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On Subdividing Regular Polygons using Structures other than Spidrons and Tiling Patterns Generated by Them

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ABSTRACT

A regular n-sided polygon can be split into n n-part spidrons. In the present paper, it is shown that there exist other linked triangular structures which are distinct from spidrons and which can also be used to subdivide regular polygons. Tiling patterns using such subdivisions are also explored in detail.

General Terms

Tiling, Algorithm, Turbo C++, Program.

Keywords

Spidron, polygon, isosceles, subdivision

1. INTRODUCTION

An n-part spidron is a figure in plane geometry consisting entirely of an alternating sequence of two isosceles triangles, each with its own base angles[2], [6], [8]. The two triangles are juxtaposed in such a way that together they from a third bigger triangle. The sequence is then drawn at an appropriate angle on a smaller scale and the process is repeated ad infinitum. n-part spidrons have been largely studied for n=6 and 8[5]. A regular n-sided polygon can be split into n n-part spidrons[7]. An 8-part spidron is displayed in Figure 1. Erdely[2], has called the same structure a semispidron.

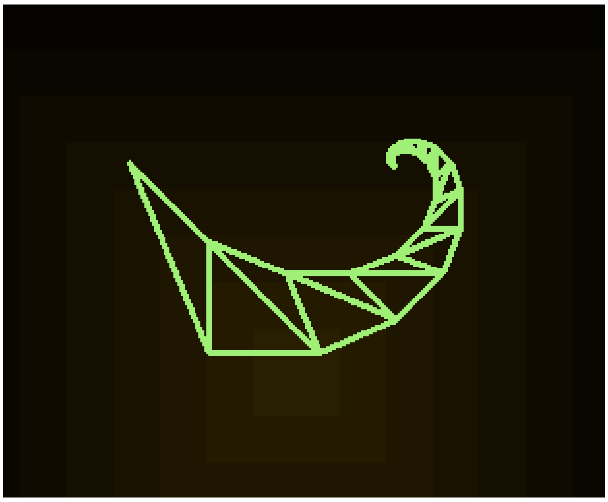

Fig 1 : An eight-part spidron

In earlier papers, the present author has developed an alternate way of constructing individual n-part spidrons so as to algorithmically juxtaposing them to produce a regular n-sided polygon (Gangopadhyay([3]) as well as new symmetric designs which inscribe regular polygons using n 6-part spidrons while exploring several tiling designs with the same (Gangopadhyay([4]). In the present paper, it is shown that there exist other linked triangular structures which are distinct from spidrons and which can also be used to subdivide regular polygons. Tiling patterns using such subdivisions are also explored in detail. These are primarily the distinctive features of this paper.

2. THE ALGORITHM

The basic building blocks for subdividing a regular polygon are linked triangular structures that are drawn recursively on a reduced scale. The process is similar to that of constructing a spidron, though the final outcome is quite distinct.

The algorithm is best explained in terms of Figures 2 and 3, which respectively represents the results of the first and second iterations for n=8. Figure 2 depicts two sets of two juxtaposed isosceles triangles. Let the base angle ABC have value ag and let AB have length s. Then AC=CD=AB=s and angle ACB=ag. Also angle CAD=angle ADC. From this it follows that angle CAD=angle ACD = ag/2. Also Bc=2s(cos(ag)) and AD=2s(cos(ag/2)). With this it is easy to construct the two triangles ABC and ACD in Figure 2. one next notes that BE = s(cos(ag))/cos(ag/2)+2s(cos(ag)cos(ag)/cos(ag/2))) and that CE =CF = s(cos(ag)/cos(ag/2)). Also angle CFE= angle CEF=ag and angle FBC =angle FCB =ag/2. With this it is easy to construct the two triangles BCF and CEF. It is to be noted that for the subdivision of an n-sided regular polygon, ag=360/n.

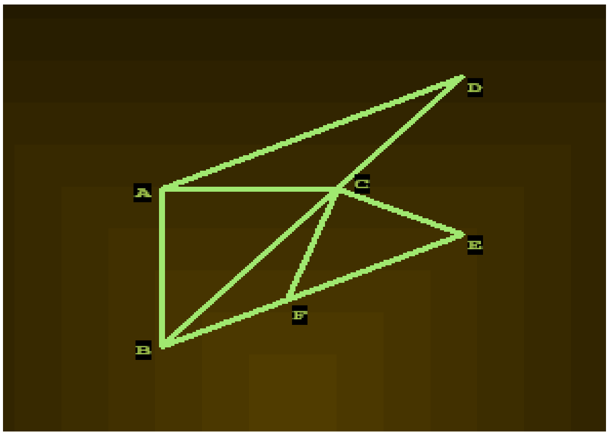

Fig 2 : The first iteration

For the second iteration one notes that the point A is replaced by point F and s is replaced by (s)cos(ag)cos(ag)/(cos(ag/2)*cos(ag/2)). Figure 3 shows the result after the second iteration.

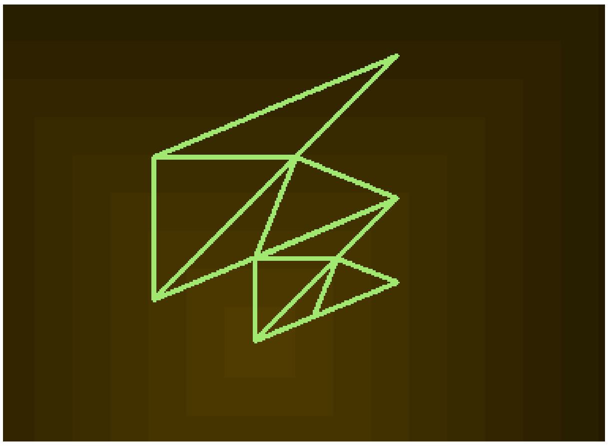

Fig 3 : The second iteration

The final result after 15 iterations is shown in Figure 4. It is to be noted that is quite distinct from an 8-part spidron shown in Figure 1. For convenience, the linked triangular structure in Figure 4 is names a 'ladder'.

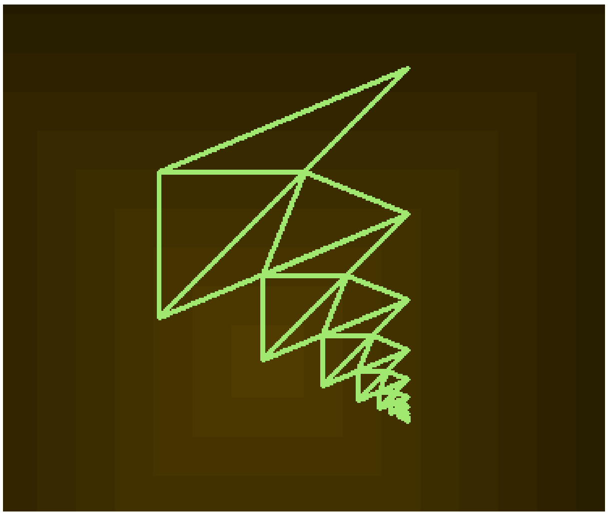

Fig 4 : The Final result

In the next section one submits a programming code in Visual C++ that captures the algorithm and generates figures 2,3 and 4.

3. THE CODE

The code uses a function spid which is declared first. The function has two parameters – s, which gives the value of the two equal sides of the first isosceles triangle and l, which specifies the number of recursive iterations. For s=90 and l=0, 1 and 15, the output of spid is given respectively in Figures 2, 3 and 4. spid2 to create the superimposed polygonal design displayed in figure 4. The functions fd, rt, lt, pu and pd are adapted from turtle graphics[1] and have their usual connotations. The function fd draws a line of specified length, rt and lt respectively rotates the pen right and left by a specified angle, pu puts the pen up(no drawing) and pd puts the pen down. Finally, the for loop in the 'main' segment of the code draws n ladders of the type shown in Figure 4 in a nested manner so that together they make up an n-sided regular polygon. The various functions and the code are given below:

```
float ang=90, px, py, ps=1;
void fd(float dist)
{float hx=cos(ang*3.1415926536/180);
float hy=sin(ang*3.1415926536/180);
float nx=px-hx*dist;
float ny=py-hy*dist;
if(ps!=1)goto label;
line(px,py,nx,ny);
label:px=nx;py=ny;
}
float rt(float l)
{ang+=l;
return ang;}
float lt(float l)
{ang-=l;
return ang;}
void pu()
{ps=0;}
void pd()
{ps=1;}
int n=8;float ag=360/n;
void spid(float,int);
void spid(float s,int l)
{
if(l==0){fd(s);lt(180-ag);fd(s);fd(2*s*cos(ag*3.14/180));
lt(180-ag/2);fd(2*s*cos(ag/2*3.14/180));
lt(180-ag/2);fd(s);rt(180-ag);
pu();fd(s*2*cos(ag*3.14/180));pd();lt(180-ag/2);
fd(s*cos(ag*3.14/180)/cos(ag/2*3.14/180)+2*s*cos(ag*3.14/
180)*cos(ag*3.14/180)/cos(ag/2*3.14/180));
lt(180-ag);
fd(s*cos(ag*3.14/180)/cos(ag/2*3.14/180));lt(2*ag);
fd(s*cos(ag*3.14/180)/cos(ag/2*3.14/180));lt(ag/2);
return;}
spid(s,0);spid((s)*cos(ag*3.14/180)*cos(ag*3.14/180)/(cos(ag
/2*3.14/180)*cos(ag/2*3.14/180)),l-1);
}
void main()
{
 initwindow(1000,800);
float s=90;px=650,py=550;
setlinestyle(SOLID_LINE,0,THICK_WIDTH);
for(int i=0;i<n;i++)
{if(i%2==0)setcolor(2);else setcolor(4);
float a=px,b=py;
```

```
spid(s,15);px=a,py=b;rt(ag/2);pu();
fd(2*s*cos(ag/2*3.14/180)+5);;lt(ag+ag/2);
pd();}
getch();
closegraph();
}
```

The output of the sample code is illustrated in Figure 5 where n has been set to 8, showing how a regular octagon can be subdivided into 8 ladders.

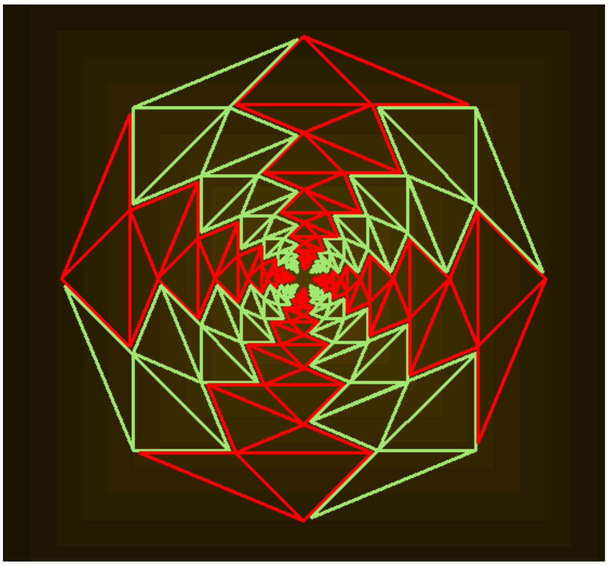

Fig 5 : Output of the sample code, n=8

Figures 6 and 7 depict the respective outputs for n=6 and n=12.

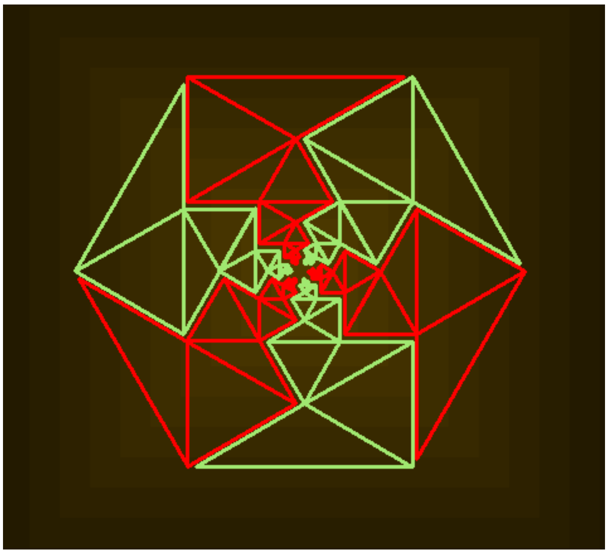

Fig. 6 : Output of the sample code for n=6

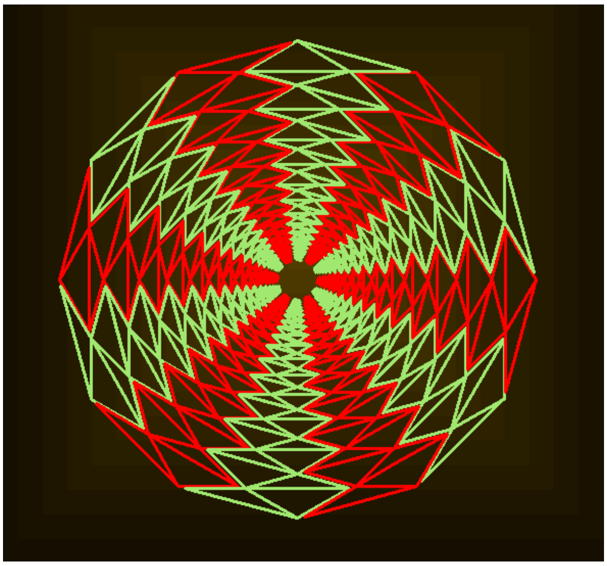

Fig. 7 : Output of the sample code for n=12

5. TILING THE PLANE USING THE SUBDIVIDED POLYGONS

Tiling with regular polygons that have been subdivided through ladders offer several interesting new patterns. These are depicted in the Figures 8-12. Figure 8 shows a tiling pattern that uses the subdivided regular hexagon in Figure 6 as the basic building block. Figures 9 and 10 depict two tiling patterns both created by using the subdivided regular octagon in Figure 5 as the basic building block, In Figure 9 the octagons just touch each other, while in Figure 10 they intersect. Figure 11 shows yet another tiling pattern created with subdivided regular hexagons. Figure 12 shows a tiling pattern that combines the effect of the tiling patterns depicted in Figures 8 and 11.

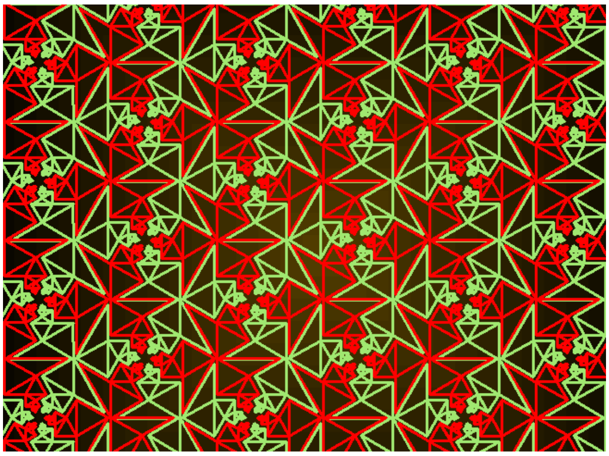

Fig. 8 : Tiling pattern with subdivided regular hexagons

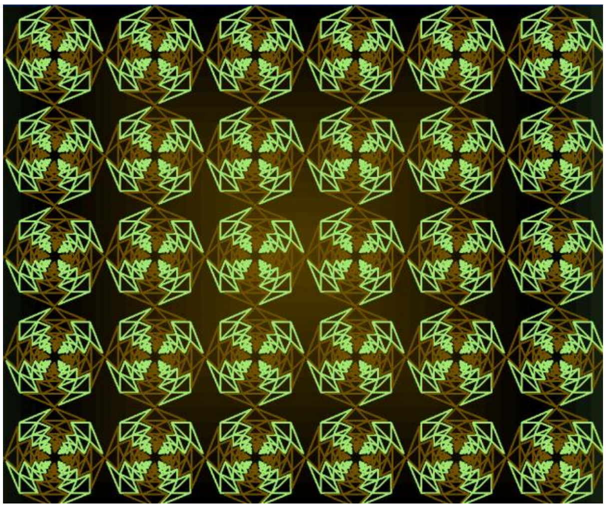

Fig. 9 : Tiling pattern with subdivided regular octagons

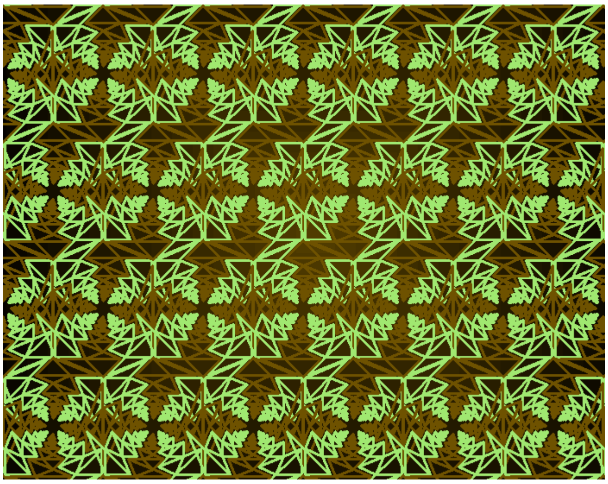

Fig. 10 : Tiling pattern with intersecting subdivided regular octagons

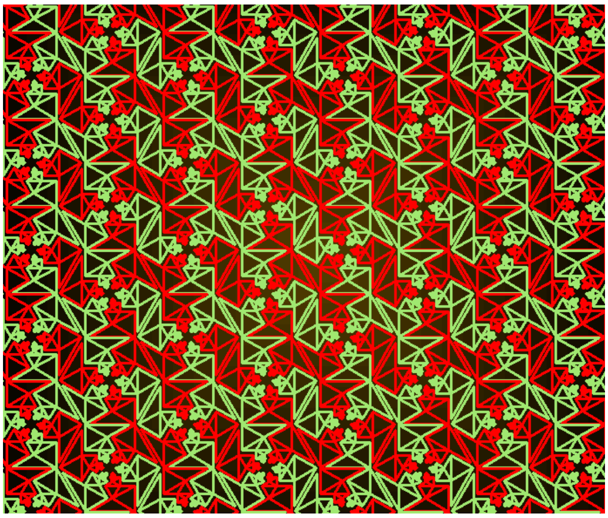

Fig. 11 : Second Tiling pattern with subdivided regular hexagons

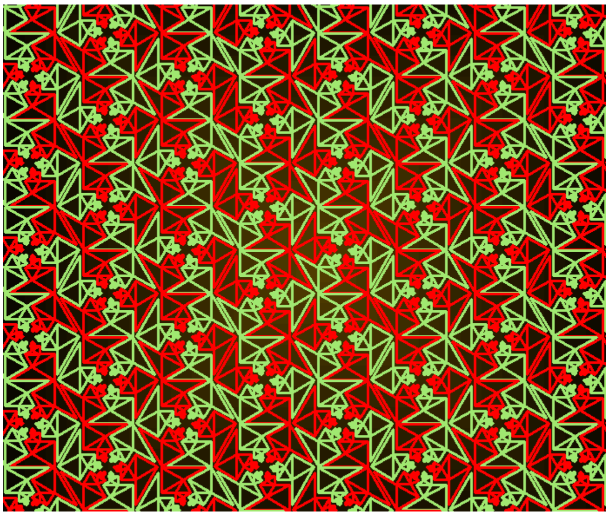

Fig. 12 : A combined tiling pattern using subdivided regular hexagons

6. CONCLUSION

This paper presents the construction of a new linked triangular structure called ladder and shows how regular polygons can also be subdivided using the same. Also tiling patterns using subdivided regular polygon are explored here. In subsequent studies one will explore other such linked triangular structures which the effect of these structures on tiling patterns will also be studied. These are the aspects that would be explored in future work.

7. ACKNOWLEDGMENTS

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8. REFERENCES

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