ABSTRACT
This paper presents a modeling and control of the nonlinear full vehicle active suspension system with passenger seat utilizing PID with artificial bee colony (ABC) algorithm technique. Five PID controllers are used. The main objective of designing the controller is to improve the performance of suspension system, while the aim of suspension system in automobiles is to isolate the road disturbance experienced by the tires from being transmitted to the passengers. The effects of the nonlinear forces which come from damper, spring, actuator and parametric uncertainty in the spring, damper and actuator have been considered, therefore robust control is utilized. The MATLAB environment is utilized to determine the performance of the proposed control scheme. A comparison is performed to illustrate the effectiveness of PID-ABC controller in terms of modifying the ride comfort and the safety of travelling passengers.

Keywords
Artificial bee colony algorithm, PID control tuning, eight degrees of freedom vehicle model, non linear active suspensions, Matlab/Simulink, simulation.

1. INTRODUCTION
Vehicles are complex mechanical systems with many degrees of freedom and may incorporate linear or nonlinear springs and various types of damping characteristic as suspension system. The primary function of a vehicle suspension system is to isolate passengers and chassis from the roughness of the road to provide a more comfortable ride and to increase the road handling [1,2].

Several engineers and researchers in the automotive industry field devoted particular attention to discuss the problem of vehicle suspension control in order to develop the characteristics of both ride comfort and driving safety. The developing on suspension system starts from passive suspension to intelligent suspension system. The intelligent or active suspension system includes control strategies, sensor technology and actuators. By using this arrangement, the significant achievements in vehicle response can be carried out [3,4].

Many types of actuators can be connected in parallel with the passive elements such as: hydraulic actuator, magnetorheological actuator or pneumatic actuator. In this work the hydraulic actuators are used. The actuator supply additional force to the suspension system in order to eliminate the vibrations coming from rough roads, this force is determined by a feedback control law utilizing data from sensors that are attached to vehicle[1,5].

Due to fast developments in the control technology, electronically controlled suspension has gained more interest to improve the riding requirements. Many types of controller systems such as adaptive control, LQR, LQG, and $H_\infty$ control are developed for vehicle suspensions model with active components to obtain more comfortable riding for passengers and guarantee road handling for the vehicle[1,2].

In this work the PID controller is utilized for modifying the performance of suspension system. Though the advanced and intelligent control algorithms have been developed over the past several years, however, the proportional, integral and derivative (PID) controller remains the most popular kind of feedback controller in use today [6].

In refer to the studies, It is found that approximately 90% of all industrial controllers are of the PID-type, it is most widely utilized in the autopilots for aircrafts, chemical process industry, electrical and electronic systems and industrial robots. The popularity of PID controller is due to the simplicity of their control law, robustness in a wide range of operating conditions and few tuning parameters are needed [6,7].

The PID controller computation involves three parameters, its performance completely depends on the tuning of these parameters, and the task of tuning these parameters properly for the PID controller is quite difficult. So in spite of the popularity of PID controller but it is often poorly tuned in practice [8,9].

Many efforts have been made to overcome this problem, where different techniques have been introduced to tune the parameters of PID controller, such as Ziegler-Nichols method, manual tuning, Cohen-Coon method [10].

On the other hand, these classical methods have some drawbacks or disadvantages, where the Cohen-Coon method is utilized only for first order models including large process delays, while the manual tuning method is that it requires experienced personal. Some shortcoming of the Ziegler and Nichols tuning method is that mostly utilized to determine optimal PID gains for the plant with certain operating point. However, if the parameters of the plant are changed because of uncertainty or nonlinearity, those methods provide poor performance and the controller becomes not robust and not in optimum state [6,11].

In recent decade, designer engineers have focused on evolutionary based approaches to improve the existing design theories to tune the parameters of PID controllers. Therefore, several optimization methods have been proposed to optimize the parameters of PID controller. The emergence of
intelligence and optimization algorithms such as genetic algorithm (GA), Particle Swarm Optimization (PSO) method, Ant Colony Optimization (ACO) method, provides new techniques to tuning the PID parameters successfully [6,8].

In this work, the mathematical model for full vehicle nonlinear active suspension systems with 8 degrees of freedom included a passenger seat with hydraulic actuators has been derived to take into account all the motions of the vehicle and the nonlinearity behaviors of active suspension system and hydraulic actuators. A new optimization algorithm ABC is proposed to optimize the parameters of PID controller to design a robust control scheme for full vehicle nonlinear active suspension model. The proposed controllers are used to improve the riding comfort and road handling during various maneuvers (travelling, braking and cornering). The performances of the proposed controller are compared with passive suspension using computer simulations through the MATLAB and SIMULINK toolbox.

2. MATHEMATICAL MODEL FOR FULL VEHICLE NONLINEAR ACTIVE SUSPENSION SYSTEMS WITH PASSENGER SEAT

A full vehicle model with eight degrees of freedom (8DOF) is considered for analysis to investigate the problem of balancing riding comfort and road handling, where a passenger seat is included in the vehicle model to predict the response of the passenger due to a road disturbances as shown in Figure(1). A number of researchers deal with the suspension models as linear system by ignoring the nonlinearities behavior of suspension systems in order to simplify models. But on the other hand, the results become more realistic when take into account the effects of nonlinearities behavior of suspension systems such as dry friction on dampers and springs. Therefore, in this work, the effects of the nonlinearities in forces which came from damper, spring and actuator have been taken into account.

The full vehicle nonlinear active suspension system with a passenger seat model used in this study is shown in Figure (1), consisting of passenger seat, sprung mass ($M_2$) referring to the part of the car that is supported on springs and unsprung mass ($M_{usi}$) which refers to the mass of wheel assembly or in the other word the unsprung mass is total mass of the components under the suspension system. The tires have been replaced with their equivalent stiffness and damping. The tires are modeled by linear springs in parallel with linear dampers, while the suspension systems and passenger seat consists the nonlinear hydraulic actuators which are connected in parallel with nonlinear springs and nonlinear dampers. The spring in model of the tire and suspension system has stiffness coefficient labeled as $k_{st}$, $k_{si}$ respectively, and the damper has damping coefficient labeled $c_{st}$, $c_{si}$.

In order to control vehicle body motions, the vehicle model sprung mass is considered to have 3DOF. These three types of motions: bounce motion, pitch motion and roll motion, while passenger seat and four unsprung mass have 1DOF for each. The mathematical model was derived based on the work done in [12,13].

Based on the Newton’s third law of motion the differential equations of the full vehicle nonlinear active suspension system can be obtained as explained below.

The heave motion of sprung mass can be written as follows

$$M_2 \ddot{z}_c = - \sum_{i=1}^{5} F_{ksi} - \sum_{i=1}^{5} F_{csi} + \sum_{i=1}^{5} F_i$$  \hspace{1cm} (1)

Where:

$F_{ksi}$ is the nonlinear forces of $i^{th}$ spring

$F_{csi}$ is the nonlinear forces of $i^{th}$ damper

$F_i$ is the applied nonlinear forces between the sprung mass and unsprung masses which is generated from $i^{th}$ hydraulic actuator. Those nonlinear forces can be written as

$$F_{ksi} = K_{si}(x_{si} - z_{usi}) + \zeta K_{si}(x_{si} - z_{usi})^3$$  \hspace{1cm} (2)

$$F_{csi} = C_{si}(x_{si} - z_{usi}) + \zeta C_{si}(x_{si} - z_{usi})^2 \text{sgn}(x_{si} - z_{usi})$$  \hspace{1cm} (3)

$$F_i = F_{hi} - F_{fi}$$  \hspace{1cm} (4)

Where:

$z_{usi}$ is the vertical displacements of unsprung masses

$\zeta$ is the empirical operator

$F_{hi}$ is the $i^{th}$ nonlinear hydraulic real force

$F_{fi}$ is the $i^{th}$ nonlinear frictional force

The force $F_{hi}$ that generated by the $i^{th}$ hydraulic actuators can be written as

$$F_{hi} = A_p P_{li}$$  \hspace{1cm} (5)

Where $P_{li}$ the pressure across the $i^{th}$actuator’s piston or the load pressure, can be written this pressure equation in term of spool valve displacement $x_{pi}$ as bellow

$$P_{li} = -\beta P_{li} - \sigma A_p x_{pi} + \sigma C d \omega x_{vi} \frac{1}{2} (P_{li} - \text{sgn}(x_{pi})P_{li})$$

$$P_{li} = \mu \text{sgn}(x_{pi})$$ for $|x_{pi}| \geq 0.01$$

$$P_{li} = \mu \sin\left(\frac{x_{pi}}{0.02}\right)$$ for $|x_{pi}| < 0.01$$

Where $\mu$ is the empirical operator

Then the rolling motions of the sprung mass can be given as

$$J_2 \ddot{\theta} = (\gamma F_{ksi} - F_{ksi^2} - F_{ksi^3} + F_{ksi^5}) \frac{b_2 F_{ksi^5}}{2} + b_2 F_{ksi^5}$$

$$((F_{csi} - F_{csi^2} - F_{csi^3} - F_{csi^5}) \frac{b_2 F_{ksi^5}}{2} + b_2 F_{ksi^5})$$

Where:

$J_2$ is the roll moments of inertia about x-axis.

$T_x$ is the cornering torque.
The pitching motion of sprung mass can be written as
\[ J_p\ddot{\theta} + (F_{kz3} + F_{kz4})\dot{\theta}_2 - (F_{kz3} + F_{kz2})\dot{\theta}_1 + l_yF_{kz5} + (F_{cxi} + F_{cu})\dot{\theta}_2 - (F_{cxi} + F_{cu})\dot{\theta}_1 + l_yF_{cui} + (F_1 + F_2)\dot{\theta}_1 - (F_3 + F_4)\dot{\theta}_2 - l_yF_2 + T_y \] .. (9)

Where:
\[ J_p \] is the pitch moments of inertia about y-axis.
\[ T_y \] is the braking torque.

While the heave motion of unsprung masses can be governed by the following equation
\[ M_{z_{ unst}}\ddot{z}_{ unst} = -k_{z1}(x_{ unst} - u_{r1}) - c_{z1}(\dot{x}_{ unst} - \dot{u}_{r1}) + F_{kz1} + F_{czi} - F_i \] .. (10)

Where \[ u_{r1} \] is the road input.

3. PID CONTROLLER

The abbreviation of PID stands for Proportional-Integral-Derivative, show the three terms operating on the error signal to produce a control signal. The PID controller is a standard control loop feedback mechanism significantly utilized in industrial control systems, \[15,16\].

The block diagram of PID controller is depicted in Figure (2). This Figure is known as parallel form or non-interacting form. The parallel controllers are mostly preferred for higher order systems\[10\].

The transfer function of PID controller in s-plan has a form
\[ G_c(s) = K_p + \frac{K_i}{s} + K_d s \] .. (11)

where: \[ K_p, K_i, \] and \[ K_d \] are the proportional, integral and derivative gains, respectively.

The PID controller attempts to minimize the error by adjusting the three parameters \[ K_p, K_i \] and \[ K_d \] without needing for specific knowledge of a plant model.

The proportional controller action is generated based on the current error, this term is represented as the multiplication of the system error and the proportional gain \[ K_p \] so can be mathematically expressed as \[16,17\]
\[ P_{term} = K_p \times Error \] .. (12)

The integral control provided a control signal to the system, these signal are generated based on the sum of recent error. The integral action is mainly utilized to eliminate static error and improve the stability of system by removing the offset introduced by the proportional control, but a phase lag is added into the system. Integral term can be mathematically expressed as \[7,16\].
\[ I_{term} = K_i \times \int Error \, dt \] .. (13)

The derivative control is proportional to the rate of change of the error, where it reflects the change of the deviation signal, introduces a correction signal before the deviation signal value becomes bigger and accelerates the response of the system in order to reduce the setting time. So the derivative action is utilized to improve the closed-loop stability, decrease the overshoot or to damp out the oscillations of system response and introduce a phase lead action that removes the phase lag introduced by the integral action \[6,7\].
\[ D_{term} = K_d \times \frac{d(Error)}{dt} \] .. (14)

So the excellent performance of PID controller can be achieved by optimization and proper combination of the...
controller gains $K_P, K_I, \text{ and } K_D$. The weighted sum of these three parameters is used to make the system stable and obtain an effective or robust transient response [16].

4. ARTIFICIAL BEE COLONY ALGORITHM (ABC)

In the last years, with the growth of computer technology, the Swarm Intelligence (SI) inspired by social insects has become one of the most interesting research areas to many scientists of related fields and attracted much attention. Swarm Intelligence is defined by Bonabeau as (any trying to design algorithms or distributed problem-solving devices inspired by the collective behavior of social insect colonies and other animal societies...). In general manner the term swarm is utilized to indicate to any restrained collection of individuals or interacting agents [7,18].

The development of swarm intelligence algorithms expanded the utilize of optimization techniques and improved the reliability of the optimization results that obtained by classic approaches. The aim of these optimization algorithms are to find a set of values for the parameters, have been suggested for solving difficult optimization problem due to their production of effective solutions for problems in a acceptable time and simple structures. There are many complex multi-variable optimization problems with arbitrarily high dimensionality which cannot be certifiably solved within bounded computation time. So search algorithms capable of finding near-optimum or at least practically good solutions within reasonable computation time have drawn the attention of the scientific community [8,19].

One of these a new algorithms it is the Artificial Bee Colony (ABC) algorithm was proposed by Dervis Karaboga in 2005 , where the natural behavior of bees and their collective activities in their colony such as memorizing, learning, and information sharing characteristics in their hives has been attracted researchers for centuries [8].

Artificial Bee Colony (ABC) is a bio-inspired optimization algorithm mimick food foraging behavior of bee colonies, where the action of the honey bees foraging is the main element of the ABC algorithm. This algorithm has attracted much attention due to it has lesser number of control variables and superior convergence, so it’s well suited to solve multidimensional optimization problems. The colony of artificial bees contains two kind of bees are employed and unemployed bees [20].

Employed bees try to find new food sources within the neighborhood of the food source that is currently exploiting or the food source that visited by it previously. It is store the information about the quality of the food source and then moves nectar sources and the position information about food sources to the hive where shares this information with other bees waiting there via performing the peculiar communication called waggle dance. The food source represents a possible solution to the optimization problem while the nectar amount of source corresponds to the fitness of the associated solution [8,18].

On the other hand, the unemployed foragers consist of onlooker bees and scout bees. The onlooker bees waiting on the dance area and decide a food source based on the information provided by employed bees. After that the employed bees deploy to the food sources within the neighborhood of the food sources that which picked out depended on the probability principle, then apply the greedy selection scheme to update food source position [7,18].

The second kind of unemployed bee is scout bees this type is associated with employed bees where if the employed bee whose food source is exhausted and could not be improved through a predetermined number of cycles it’s becomes a scout bee. The scout bees starts carries out random search to find a new food source depending on an internal motivation or possible external clues or randomly. After a new food source is randomly generated this scout bee becomes an employed bee again [8,21].

In the ABC algorithm first half of the colony constitutes of the employed bees and the second half constitutes the onlooker bees, it is assumed that for every food source there is only one employed bee. In other words the number of employed bees is equal to the number of food sources around the hive, so the number of the employed bees or the onlooker bees is equal to the number of solutions in the population [22].

5. USING THE ARTIFICIAL BEE COLONY ALGORITHM TO TUNE THE PARAMETERS OF PID CONTROLLER

The steps of using the ABC algorithm to design the parameters of PID controller are described below [10,19]:

**Step 1:** Limits of PID parameters are read at this step.

**Step 2:** ABC parameters such as maximum cycle number, colony dimension, limit parameter and number of variables are initialized.

**Step 3:** An initial population of FS individuals are produced randomly. Each solution $K_I,t = [1,2,...,F_S]$ is a D-dimensional vector corresponded to number of PID
parameters optimized i.e., \( D = [K_p, K_i, K_d] \). In general the position of \( l_{th} \) solution is represented as:

\[
K\text{(FS,} \; D) = \begin{bmatrix}
K_p^1 & K_i^1 & K_d^1 \\
K_p^2 & K_i^2 & K_d^2 \\
\vdots & \vdots & \vdots \\
K_p^{FS} & K_i^{FS} & K_d^{FS}
\end{bmatrix}
\]

**Step 4:** The fitness value of each individual solution is evaluated utilizing:

\[
f_{it} = \frac{1}{1 + \sum_{k=1}^{n} e^2_k(t)} \quad (15)
\]

\[
e(t) = r(t) - \dot{z}(t) \quad (16)
\]

Where \( \dot{z} \) is the derivative of the vertical displacement.

**Step 5:** The cycle counter is set to 1.

**Step 6:** Solutions are modified and replaced with a new solution \( K_{new} \) by employed bees, where each employed bee placed at a solution that is different from others to find the better solution that which neighborhood of its present position utilizing the following equation:

\[
k_{new}(l,j) = k(l,j) + \varphi(l,j)(k(l,j) - k(n,j)) \quad (17)
\]

Where:
- \( n = 1,2, \ldots, FS \) and \( j = 1,2, \ldots, D \) are randomly selection indexes.
- \( K_{new} \) is the candidate new solution.
- \( \varphi \) is a uniformly distributed real number determined randomly within the range \([-1, 1]\).
- \( k \) is the \( j^{th} \) dimension of the neighbor employed bee.
- \( l \) is represents the current iteration.

Then the fitness of the new solution is computed and compared with old one to apply a greedy selection to select the best one of them.

**Step 7:** Determine the selection probability values \( P_t \) for each employed bee as described in Eq.(18)

\[
P_t = \frac{f_{it}}{\sum_{m=1}^{FS} f_{im}} \quad (18)
\]

Each onlooker bee assign to an employed bee at random according to a probability value, now each onlooker bee produce new solution \( K_{new} \) as in Eq.(17). Then the fitness of each new solution is calculated as give in Eq.(15) and a greedy selection is applied between the new and old solution to save the better one and ignore the other.

**Step 8:** If a particular food source \( K_t \) is not improved over a number of iterations, that source abandoned. This source is replaced with a new one that generated randomly by scout bees, this process is done according to the “limit” parameter.

**Step 9:** Keep track of the best solution so far.

**Step 10:** increase the cycle counter.

Steps between 6 and 10 are repeated until reach the Maximum Cycle Number(MCN). If the maximum numbers of iterations have elapsed, stop and return the better solution found so far.

**6. SIMULATION AND RESULTS**

The proposed system is simulated using the MATLAB/SIMULINK Software. Five PID controllers are utilized one for each suspension system. The output of these controllers are applied as valves control signals of the hydraulic actuators \( u_m1, u_m2, u_m3, u_m4 \) and \( u_m5 \) and the input of PID controller is the difference between the reference input set with amplitude zero and the derivative of vertical displacement. The input to the ABC algorithm is the error which is used to determine the fitness function and the outputs values \( (K_p^*, K_i^* \text{ and } K_d^*) \) are used to tune the parameters of the PID controller as shown in Figure (3).

The parameters \( K_p, K_i, \text{and } K_d \) of each PID controller are tuned by ABC algorithm, the ranges of those parameters are selected as \( K_p \in [8000, 10500], \; K_i \in [1000, 4000], \text{and } \) \( K_d \in [100, 800] \).

The parameters of ABC algorithm for each PID controller are selected as shown in Table(1), while the optimal values of the PID controller parameters are given in Table (2).

**Table 1: Parameters of foraging algorithm**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>colony size</td>
<td>100</td>
</tr>
<tr>
<td>Number of food sources(FS)</td>
<td>50</td>
</tr>
<tr>
<td>Maximum number of cycles</td>
<td>20</td>
</tr>
<tr>
<td>Number of optimization parameters</td>
<td>3</td>
</tr>
</tbody>
</table>

The performances of the suspension system measured in terms of ride quality and road handling, so when supplying the control signal, just the vertical displacements of \( j^{th} \) sprung mass \( z_{si} \) and body acceleration \( \ddot{z}_c \) will be reduced to minimum while the vertical displacements of \( l^{th} \) unsprung masses \( z_{usi} \) will remain as it is.

**Table 2: Optimal values of PID controllers parameters**

<table>
<thead>
<tr>
<th>Controller</th>
<th>( K_p )</th>
<th>( K_i )</th>
<th>( K_d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_1 )</td>
<td>1.0053*e^4</td>
<td>761.4882</td>
<td>3.6122*e^3</td>
</tr>
<tr>
<td>( C_2 )</td>
<td>1.0327*e^4</td>
<td>756.369</td>
<td>3.8976*e^3</td>
</tr>
<tr>
<td>( C_3 )</td>
<td>1.0247*e^4</td>
<td>533.6652</td>
<td>3.8* e^3</td>
</tr>
<tr>
<td>( C_4 )</td>
<td>1.039*e^4</td>
<td>679.4983</td>
<td>3.792*e^3</td>
</tr>
<tr>
<td>( C_5 )</td>
<td>1.0358*e^4</td>
<td>479.1239</td>
<td>3.9619*e^3</td>
</tr>
</tbody>
</table>
Figures (4-5) show the comparison between the outputs response of full vehicle active suspension system with PID controller and the passive suspension system, where the road profile is selected as square inputs.

Fig 4: Time response of vertical displacement at (a) P1; (b)P2; (c)P3; (d)P4.
Fig 5: Time response of (a) vertical displacement at P5; (b) vertical displacement at Pc; (c) Roll angle; (d) Pitch angle.

7. ROBUSTNESS TEST

To confirm the effectiveness of the proposed controller, the robustness of the controller should be tested by applying different types of disturbance inputs on the system and the controller should be insensitive to this varying in the disturbance signal. So the four types of disturbance implemented in order to test the robustness of the PID controller with optimal parameters have been applied in compare to corresponding passive system.

- **Square wave signal with different amplitude is applied as the road profile**
  The amplitude of the square signal is changed from 0.01m to 0.1m with fixed frequency (0.1 Hz) is applied as road profile input, where the cost function calculated for each value by using the following equation:
  \[ J_c = 0.5 \sum_{i=1}^{n} x_i^2 \]  
  The response of the cost function against the different amplitude values of square signal input for passive and active vehicle is shown in Figure (6.a)

- **Sine wave with different amplitude is applied as the road profile**
  The sine input signal is proves as input road profile with frequency (0.1 Hz), its amplitude is varied from 0.01m to 0.1m. Figure(6.b) illustrates the response of the cost function against the different amplitude values of sine signal input.

- **A bending inertia torque (T_x) is applied with different values**
  The values of bending torque changed from 1000 Nm to 10000Nm with random road profile input has been applied to the system. The response of cost function is plotted as function of \( T_x \) as shown in Figure (6.c).

- **The braking inertia torque (T_y) is applied with different values**
  The values of braking torque from 1000 Nm to 10000Nm with random signal applied as road profile has been applied to the system. The response of the cost function is plotted as function of \( T_y \) as shown in Figure(6.d).
Fig 6: Cost functions against the different amplitude of (a) sine wave; (b) square wave; (c) bending torque; (d) braking torque.

8. CONCLUSION

In this work, a new optimization algorithm known as ABC has been exploited as a tuning design method for optimizing the PID controller parameters to improve the performance of full vehicle nonlinear active suspension system with 8-DOF in terms of passenger ride comfort.

From the simulation results, it can be seen that the active suspension system with proposed ABC-PID controller is more effective in controlling the vehicle oscillation and more robust in leading the system to reach steady state value when compared to the passive suspension system. However, the amplitude of the output transient responses of the controlled system with PID controller is not small, which can be noticeable by the passengers so, the control objectives cannot be met totally.

The robustness of the proposed controller was tested by applying four types of disturbances, the simulation results show that the performances of the cost function for the system with the PID controller have been slightly improved, while for the case when the square input applied with different amplitude values and the case when bending torque applied as disturbances type the performance of the cost function of the system with proposed controller is near to the performance of the cost function of the passive system which means when different values of the square wave or bending torque are applied, the suggested controller approximation will lose its robustness.

9. REFERENCES


10. Nomenclature

\[ M_s \] is the sprung mass(Kg).

\[ M_{uls} \] is the passenger seat mass(kg).

\[ M_{uls1},M_{uls2} \] are the front left and front right side unsprung mass respectively(kg).

\[ M_{ulr3},M_{ulr4} \] are the rear left and rear right side unsprung mass respectively(kg).

\[ k_{cs} \] is the passenger seat stiffness(N/m).

\[ k_{s1},k_{s2} \] are the front left and front right side spring stiffness respectively(N/m).

\[ k_{s3},k_{s4} \] are the rear left and rear right side spring stiffness respectively(N/m).

\[ k_{t1},k_{t2} \] are the front left and front right side tire stiffness respectively(N/m).

\[ k_{t3},k_{t4} \] are the rear left and rear right side tire stiffness respectively(N/m).

\[ c_{s5} \] is the passenger seat damping coefficient(N/m).

\[ c_{s1},c_{s2} \] are the front left and front right side damping coefficient respectively (N/m).

\[ c_{s3},c_{s4} \] are the rear left and rear right side damping coefficient respectively (N/m).

\[ c_{t1},c_{t2} \] are the front left and front right side tire damping coefficient respectively (N/m).

\[ c_{t3},c_{t4} \] are the rear left and rear right side tire damping coefficient respectively (N/m).

\[ l_1 \& l_2 \] are the distance between the center of gravity of the sprung mass and center of front and rear wheel axle respectively(m).

\[ l_p \] is the distance of passenger seat position from the center of gravity(m).
b is the distance between the front or rear wheels (m).

\(b_x\) is the distance of passenger seat position from the center of gravity of the vehicle (m).

### 11. Appendix

Variable design parameters value

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Unit</th>
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<td>(k_{x1}, k_{x2})</td>
<td>19960</td>
<td>N/m</td>
</tr>
<tr>
<td>(k_{x3}, k_{x4})</td>
<td>17500</td>
<td>N/m</td>
</tr>
<tr>
<td>(k_{x5})</td>
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<td>Kg</td>
</tr>
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</tr>
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</tr>
<tr>
<td>(l_x)</td>
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<td>m</td>
</tr>
<tr>
<td>(l_1)</td>
<td>1.011</td>
<td>m</td>
</tr>
<tr>
<td>(l_2)</td>
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<tr>
<td>(b)</td>
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<td>m</td>
</tr>
<tr>
<td>(\zeta)</td>
<td>0.1</td>
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