inc}(A)$	$R^{*inc}(A)$	$BN_{*inc}(A)$	$\alpha^{*inc}(A)$	$R_{*G\text{-}inc}(A)$	$R^{*G\text{-}inc}(A)$	$BN_{*G\text{-}inc}(A)$	$\alpha^{*G\text{-}inc}(A)$
ϕ	ϕ	ϕ	ϕ	0	ϕ	{a}	{a}	0
{a}	ϕ	X	X	0	ϕ	{a}	{a}	0
{b}	ϕ	X	X	0	ϕ	X	X	0
{c}	ϕ	{c}	{c}	0	ϕ	X	X	0
{d}	ϕ	X	X	0	ϕ	X	X	0
{a, b}	ϕ	X	X	0	ϕ	X	X	0
{a, c}	ϕ	X	X	0	ϕ	X	X	0
{a, d}	ϕ	X	X	0	ϕ	X	X	0

{b, c}	ϕ	X	X	0	ϕ	X	X	0
{b, d}	ϕ	X	X	0	{b, d}	X	{a, c}	0.5
{c, d}	ϕ	X	X	0	ϕ	X	X	0
{a, b, c}	ϕ	X	X	0	ϕ	X	X	0
{a, b, d}	ϕ	X	X	0	{a, b, d}	X	{c}	0.75
{a, c, d}	ϕ	X	X	0	ϕ	X	X	0
{b, c, d}	{b, c, d}	X	{a}	0.75	{b, d}	X	{a, c}	0.5
X	X	X	ϕ	1	{a, b, d}	X	{c}	0.5

Table 4. Comparison between the boundary and accuracy by using Kandil et al. Definition 5.2 [5] and the present Definition 4.2 in case of decreasing sets

A	Kandil'smethod				The current method in Definition 4.2			
	$R_{*dec}(A)$	$R^{dec}(A)$	$BN_{*dec}(A)$	$\alpha^{*dec}(A)$	$R_{*G\text{-}dec}(A)$	$R^{*G\text{-}dec}(A)$	$BN_{*G\text{-}dec}(A)$	$\alpha^{*G\text{-}dec}(A)$
ϕ	ϕ	ϕ	ϕ	0	ϕ	{c}	{c}	0
{a}	ϕ	{a}	{a}	0	ϕ	X	X	0
{b}	ϕ	X	X	0	ϕ	{c}	{c}	0
{c}	ϕ	X	X	0	ϕ	X	X	0
{d}	ϕ	X	{c}	0	ϕ	X	X	0
{a, b}	ϕ	X	X	0	ϕ	X	X	0
{a, c}	ϕ	X	X	0	ϕ	X	X	0
{a, d}	ϕ	X	X	0	ϕ	X	X	0
{b, c}	ϕ	X	X	0	ϕ	X	X	0
{b, d}	ϕ	X	X	0	ϕ	X	X	0
{c, d}	ϕ	X	X	0	ϕ	X	X	0
{a, b, c}	ϕ	X	X	0	ϕ	X	X	0
{a, b, d}	{a, b, d}	X	{c}	0.75	ϕ	X	X	0
{a, c, d}	ϕ	X	X	0	ϕ	X	X	0
{b, c, d}	ϕ	X	X	0	ϕ	X	X	0
X	X	X	ϕ	1	ϕ	X	X	0

Table 5. Comparison between the boundary and accuracy by using Kandil et al. Definition 3.2 [6] and the present Definition 5.2 in case of increasing sets

A	Kandil's method				The current method in Definition 5.2			
	$R_{**inc}(A)$	$R^{**inc}(A)$	$BN_{**inc}(A)$	$\alpha^{**inc}(A)$	$R_{**G\text{-}inc}(A)$	$R^{**G\text{-}inc}(A)$	$BN_{**G\text{-}inc}(A)$	$\alpha^{**G\text{-}inc}(A)$
ϕ	ϕ	ϕ	ϕ	0	ϕ	{a, b}	ϕ	0
{a}	ϕ	X	X	0	ϕ	{a}	{a}	0
{b}	ϕ	X	X	0	ϕ	{b}	{b}	0
{c}	ϕ	{c}	{c}	0	ϕ	X	X	0
{d}	ϕ	X	X	0	{d}	X	{a, b, c}	0.25
{a, b}	ϕ	X	X	0	ϕ	X	X	0
{a, c}	ϕ	X	X	0	ϕ	X	X	0
{a, d}	ϕ	X	X	0	{a, d}	X	{b, c}	0.5

{b, c}	ϕ	{b, c}	{b, c}	0	ϕ	X	X	0
{b, d}	ϕ	X	X	0	{b, d}	X	{a, c}	0.5
{c, d}	{c, d}	X	{a, b}	0.5	ϕ	X	X	0
{a, b, c}	ϕ	X	X	0	ϕ	X	X	0
{a, b, d}	ϕ	X	X	0	{a, b, d}	X	{c}	0.75
{a, c, d}	{a, c, d}	X	{b}	0.75	{a, d}	X	{b, c}	0.5
{b, c, d}	{b, c, d}	X	{a}	0.75	{b, d}	X	{a, c}	0.5
X	X	X	ϕ	1	{a, b, d}	X	{c}	0.75

Table 6. Comparison between the boundary and accuracy by using Kandil et al. Definition 3.2 [6] and the present Definition 5.2 in case of decreasing sets

A	Kandil's method				The current method in Definition 5.2			
	$R_{**dec}(A)$	$R^{**dec}(A)$	$BN_{**dec}(A)$	$\alpha^{**dec}(A)$	$R_{**G\text{-}dec}(A)$	$R^{**G\text{-}dec}(A)$	$BN_{**G\text{-}dec}(A)$	$\alpha^{**G\text{-}dec}(A)$
ϕ	ϕ	ϕ	ϕ	0	ϕ	{c}	{c}	0
{a}	ϕ	{a}	{a}	0	ϕ	X	X	0
{b}	ϕ	{b}	{b}	0	ϕ	X	X	0
{c}	ϕ	X	X	0	ϕ	{c}	{c}	0
{d}	ϕ	X	X	0	{d}	X	{a, b, c}	0.25
{a, b}	ϕ	{a, b}	{a, b}	0	ϕ	X	X	0
{a, c}	ϕ	X	X	0	ϕ	X	X	0
{a, d}	{a, d}	{a, d}	ϕ	1	{d}	X	{a, b, c}	0.25
{b, c}	ϕ	X	X	0	ϕ	X	X	0
{b, d}	ϕ	X	X	0	{d}	X	{a, b, c}	0.25
{c, d}	ϕ	X	X	0	{Sc, d}	X	{a, b}	0.5
{a, b, c}	ϕ	X	X	0	ϕ	X	X	0
{a, b, d}	{a, b, d}	X	{c}	0.75	{d}	X	{a, b, c}	0.25
{a, c, d}	{a, d}	X	{b, c}	0.5	{c, d}	X	{a, b}	0.5
{b, c, d}	ϕ	X	X	0	{c, d}	X	{a, b}	0.5
X	X	X	ϕ	1	{c, d}	X	{a, b}	0.5

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