Effects of Variable Viscosity and Thermal Conductivity on MHD Convective Flow through a Porous Medium

Gopal Chandra Hazarika
Professor, Dept. of Mathematics,
Dibrugarh University,
Dibrugarh, Assam

Kalpana Goswami
Asstt. Prof, Dept. Of Mathematics
Kherajkhat College, Lakhimpur,
Assam, India

ABSTRACT
Effects of variable viscosity and thermal conductivity on MHD convective flow through a porous medium with chemical reaction have been investigated. The governing equations have been expressed in non dimensional form using non-dimensional variables, constants and similarity parameters; the resulting boundary value problem has been solved using shooting method. Velocity, temperature and the concentration profile are presented graphically. The coefficient of skin friction, rate of heat transfer in form of Nusselt number and the coefficient of mass transfer in terms of Sherwood number are also obtained and presented in tabular form. The effects of all the parameters are significant.

Keywords
Heat and Mass Transfer, MHD**,** Variable Viscosity and Thermal Conductivity, Free Convection.

1. INTRODUCTION
There is a wide application of heat and mass transfer processes in engineering sciences. As regards to Magneto hydrodynamics, a number of works has been done on the generalisation of viscous flow and heat transfer solution. Flows through porous medium has got its importance in many branches of engineering such as reservoir engineering, soil mechanics, chemical engineering etc.. Convection problems of electrically conducting fluid have got much importance because of its various applications in Geophysics and Engineering, Missile technology etc.. Due to rapid growth in the fluid mechanics research and its importance in many branches of science, in recent years a remarkable attention has been shown on the study of Thermal convection in porous medium. . Problems through porous medium are heavily based in Darcy's experimental law (1857). MHD principle finds its application in medicine and biology also. The present stage of MHD is due to the pioneer contribution of notable authors like Cowling [1], Shercliff [2], Ferraro and Plumpton [3] and Cramer and Pai[4]. Lai and Kulacki[5] has carried out model studies on the problems of free convective flow of incompressible viscous fluid taking in to account the thermal radiation.

The heat and mass transfer effects on a flow along a vertical plate in presence of magnetic field was investigated by Elbashbeshy [6]. Kafousias et al. [7] have studied free connective flow past vertical plates with suction . Jha and Singh[8] presented an analytical study for free convection and mass transfer flow past an infinite vertical plate moving impulsively in its own plane taking Soret effects into account. Ramana Kumari and Bhaskara Reddy [9] have studied MHD free convective flow of a viscous incompressible electrically conducting fluid past an infinite vertical porous plate with variable suction. Hayat et al.[10] analyzed a mathematical model in order to study the heat and mass transfer characteristics in mixed convection boundary layer flow about a linearly stretching vertical surface in a porous medium filled with a visco-elastic fluid, by taking into account the diffusion thermo (Dufour) and thermal-diffusion (Soret) effects. The experimental investigation of the thermal diffusion effect on mass transfer related problems was first performed by Charles Soret in 1879. Eckert and Drake (1972) have emphasized that the Soret effect assumes significant role in cases concerning isotope separation and in mixtures between gases with very light molecular weight (H_2, He) and the medium molecular weight (N_2, air).

In many times chemical reaction plays an important role in chemical industry. The study of effects of chemical reaction on heat and mass transfer in a flow is important to the engineers and scientists because of its occurrence in many branches of science and technology. In processes such as drying , distribution of temperature and moistures over agricultural fields, energy transfer in a wet cooling tower and flow in a cooler heat and mass transfer occur simultaneously. Many investigators have studied the effect of chemical reaction in different heat and mass transfer problems of whom the names of Muthecumaraswamy and Meenakshisundaram [11], Hazarika[12] and Hazarika[13]. In view of the importance of the combined effects of magnetic field,thermal diffusion and chemical reaction Ahmed and Talukdar[14]have studied the problem of two dimensional MHD free convective poiseuille flow through a porous medium bounded by two infinite vertical porous plates with chemical reaction taking into account the Soret effect.

The aim of the present work is to study the effects of variable viscosity and thermal conductivity to the problem discussed by Ahmed and Talukdar [14]. Solutions of the governing flow equations are obtained by Shooting method. The effects of the viscosity and thermal conductivity parameters on velocity, temperature, concentration, rate of heat transfer and rate of mass transfer are investigated. The effects of various parameters are quite significant.

2. MATHEMATICAL FORMULATIONS
Consider a steady, free convection flow of an incompressible viscous electrically conducting and chemical reacting fluid through porous medium bounded by two infinite vertical porous plates. Let distance between the plates be 'h' and a uniform transverse magnetic field is applied in the y-direction under the following assumptions:

(i)All fluid properties are considered constant except the density in the buoyancy force term ,the viscosity and thermal conductivity.

(ii)The Ohmic dissipation term in the energy equation is negligible.

(iii)The magnetic Reynolds number is so small that the induced magnetic field can be neglected in comparison to the applied magnetic field.

(iv)The plates are isothermal and electrically non conducting.

(v)No external electric field is applied for which $\bar{E}=\bar{0}$.

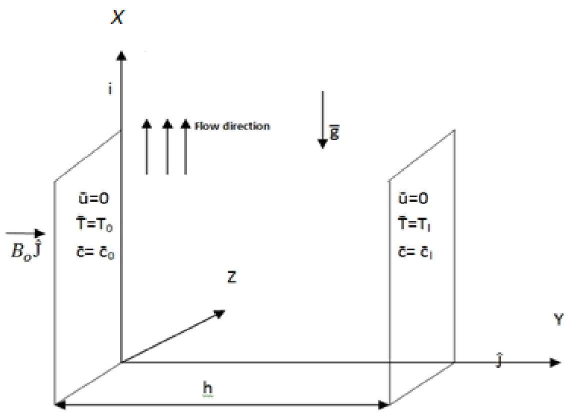

Fig-1: Flow Configuration

Let us consider a co-ordinate system with x−axis vertically upwards along the length of the plate, y-axis perpendicular to it directed into the fluid region and z-axis normal to the plate. Let $\bar{q}=u\hat{\imath}+v\hat{\jmath}$ be the fluid velocity at a point $(\bar{x},\bar{y},\bar{z})$ and $\bar{B}=B_0\hat{\jmath}$ be applied magnetic field, $\hat{\imath}$, $\hat{\jmath}$ be the unit vectors along x-axis and y-axis respectively. As the plates are of infinite length, all physical quantities except pressure are independent of $\bar{x}$.

The governing equations are:

Equation of continuity: div $\bar{q}=0$ (1)

Gauss law of magnetism: div $\bar{B}=0$ (2)

Momentum equation:

$$\left(\vec{q}.\vec{\nabla}\right)\vec{q}=-\frac{1}{\rho}\vec{\nabla}p+\frac{\vec{J}\times\vec{B}}{\rho}+\upsilon\nabla^2\vec{q}+\vec{g} \qquad (3)$$

Energy equation:

$$\rho c_p\left[\left(\vec{q}.\vec{\nabla}\right)\bar{T}\right]=k\nabla^2\bar{T}+\varphi+\frac{\vec{J}^2}{\sigma} \qquad (4)$$

Species continuity Equation:

$$\left(\vec{q}.\vec{\nabla}\right)\bar{C}=D_m\nabla^2\bar{C}+D_T\nabla^2\bar{T} \qquad (5)$$

Ohm's law:

$$\bar{J}=\sigma[\bar{E}+\bar{q}\times\bar{B}] \qquad (6)$$

All the physical quantities are defined in the nomenclature.

With the above assumptions and under the usual boundary layer conditions the equations are reduces to:

The equation of continuity gives $\frac{d\bar{v}}{d\bar{y}}=0$

So, $\bar{v}$ = -v_0 (say) velocity of suction or injection. (7)

The momentum equation:

$$\overrightarrow{(q.\nabla)}\,\vec{q}=-\frac{1}{\rho}\vec{\nabla}p+\frac{\vec{J}\times\vec{B}}{\rho}+\frac{1}{\rho}\nabla.(\mu\nabla\vec{q})+\vec{g} \qquad (8)$$

Now, $\bar{g}$ is the body force per unit volume due to gravity i.e. buoyancy force or lift force per unit volume.

Now, $\bar{g}=g\beta(\bar{T}-\bar{T}_s)+g\bar{\beta}(\bar{c}-\bar{c}_s)+\frac{\upsilon}{K}\bar{q}$

$=g\beta(\bar{T}-\bar{T}_s)+g\bar{\beta}(\bar{c}-\bar{c}_s)-\frac{\upsilon}{K}\bar{u}$

Therefore, the momentum equation becomes

$$-v_0\frac{d\bar{u}}{d\bar{y}}=\frac{1}{\rho}\frac{d\mu}{d\bar{y}}\frac{d\bar{u}}{d\bar{y}}+\frac{\mu}{\rho}\frac{d^2\bar{u}}{d\bar{y}^2}+g\beta(\bar{T}-\bar{T}_s)+g\bar{\beta}(\bar{c}-\bar{c}_s)-\frac{\upsilon}{K}\bar{u}-\frac{\sigma}{\rho}B_0^2\bar{u} \qquad (8),$$

The energy equation : The equation of energy for steady motion of an incompressible viscous electrically conducting fluid in presence of a magnetic field can be written as,

$$\rho C_p[\overrightarrow{(q.\nabla)}\vec{T}]=\nabla.(\lambda\nabla\vec{T})+\emptyset+\frac{\vec{J}^2}{\sigma}$$

In our problem, the energy equation becomes,

$$-v_0\frac{d\bar{T}}{d\bar{y}}=\frac{1}{\rho c_p}\frac{d\lambda}{d\bar{y}}\frac{d\bar{T}}{d\bar{y}}+\frac{\lambda}{\rho c_p}\frac{d^2\bar{T}}{d\bar{y}^2}+\frac{\upsilon}{c_p}\left(\frac{d\bar{u}}{d\bar{y}}\right)^2+\frac{\sigma}{\rho c_p}(\bar{u}^2+\bar{v}^2)B_0^2 \qquad (9)$$

Species continuity equation: In our problem the species continuity equation with chemical reaction becomes,

$$-v_0\frac{d\bar{c}}{d\bar{y}}=\frac{d}{d\bar{y}}\left(D_M\frac{d\bar{c}}{d\bar{y}}\right)+\frac{d}{d\bar{y}}\left(D_T\frac{d\bar{T}}{d\bar{y}}\right)+\xi(\bar{c}_s-\bar{c}) \qquad (10)$$

where,
g is the acceleration due to gravity
$\bar{T}$ is the temperature
$\bar{T}_s$ is the temperature at the static condition
$\bar{c}$ is the species concentration
$\bar{c}_s$ is the concentration at the static condition
υ is the kinematic viscosity
k is the permeability of the porous media
β is the coefficient of volume expansion for heat transfer
$\bar{\beta}$ is the coefficient of volume expansion for mass transfer
λ *is the* thermal conductivity.

The relevant boundary conditions are,

$\bar{y}=0$: $\bar{u}=0, \bar{T}=\bar{T}_0, \bar{c}=\bar{c}_0$ (11)
$\bar{y}=h$: $\bar{u}=0, \bar{T}=\bar{T}_l, \bar{c}=\bar{c}_l$ (12)

To normalize the flow model, we introduce the following non-dimensional variables and similarity parameters:

$$\left.\begin{aligned} & y=\frac{\bar{y}}{h},\ u=\frac{\bar{u}}{v_0},\ \theta=\frac{\bar{T}-\bar{T}_s}{\bar{T}_0-\bar{T}_s},\ \varphi=\frac{\bar{C}-\bar{C}_s}{\bar{C}_0-\bar{C}_s}, \\ & \Pr=\frac{\mu C_p}{k},\ So=\frac{D_T\left(\bar{T}_0-\bar{T}_s\right)}{\nu\left(\bar{C}_0-\bar{C}_s\right)},\ Sc=\frac{\nu}{D_M}, \\ & M=\frac{\sigma B_0{}^2h^2}{\mu},\ G_r=\frac{g\beta h\left(\bar{T}_0-\bar{T}_s\right)}{v_0{}^2}, \\ & m=\frac{\bar{T}_l-\bar{T}_s}{\bar{T}_0-\bar{T}_s},\ n=\frac{\bar{C}_l-\bar{C}_s}{\bar{C}_0-\bar{C}_s}, \end{aligned}\right\} \qquad (13)$$

We use the viscosity of the fluid as a linear function of the temperature as given by Lai and Kulacki [5]

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}\left[1+\delta\left(\bar{T}-\bar{T}_\infty\right)\right]$$

In a similar manner the thermal conductivity k is also taken as

$$\frac{1}{k} = \frac{1}{k_\infty}[1+\zeta(\bar{T}-\bar{T}_\infty)]$$

From the equation [13], $\mu = \frac{1}{b(\bar{T}-\bar{T}_r)}$,where b=$\frac{\delta}{\mu_\propto}$,$\bar{T}_r = \bar{T}_\propto - \frac{1}{\delta}$, where $\mu_\propto$ is the free stream viscosity , b and $\bar{T}_\propto$ are constants whose values depend upon the reference state and thermal property of
the fluid , $\bar{T}_r$ is transformed reference temperature related to viscosity parameter, δ is a constant based on the thermal property of the fluid and b<0 for gas and b>0 for liquid.
And from equation [14], $\lambda = \frac{1}{c(\bar{T}-\bar{T}_c)}$, where $\bar{c} = \frac{\beta}{\lambda_\propto}, and\ \bar{T}_c = \bar{T}_\propto - \frac{1}{\beta}$, where c and $\bar{T}_c$ are constants and their values depend upon the reference state and thermal properties of the fluid i.e. β.
The dimensionless temperature $\theta(\eta)$ is defined as
$\theta(\eta) = \frac{(\bar{T}-\bar{T}_s)}{(\bar{T}_0-\bar{T}_s)}$
Hence, $\mu = -\frac{\mu_\propto \theta_r}{\theta-\theta_r}$
and, $\lambda = -\frac{\lambda_\propto \theta_c}{\theta-\theta_c}$

Finally the dimensionless governing equations and relevant boundary conditions are:
The momentum equation:

$$-\frac{du}{dy} = \frac{1}{R_e}\frac{\theta_r}{(\theta-\theta_r)^2}\frac{d\theta}{dy}\frac{du}{dy} - \frac{1}{R_e}\frac{\theta_r}{(\theta-\theta_r)}\frac{d^2u}{dy^2} + G_r\theta + G_m\varphi - \frac{u}{R_e\alpha} - \frac{M}{R_e}u \quad (15)$$

The energy equation:

$$-\frac{d\theta}{dy} = \frac{1}{R_eP_r}\frac{\theta_c}{(\theta-\theta_c)^2}(\frac{d\theta}{dy})^2 + \frac{1}{R_eP_r}\frac{d^2\theta}{dy^2} + \frac{Ec}{R_e}(\frac{du}{dy})^2 + \frac{MEc}{R_e}(1+u^2) \quad (16)$$

The species continuity equation in non-dimensional form is,

$$-\frac{d\varphi}{dy} = \frac{1}{R_eS_c}\frac{d\theta}{dy}\frac{d\varphi}{dy} + \frac{1}{R_eS_c}\frac{d^2\varphi}{dy^2} + \frac{S_0}{R_e}\frac{\theta_r}{(\theta-\theta_r)^2}(\frac{d\theta}{dy})^2 + \frac{S_0}{R_e}\frac{d^2\theta}{dy^2} - ChR_e\varphi \quad (17)$$

with boundary conditions,
y=0: u=0, $\theta = 1, \varphi = 1$ (18)
y=1: u=0, $\theta = m, \varphi = n$ (19)

The physical quantities of interest in this problem are the skin friction coefficient C_f, Nusselt number Nu and Sherwood number Sh which indicates physically wall shear stress, rate of heat transfer and rate of mass transfer respectively. These are expressed as below:

Co-efficient of skin friction:

The skin friction in the non-dimensional form on the plate y=0 is given by

$\mathrm{C_f} = \frac{2\,\tau_w}{\rho\, v_0^2}\Big)_{\bar{y}=0}$, where $\tau_w = \mu\frac{\partial\bar{u}}{\partial\bar{y}}\Big)_{\bar{y}=0}$ is the shearing stress

$$= 2\mu\left(\frac{\partial\bar{u}}{\partial\bar{y}}\right)_{\bar{y}=0}\left(\frac{1}{\rho v_0^2}\right)_{\bar{y}=0}$$

$$= 2\left(\frac{\mu}{\rho v_0^2}\right)_{y=0}\left(\frac{v_0}{h}\frac{\partial u}{\partial y}\right)_{y=0}$$

$$= \left[\frac{2}{Re}\frac{\partial u}{\partial y}\right]_{y=0}$$

$= \left[\frac{2}{Re}\frac{du}{dy}\right]_{y=0}$,where Re is the Reynolds number.

Co-efficient of rate of heat transfer:

The Co-efficient of rate of heat transfer from the plate to the fluid in terms of Nusselt number in the non-dimensional form on the plate y=0 is given by

$Nu = \frac{hq_w}{k(\bar{T}_0-\bar{T}_s)}$, where q_w is the heat transfer from the plate is given by
$q_w = -k\frac{\partial\bar{T}}{\partial\bar{y}}\Big)_{\bar{y}=0}$
Using the non-dimensional variables, we get

$$Nu = -\frac{d\theta}{dy}\Big]_{y=0}$$

Co-efficient of rate of mass transfer:

The mass flux at the plate is given by $M_w = -D_M\left[\frac{\partial\bar{c}}{\partial\bar{y}}\right]_{\bar{y}=0}$

The Co-efficient of rate of mass transfer from the plate y=0 to the fluid in terms of Sherwood number in non-dimensional form is given by,

$$S_h = \frac{-D_M}{v_0(\bar{c}_0-\bar{c}_s)}\left(\frac{\partial\bar{c}}{\partial\bar{y}}\right)_{\bar{y}=0}$$

$$= -\frac{1}{S_cR_e}\frac{d\theta}{dy}\Big)_{y=0},$$

where, $S_c = \frac{v}{D_M}$ is the Schmidt number.
$R_e = \frac{v_0h}{v}$ is the Reynolds number.

3. METHOD OF SOLUTION

Numerical solutions are obtained for the velocity field (u), temperature field (θ) and concentration field $(\emptyset)$. Calculations are carried out for different values of magnetic parameter (M), thermal conductivity parameter (θ_c), viscosity parameter (θ_r), Reynolds number (Re), Schmidt number (S_c), Soret number (So) on the velocity, temperature and species concentration.

The values of the parameters are taken as

Re =.75 ; M = 1 ; Pr=.7; θ_c= -14 ; θ_r=-12 ; s_0 = 1; Gr=5 ; Gm=3 ; Sc = .22 ; Ec = .05 ; α = .5 ; ch = .4 , unless otherwise stated.

The velocity profile against y are presented in figures 1, 2, 3 respectively for different values of magnetic parameter (M), thermal conductivity parameter (θ_c), viscosity parameter (θ_r).

From figure it is clear that low viscosity or thermal diffusivity causes the fluid velocity to increase whereas due to application of the transverse magnetic field fluid motion is retarded due to thermal conductivity.

Figures 4, 6, 7 exhibit the variation of temperature field θ versus y under the influence of Reynolds number Re, Hartmann number M and thermal conductivity parameter θ_c.

From the figure it is observed temperature increases with the increasing values of Re and M, but there is a decrease with the decreasing values of θ_c(thermal conductivity parameter).

Figures 5,8,9 present the variation of the species concentration $\emptyset$ under the influence of chemical reaction Ch, Schmidt number Sc and Soret number So. The figures exhibit that species concentration decreases with the increase of chemical reaction and Schmidt number while it increases for increasing values of Soret number.

Table-1 to Table-5 give the computed values of the missing initial values namely $u'(0)$, θ' (0) and $\emptyset'$ (0) and the co-efficient of skin friction c_f, Nusselt number Nu and the Sherwood number Sh at the plate y=0 for various values of Ch, M, Sc, θ_c and θ_r.

From these tables it is clear that the skin friction decreases with the increasing values of viscosity parameter, thermal conductivity, chemical reaction, magnetic parameter and Schmidt number whereas Nusselt number increases with the increasing values of chemical reaction , Schmidt number, thermal conductivity but decreases with the magnetic and viscosity parameter.

Further it is seen that rate of mass transfer increases with the increase of Schmidt number (Sc), thermal conductivity (θ_c) and viscosity parameter (θ_r) but decreases with the increase of chemical reaction and magnetic parameter.

4. CONCLUSION

Finally based on the above study we may conclude as below:

1. Magnetic field ,Chemical reaction and viscosity have retarding effect in the flow.

2. The temparature falls due to chemical reaction and thermal conductivity , but rises due to Reynold's number and Magnetic field.

3. The species concentration decreases due to Schmidt number but increases due to Soret number.

4. The coefficient of skin friction (which represents the wall shear stress of fluid) decreases due to viscosity, conductivity, chemical reaction, Magnetic field and Schimdt number.

5. The rate of heat transfer decreases due to chemical reaction ,Magnetic field and viscosity whereas it increases due to thermal conductivity and Schimdt number.

6. Rate of mass transfer decreases due to chemical reaction and magnetic field whereas it increases due to Schimdt number, viscosity and thermal conductivity parameter.

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6. APPENDIX

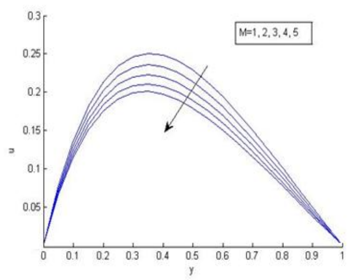

Fig.2: Velocity profile for different values of M

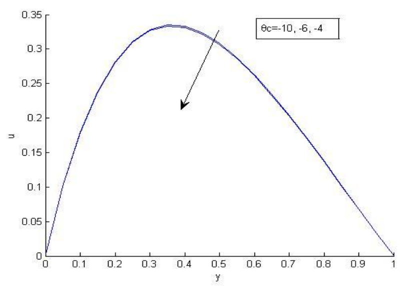

Fig.3: Velocity profile for different θ_c

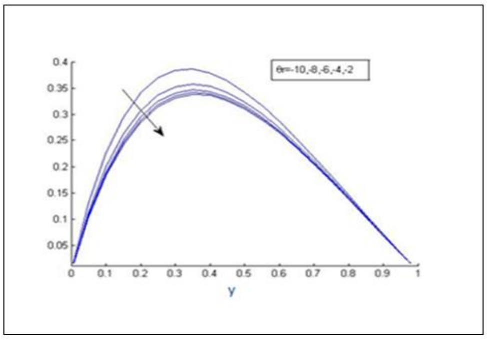

Fig – 4: Velocity profile for different θ_r

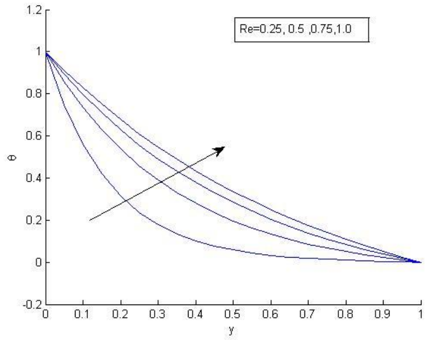

Fig-5: Temperature profile for different R_e

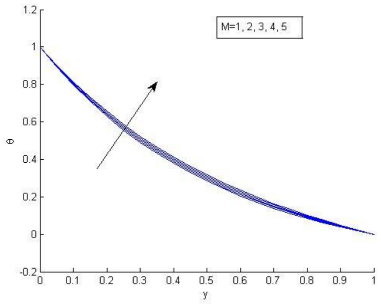

Fig – 6: Temperature profile for different M

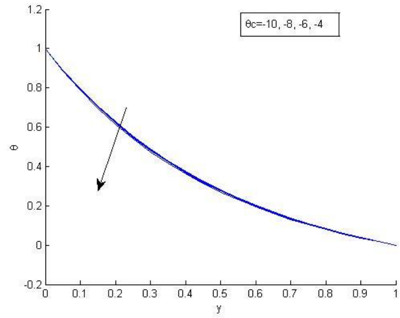

Fig-7: Temperature profile for different θ_c

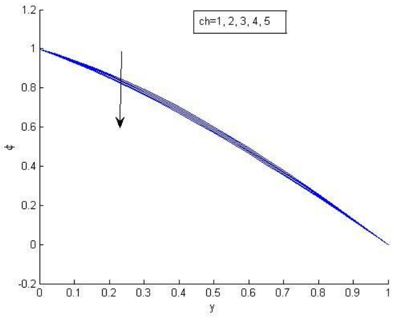

Fig –8:Species concentration profile for different Ch

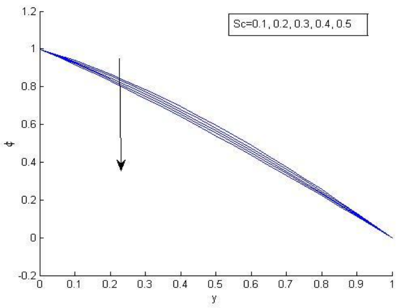

Fig-9: Species concentration profile for different S_c

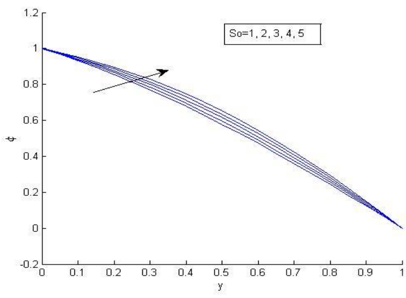

Fig – 10: Species concentration profile for different S_0

Table-1

Ch	$u'(0)$	$\theta'(0)$	$\emptyset'(0)$	C_f	Nu	Sh
1.00	1.754573	-2.240041	-0.540421	3.740320	2.586576	-2.618285
2.00	1.748625	-2.240273	-0.574856	3.727640	2.586845	-2.785118
3.00	1.742799	-2.240499	-0.608826	3.715221	2.587106	-2.949699
4.00	1.737092	-2.240720	-0.642346	3.703055	2.587360	-3.112099
5.00	1.731500	-2.240934	-0.675428	3.691133	2.587608	-3.272381
6.00	1.726019	-2.241143	-0.708087	3.679449	2.587849	-3.430608

Table-2:

M	$u'(0)$	$\theta'(0)$	$\emptyset'(0)$	C_f	Nu	Sh
1.00	1.737092	-2.240720	-0.642346	3.703055	2.587360	-3.112099
2.00	1.667400	-2.193575	-0.649244	3.554488	2.532922	-3.145522
3.00	1.606262	-2.146377	-0.656327	3.424156	2.478423	-3.179836
4.00	1.552096	-2.099138	-0.663589	3.308688	2.423876	-3.215021
5.00	1.503695	-2.051869	-0.671028	3.205508	2.369295	-3.251061
6.00	1.460117	-2.004580	-0.678641	3.112612	2.314689	-3.287947

Table 3:

Sc	$u'(0)$	$\theta'(0)$	$\emptyset'(0)$	C_f	Nu	Sh
0.10	1.749342	-2.240245	-0.567208	3.729168	2.586812	-6.045745
0.20	1.739101	-2.240642	-0.629843	3.707336	2.587271	-3.356675
0.30	1.729188	-2.241022	-0.692281	3.686205	2.587709	-2.459621
0.40	1.719592	-2.241385	-0.754528	3.665749	2.588129	-2.010586
0.50	1.710301	-2.241733	-0.816590	3.645942	2.588530	-1.740769

Table-4:

M	$u'(0)$	$\theta'(0)$	$\emptyset'(0)$	C_f	Nu	Sh
1.00	1.737092	-2.240720	-0.642346	3.703055	2.587360	-3.112099
2.00	1.667400	-2.193575	-0.649244	3.554488	2.532922	-3.145522
3.00	1.606262	-2.146377	-0.656327	3.424156	2.478423	-3.179836
4.00	1.552096	-2.099138	-0.663589	3.308688	2.423876	-3.215021
5.00	1.503695	-2.051869	-0.671028	3.205508	2.369295	-3.251061

Table 5:

θ_r	$u'(0)$	$\theta'(0)$	$\emptyset'(0)$	C_f	Nu	Sh
-12.00	1.737092	-2.240720	-0.642346	3.703055	2.587360	-3.112099
-10.00	1.760063	-2.240309	-0.640597	3.695174	2.586886	-3.056601
-8.00	1.794395	-2.239689	-0.638056	3.683537	2.586170	-2.976823
-6.00	1.851296	-2.238648	-0.634029	3.664615	2.584967	-2.852389
-4.00	1.963961	-2.236532	-0.626672	3.628459	2.582524	-2.631341
-2.00	2.294018	-2.229943	-0.608991	3.531872	2.574916	-2.130917