On Maximal Soft δ-open (Minimal soft δ-closed) Sets in Soft Topological Spaces

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ABSTRACT

In soft topological space there are some existing related concepts such as soft open, soft closed, soft subspace, soft separation axioms, soft connectedness, soft locally connectedness. In this paper, a new class of soft sets called maximal soft δ-open sets and minimal soft δ-closed sets which are fundamental results for further research are defined on soft topological space and continued in investigating the properties of these new notions of open sets with example and counter examples.

Keywords

Soft regular open sets, soft regular closed sets, soft δ-cluster point, soft δ-open sets, soft δ-closed sets, soft maximal open sets, soft minimal closed sets, soft maximal δ-open sets, soft minimal δ-closed sets etc.

1. INTRODUCTION

Molodtsov [1] initiated a novel concept of soft set theory, which is a completely new approach for modeling vagueness and uncertainty. He successfully applied the soft set theory into several directions such as smoothness of functions, game theory, Riemann Integration, theory of measurement, and so on. Soft set theory and its applications have shown great development in recent years. This is because of the general nature of parametrization expressed by a soft set. Shabir and Naz [2] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. Later, Zorlutuna et al.[3], Aygunoglu and Aygun [4] and Hussain et al are continued to study the properties of soft topological space. They got many important results in soft topological spaces. Weak forms of soft open sets were first studied by Chen [5]. He investigated soft semi-open sets in soft topological spaces and studied some properties of it. Yumak and Kaymakci [14] are defined soft β-open sets and continued to study weak forms of soft open sets in soft topological space. Later, Yumak and Kaymakci [9] defined soft b-open (soft b-closed) sets and Akdag and Ozkan [6] soft α-open (soft α-closed) sets respectably.

In the present study, first of all, some new concepts such as maximal soft open sets, minimal soft closed sets, maximal soft interior and minimal soft closure are focused in soft topological spaces and investigated some of their properties. Secondly,the concepts of soft δ-open sets, soft δ-closed sets, soft δ-interior, soft δ-closure are defined in soft topological spaces and investigated some of their properties on soft topological spaces and obtain some characterizations of these concepts.

2. PRELIMINARIES

Throughout the paper, the space X and Y stand for soft topological spaces with (X,τ, E) and (Y,ν, K) assumed unless otherwise stated. Moreover, throughout this paper, a soft mapping $f : X \to Y$ stands for a mapping, where $f : (X,\tau, E) \to (Y,\nu, K)$, $u : X \to Y$ and $p : E \to K$ are assumed mappings unless otherwise stated.

Definition: 2.1[1]: Let X be an initial universe and E be a set of parameters. Let *P*(X) denotes the power set of X and A be a non-empty subset of E. A pair (F,A) is called a soft set over X, where F is a mapping given by $F: A \to P(X)$ defined by $F(e) \in P(X)$ $\forall e \in A$. In other words, a soft set over X is a parameterized family of subsets of the universe X. For $e \in A$, F(e) may be considered as the set of e-approximate elements of the soft set (F,A).

Definition 2.2[10]: A soft set (F,A) over X is called a null soft set, denoted by $\tilde{\varphi}$, if $e \in A, F(e)=\varphi$.

Definition 2.3[10]:A soft set (F,A) over X is called an absolute soft set, denoted by $\tilde{A}$, if $e \in A$, F(e)=X. If A=E, then the A-universal soft set is called a universal soft set, denoted by $\tilde{X}$.

Theorem 2.4[2]: Let Y be a non-empty subset of X, then $\tilde{Y}$ denotes the soft set (Y,E) over X for which Y(e)=Y, for all $e \in E$.

Definition 2.5 [10]: The union of two soft sets (F,A) and (G,B) over the common universe X is the soft set (H,C), where C=A $\tilde{\cup}$ B and for all $e \in C$,

$$H(e)=\begin{cases} F(e), if e \in A-B \\ G(e), if e \in B-A \\ F(e) \cup G(e), if e \in A \cap B \end{cases}$$

We write (F,A) $\tilde{\cup}$ (G,B) = (H,C)

Definition 2.6 [10]: The intersection (H,C) of two soft sets (F,A) and (G,B) over a common universe X, denoted by

(F,A) $\tilde{\cap}$ (G,B), is defined as C=A∩B and H(e)= F(e)∩ G(e) for all e∈C.

Definition 2.7 [10]: Let (F,A) and (G,B) be two soft sets over a common universe X. Then, (F,A) $\tilde{\subseteq}$ (G,B) if A ⊆ B, and F(e) ⊆ G(e) for all e∈A.

Definition 2.8 [2]: Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X if satisfies the following axioms.

(1) $\tilde{\varphi}$, $\tilde{X}$ belong to τ,

(2) the union of any number of soft sets in τ belongs to τ,

(3) the intersection of any two soft sets in τ belongs to τ.

The triplet (X,τ,E) is called a soft topological space over X. Let (X,τ,E) be a soft topological space over X, then the members of τ are said to be soft open sets in X. A

soft set (F,A) over X is said to be a soft closed set in X, if its relative complement $(F,A)^c$ belongs to τ.

Definition 2.9 [11]: For a soft set (F,A) over X, the relative complement of (F,A) is denoted by $(F,A)^c$ and is defined by $(F,A)^c = (F^c, A)$, where $F^c : A \to P(X)$ is a mapping given by $F^c(e)=X-F(e)$ for all e∈A.

Definition 2.10: A soft set (F,A) in a soft topological space X is called **(i)** soft regular open (resp.soft regular closed) set [16] if (F,A) = Int(Cl(F,A)) [resp. (F,A) = Cl(Int (F,A))].

(ii) soft semi-open (resp.soft semi-closed) set [5] if (F,A) $\tilde{\subseteq}$ Cl(Int(F,A)) [resp. Int(Cl(F,A)) $\tilde{\subseteq}$ (F,A).

(iii) soft pre-open (resp.soft pre-closed)[12] if (F,E) $\tilde{\subseteq}$ int(cl(F,E)) [resp. Cl(Int(F,A)) $\tilde{\subseteq}$ (F,A).

(iv) soft α-open (resp.soft α-closed)[6] if (F,E) $\tilde{\subseteq}$ Int(Cl(Int(F,E))) [resp. Cl(Int(Cl(F,A))) $\tilde{\subseteq}$ (F,A)].

(v) soft β-open (resp.soft β-closed) set [14] if (F,A) $\tilde{\subseteq}$ Cl(Int(Cl(F,A))) [resp. Int(Cl(Int(F,A))) $\tilde{\subseteq}$ (F,A)].

Definition 2.11[15]: The soft set (F,A) in a soft topological space (X,τ,E) is called a soft point in X, denoted by P_λ^F, if for λ∈A, F(λ) ≠ φ and F(β) = φ, for β∉A.

3. MAXIMAL SOFT OPEN, MINIMAL SOFT CLOSED, MAXIMAL SOFT δ-OPEN AND MINIMAL SOFT δ-CLOSED SETS

In this section I introduce the concept of maximal soft open set and minimal soft closed sets and study their basic properties. I also introduce the notion of soft maximal δ-open set and minimal soft δ-closed sets and study some fundamental theorems and their applications.

Definition 3.1: A nonempty proper soft open set (G,E) in a soft topological space (X,τ,E) is said to be soft maximal open set if any soft open set which contains (G,E) is either (G,E) or $\tilde{X}$.

Definition 3.2: A nonempty proper soft closed set (F,E) of any soft topological space (X,τ) is said to be minimal soft closed set if any soft closed set which is contained in (F,E) is either $\tilde{\varphi}$ or (F,E).

The class of all maximal soft open sets & minimal soft closed sets will be denoted by $M_aSO(X)$ & $M_iSC(X)$ respectably.

Example 3.3: Let X={x_1, x_2, x_3, x_4},E ={e_1, e_2, e_3} and τ ={ $\tilde{\varphi}$, $\tilde{X}$, (F_1,E),(F_2,E), (F_3,E)} where, (F_1,E),(F_2,E), (F_3,E) are soft sets over X, defined as follows:

(F_1,E) ={ (e_1,{x_1,x_2,x_4}),(e_2,{ x_2, x_3}),(e_3,{ x_3, x_4})}

(F_2,E) ={ (e_1,{ x_3}),(e_2,{ x_1}),(e_3,{ x_1, x_2})}

(F_3,E) ={ (e_1, X),(e_2,{ x_1,x_2,x_3}),(e_3,X)}

Then τ ={ $\tilde{\varphi}$, $\tilde{X}$, (F_1,E),(F_2,E), (F_3,E)} forms a soft topology on X and (X,τ,E) is a soft topological space. Here, we observe that (F_1,E),(F_2,E) and (F_3,E) are each non-empty proper soft open set in (X,τ,E) . Here (F_1,E) and (F_2,E) are not soft maximal open set in X since both (F_1,E) and (F_2,E) are contained by (F_3,E). But (F_3,E) is soft maximal open set in X as (F_3,E) is either contained by (F_3,E) or $\tilde{X}$.

Theorem 3.4: A proper non empty soft subset (G,E) in a Soft Topological Space (X,τ,E) is maximal soft open iff $(G,E)^c$ = $\tilde{X}$ -(G,E) is a minimal soft closed set in (X,τ,E).

Proof: Let (G,E) be a maximal soft open set in (X,τ,E) and let (F,E) be any soft closed set in (X,τ,E) such that (F,E) $\tilde{\subset}$ $(G,E)^c$ = $\tilde{X}$ -(G,E).Then $\tilde{X}$ -(F,E) = $(F,E)^c$ $\tilde{\in}$ τ. Now, (F,E) $\tilde{\subset}$ $(G,E)^c$ implies that (G,E) $\tilde{\subset}$ $(F,E)^c$. Since, (G,E) is maximal soft open set, so, $(F,E)^c$ = $\tilde{X}$ or $(F,E)^c$ = (G,E).This implies that (F,E)= $\tilde{\varphi}$ or (F,E) = $(G,E)^c$. This shows that $(G,E)^c$ is minimal soft closed set in (X,τ,E).

Conversely, let, $(G,E)^c$ is minimal soft closed set in (X,τ,E) & let (G_1,E) be any soft open set in (X,τ, E) such that $(G,E) \tilde{\subseteq} (G_1,E)$.Then $(G_1,E)^c$ is soft closed set and $(G_1,E)^c \tilde{\subseteq} (G,E)^c$. Since, by hypothesis, $(G,E)^c$ is minimal soft closed set, this implies that $(G_1,E)^c = \tilde{\varphi}$ or $(G_1,E)^c = (G,E)^c$. i.e. $(G_1,E) = \tilde{X}$ or $(G_1,E) = (G,E)$.Thus (G,E) is maximal soft open set in (X,τ,E).

Remark 3.5: In view of theorem 3.4, $(F_3,E)^c$ is soft minimal closed set of X in example 3.3.

Definition 3.6: A soft point P_λ^F in a soft topological space (X,τ,E) is called a soft δ-cluster point of a soft Set (G,A) if for each soft regular open set (U,A) containing P_λ^F, $(G,A) \tilde{\cap} (U,A) \neq \tilde{\varphi}$.

The set of all soft δ-cluster points of (G,A) is called soft δ-closure of (G,A) and is denoted by $[(G,A)]_\delta$ or $SCl_\delta(G,A)$.Soft δ-interior of a soft set (F,A) denoted by $SInt_\delta(F,A) = \{ P_\lambda^F \in X$: for some soft open subset (G,A) of X, $P_\lambda^F \in (G,A) \tilde{\subseteq} Int(Cl(G,A)) \tilde{\subseteq} (F,A)\}$.

Definition 3.7: A soft set (G,A) in a soft topological space (X,τ,E) is called soft δ-closed set iff $(G,A) = SCl_\delta(G,A)$ and it's compliment $\tilde{X} - (G,A)$ is called soft δ-open sets in X.

Or, equivalently, if (G,A) is the union of soft regular open sets, then (G,A) is said to be soft δ-open sets in X.

The collection of all soft δ-open sets & soft δ-closed sets are respectably, denoted by $S\delta OS(X)$ & $S\delta CS(X)$.

Definition 3.8: A nonempty proper soft δ-open set (G,E) of any soft space (X,τ,E) is said to be maximal soft δ-open set if any soft δ-open set which contains (G,E) is either (G,E) or $\tilde{X}$.

Definition 3.9: A nonempty proper soft δ-closed set (F,E) of any soft space (X,τ,E) is said to be soft minimal δ-closed set if any soft δ-closed set contained in (F,E) is either $\tilde{\varphi}$ or (F,E). Or, equivalently, if $(F,E)^c$ is maximal soft δ-open set in (X,τ).

The family of all maximal soft δ-open and minimal soft δ-closed sets are respectably denoted by $M_aS\delta\text{-}O(X)$ & $M_iS\delta\text{-}C(X)$.

Definition 3.10: The union (resp.intersection) of all maximal soft δ-open (resp. minimal soft δ-closed) sets of X contained in (containing) a soft set (F,A) is called maximal soft δ-interior (resp.soft minimal δ-closure) and is denoted by $M_aS\delta\text{-}Int(F,A)$ [resp.$M_iS\delta\text{-}Cl(F,A)$] i.e, $M_aS\delta\text{-}Int(F,A) = Sup\{(F,A)_\alpha: (F,A)_\alpha \tilde{\subseteq} (F,A)$ and $(F,A)_\alpha \in M_aS\delta\text{-}O(X), \alpha\in\Lambda$,an index set$\}$ and $M_iS\delta\text{-}Cl(F,A)= Inf\{(G,A)_\alpha:(G,A)^c_\alpha \tilde{\supseteq} (F,A)$ and $(G,A)^c_\alpha \in M_aS\delta\text{-}O(X), \alpha\in\Lambda$,an index set$\}$

Lemma 3.11: The complement of $M_aS\delta\text{-}Int(F,A)$ is $M_iS\delta\text{-}Cl(F,A)$ and vicevarsa.

Proof: The proof follows from the definition 3.10 with the help of De Morgans law.

Remark 3.12: Obviously, every soft maximal δ-open set is soft δ-open set but the converse is not always true which is shown by means of the following example 3.13 and in this example we show that soft maximal open sets and soft maximal δ-open sets are two independent notions.

Example 3.13: Let $X=\{x_1, x_2, x_3, x_4\}$,$E =\{e_1, e_2, e_3\}$ and $\tau =\{ \tilde{\varphi}, \tilde{X}, (F_1,E),(F_2,E), (F_3,E), (F_4,E), (F_5,E), (F_6,E), (F_7,E), (F_8,E), (F_9,E), (F_{10},E), (F_{11},E), (F_{12},E), (F_{13},E), (F_{14},E), (F_{15},E)\}$ where, $(F_1,E),(F_2,E), (F_3,E), (F_4,E), (F_5,E), (F_6,E), (F_7,E), (F_8,E), (F_9,E), (F_{10},E), (F_{11},E), (F_{12},E), (F_{13},E), (F_{14},E), (F_{15},E)$ are soft sets over X, defined as follows:

$(F_1,E) =\{ (e_1,\{ x_1\}),(e_2,\{ x_2, x_3\}),(e_3,\{ x_1, x_4\})\}$

$(F_2,E) =\{ (e_1,\{ x_2, x_4\}),(e_2,\{ x_1, x_3, x_4\}),(e_3,\{ x_1, x_2, x_4\})\}$

$(F_3,E) =\{ (e_1, \varphi),(e_2,\{x_3\}),(e_3,\{ x_1\})\}$

$(F_4,E) =\{ (e_1, \{ x_1,x_2,x_4\}),(e_2, X),(e_3,X)\}$

$(F_5,E) =\{ (e_1,\{ x_1,x_3\}),(e_2,\{ x_2, x_4\}),(e_3,\{ x_2\})\}$

$(F_6,E) =\{ (e_1,\{ x_1\}),(e_2,\{ x_2\}),(e_3, \varphi)\}$

$(F_7,E) =\{ (e_1,\{ x_1,x_3\}),(e_2,\{ x_2, x_3, x_4\}),(e_3,\{ x_1, x_2,x_4\})\}$

$(F_8,E) =\{ (e_1,\varphi),(e_2,\{ x_4\}),(e_3,\{ x_2\})\}$

$(F_9,E) =\{ (e_1,X),(e_2,X),(e_3,\{ x_1, x_2, x_3\})\}$

$(F_{10},E) =\{ (e_1,\{ x_1,x_3\}),(e_2,\{ x_2, x_3, x_4\}),(e_3,\{ x_1, x_2\})\}$

$(F_{11},E) =\{ (e_1,\{ x_2, x_3 ,x_4\}),(e_2,X),(e_3,\{ x_1,x_2, x_3\})\}$

$(F_{12},E) =\{ (e_1,\{ x_1\}),(e_2,\{ x_2, x_3, x_4\}),(e_3,\{x_1 ,x_2, x_4\})\}$

$(F_{13},E) =\{ (e_1,\{ x_1\}),(e_2,\{ x_2, x_4\}),(e_3,\{ x_2\})\}$

$(F_{14},E) =\{ (e_1,\{ x_3, x_4\}),(e_2,\{ x_1, x_2\}),(e_3, \varphi)\}$

$(F_{15},E) =\{ (e_1,\{ x_1\}),(e_2,\{ x_2, x_3\}),(e_3,\{ x_1\})\}$

Then, τ defines a soft topology on X, and thus (X,τ,E) is a soft topological space over X.Clearly, soft closed sets are $\tilde{\varphi}, \tilde{X}, (F_1, E)^c, (F_2, E)^c, (F_3, E)^c, (F_4, E)^c, (F_5, E)^c, (F_6, E)^c, (F_7,E)^c, (F_8,E)^c, (F_9, E)^c, (F_{10},E)^c, (F_{11}, E)^c, (F_{12}, E)^c, (F_{13}, E)^c, (F_{14}, E)^c, (F_{15}, E)^c$. Here, soft regular open sets are $\tilde{\varphi}, \tilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_5, E), (F_6, E)$ and (F_8,E). Soft δ-open sets are

$\tilde{\varphi}$, $\tilde{X}$, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E) and (F_8,E). We find that (F_5,E) is maximal soft δ-open set but not maximal soft open set in X and (F_9,E) is maximal soft open set but not maximal soft δ-open set in X.

Theorem 3.14: A proper non empty soft subset (G,E) in a Soft Topological Space (X,τ,E) is maximal soft δ-open iff $(G,E)^c$ = $\tilde{X}$ -(G,E) is a minimal soft δ-closed set in (X,τ,E).

Proof: Let (G,E) be a maximal soft δ-open set in (X,τ,E) & let (F,E) be any soft δ-closed set in (X,τ,E) such that (F,E) $\tilde{\subset}$ $(G,E)^c$ = $\tilde{X}$ -(G,E).Then $\tilde{X}$ -(F,E) = $(F,E)^c$ $\tilde{\in}$ τ. Now, (F,E) $\tilde{\subset}$ $(G,E)^c$ implies that (G,E) $\tilde{\subset}$ $(F,E)^c$. Since, (G,E) is maximal soft δ-open set, so, $(F,E)^c$ = $\tilde{X}$ or $(F,E)^c$ = (G,E).This implies that (F,E) = $\tilde{\varphi}$ or (F,E) = $(G,E)^c$. This shows that $(G,E)^c$ is minimal soft δ-closed set in (X,τ,E).

Conversely, let, $(G,E)^c$ is minimal soft δ-closed set in (X,τ,E) & let (G_1,E) be any soft δ-open set in (X,τ, E) such that (G,E) $\tilde{\subset}$ (G_1,E).Then $(G_1,E)^c$ is soft δ-closed set & $(G_1,E)^c$ $\tilde{\subset}$ $(G,E)^c$. Since, by hypothesis, $(G,E)^c$ is minimal soft δ-closed set, this implies that $(G_1,E)^c$ = $\tilde{\varphi}$ or $(G_1,E)^c$ = $(G,E)^c$. i.e. (G_1,E) = $\tilde{X}$ or (G_1,E) = (G,E).Thus (G,E) is maximal soft δ-open set in (X,τ,E).

Remark 3.15: In view of above theorem 3.14, $(F_5,E)^c$ is minimal soft δ-closed set over the soft space X in example 3.13.

Theorem 3.16: In any soft topological space (X,τ,E), if (G,A) be a maximal soft δ-open set in X and (G_1,A) be any soft δ-open set, then (G,A) $\tilde{\cup}$ (G_1,A) = $\tilde{X}$ or (G_1,A) is contained in (G,A) i.e, (G_1,A) $\tilde{\subset}$ (G,A) .

Proof : Let (G,A) be a maximal soft δ-open set and (G_1,A) be any soft δ -open set in X. If (G,A) $\tilde{\cup}$ (G_1,A) = $\tilde{X}$, then the proof follows. Suppose, (G,A) $\tilde{\cup}$ (G_1,A) ≠ $\tilde{X}$. Now, we have, (G,A) $\tilde{\subseteq}$ (G,A) $\tilde{\cup}$ (G_1,A). Since (G,A) is maximal soft δ-open set, so by definition , (G,A) $\tilde{\cup}$ (G_1,A) = $\tilde{X}$ or (G,A) $\tilde{\cup}$ (G_1,A) = (G,A) ,but by hypothesis, (G,A) $\tilde{\cup}$ (G_1,A) ≠ $\tilde{X}$, so, we must have, (G,A) $\tilde{\cup}$ (G_1,A) = (G,A) so that (G_1,A) $\tilde{\subset}$ (G,A) ⇒ (G_1,A) is contained in (G,A) .

Theorem 3.17: If (G_1,A) and (G_2,A) be two maximal soft δ-open sets in X, then (G_1,A) $\tilde{\cup}$ (G_2,A) = $\tilde{X}$ or (G_1,A) = (G_2,A).

Proof: Let (G_1,A) and (G_2,A) be two maximal soft δ-open sets in X. If (G_1,A) $\tilde{\cup}$ (G_2,A) = $\tilde{X}$, then the theorem is immediate. Suppose, (G_1,A) $\tilde{\cup}$ (G_2,A) ≠ $\tilde{X}$, then we have to prove that (G_1,A) = (G_2,A).

Now, as we have (G_1,A) $\tilde{\subseteq}$ (G_1,A) $\tilde{\cup}$ (G_2,A) and (G_1,A) is maximal soft δ-open set, so, by definition of maximal soft δ-open set, it follow that (G_1,A) $\tilde{\cup}$ (G_2,A) = $\tilde{X}$ or (G_1,A) = (G_1,A) $\tilde{\cup}$ (G_2,A). But by hypothesis, (G_1,A) $\tilde{\cup}$ (G_2,A) ≠ $\tilde{X}$ .So, (G_1,A) = (G_1,A) $\tilde{\cup}$ (G_2,A)

⇒ (G_2,A) $\tilde{\subseteq}$ (G_1,A)………………. (i).

Similarly, as (G_2,A) $\tilde{\subseteq}$ (G_1,A) $\tilde{\cup}$ (G_2,A) and (G_2,A) is maximal soft δ-open set, so, by definition of maximal soft δ-open set, it follows that (G_1,A) $\tilde{\cup}$ (G_2,A)= $\tilde{X}$ or (G_2,A) =(G_1,A) $\tilde{\cup}$ (G_2,A).

But by hypothesis, (G_1,A) $\tilde{\cup}$ (G_2,A) ≠ $\tilde{X}$.

So,(G_2,A)=(G_1,A) $\tilde{\cup}$ (G_2,A)⇒(G_1,A) $\tilde{\subseteq}$ (G_2,A)……………………(ii)

From (i) & (ii), it follows that (G_1,A) = (G_2,A).

Example 3.18: In the example 3.13, we find that the soft sets (F_4,E) and (F_5,E) are both maximal soft δ-open sets and (F_4,A) $\tilde{\cup}$ (F_5,A) = $\tilde{X}$ but (F_4,A) ≠ (F_5,A).

Lemma 3.19: Union of arbitrary family of maximal soft δ-open sets is either maximal soft δ-open set or soft whole set $\tilde{X}$.

Proof : The proof is immediate from above theorem 3.17.

Theorem 3.20: Let (F,A) be a minimal soft δ-closed set in X and (F_1,A) be any soft δ-closed set. Then, (F,A) $\tilde{\cap}$ (F_1,A) = $\tilde{\varphi}$ or (F,A) $\tilde{\subset}$ (F_1,A).

Proof: Let (F,A) be a minimal soft δ-closed set and (G_1,A) be any soft δ-closed set in X. If (F,A) $\tilde{\cap}$ (F_1,A) = $\tilde{\varphi}$, then the proof follows. Suppose, (F,A) $\tilde{\cap}$ (F_1,A) ≠ $\tilde{\varphi}$, then we have to prove that (F,A) $\tilde{\subset}$ (F_1,A). Now let, (F,A) $\tilde{\cap}$ (F_1,A) ≠ $\tilde{\varphi}$ ⇒ (F,A) $\tilde{\cap}$ (F_1,A) $\tilde{\subseteq}$ (F,A). Since (F,A) is minimal soft δ-closed set, so by definition of minimal soft δ-closed set, (F,A) $\tilde{\cap}$ (F_1,A) = $\tilde{\varphi}$ or (F,A) $\tilde{\cap}$ (F_1,A) = (F,A),but by hypothesis, (F,A) $\tilde{\cap}$ (F_1,A) ≠ $\tilde{\varphi}$, so, we must have, (F,A) $\tilde{\cap}$ (F_1,A) = (F,A) implying that (F,A) $\tilde{\subset}$ (F_1,A).

Theorem 3.21: If (F_1,A) and (F_2,A) be two minimal soft δ-closed sets in X, then (F_1,A) $\tilde{\cap}$ (F_2,A) = $\tilde{\varphi}$ or (F_1,A) = (F_2,A).

Proof : Let (F_1,A) and (F_2,A) be two minimal soft δ-closed sets in X. If (F_1,A) $\tilde{\cap}$ (F_2,A) = $\tilde{\varphi}$, then the theorem is immediate. If (F_1,A) $\tilde{\cap}$ (F_2,A) ≠ $\tilde{\varphi}$, then we have to prove that (F_1,A) = (F_2,A).Now, as (F_1,A) $\tilde{\cap}$ (F_2,A) $\tilde{\subseteq}$ (F_1,A) and

(F_1,A) is minimal soft δ-closed set, so, by definition of minimal soft δ-closed set, it follow that $(F_1,A) \tilde{\cap} (F_2,A) = \tilde{\varphi}$ or $(F_1,A) = (F_1,A) \tilde{\cap} (F_2,A)$. But by hypothesis, $(F_1,A) \tilde{\cap} (F_2,A) \neq \tilde{\varphi}$.So, $(F_1,A) = (F_1,A) \tilde{\cap} (F_2,A)$. Thus, $(F_1,A) \tilde{\subseteq} (F_2,A)$………(i)

Similarly, as $(F_1,A) \tilde{\cap} (F_2,A) \tilde{\subseteq} (F_2,A)$ and (F_2,A) is minimal soft δ-closed set, so, by definition of minimal soft δ-closed set, it follows that $(F_1,A) \tilde{\cap} (F_2,A) = \tilde{\varphi}$ or $(F_2,A) = (F_1,A) \tilde{\cap} (F_2,A)$. But by hypothesis, $(F_1,A) \tilde{\cap} (F_2,A) \neq \tilde{\varphi}$.So, $(F_2,A) = (F_1,A) \tilde{\cap} (F_2,A)$. Thus $(F_2,A) \tilde{\subseteq} (F_1,A)$…………(ii)
Thus, from (i) & (ii), it follows that $(F_1,A) = (F_2,A)$.

Example 3.22: In the example 3.13, we find that the soft sets $(F_4,A)^c$ and $(F_5,A)^c$ are both minimal soft δ-closed sets and $(F_4,A)^c \tilde{\cap} (F_5,A)^c = \tilde{X}$ but $(F_4,A)^c \neq (F_5,A)^c$.

Lemma 3.23: Intersection of arbitrary family of minimal soft δ-closed sets is either minimal soft δ-closed set or soft null set $\tilde{\varphi}$.

Proof : The proof follows from above theorem 3.21.

Theorem 3.24: Let (F_1,A),(F_2,A) and (F_3,A) be three non-empty proper maximal soft δ-open sets in X such that $(F_1,A) \neq (F_2,A)$. If $(F_1,A) \tilde{\cap} (F_2,A) \tilde{\subset} (F_1,A)$, then either $(F_1,A) = (F_3,A)$ or $(F_2,A) = (F_3,A)$.

Proof: Given that $(F_1,A) \tilde{\cap} (F_2,A) \tilde{\subset} (F_3, A)$. If $(F_1, A) = (F_3, A)$, then there is nothing to prove.Now suppose, $(F_1, A) \neq (F_3, A)$, then we have to prove that $(F_2, A) = (F_3, A)$.

Now, $(F_2, A) \tilde{\cap} (F_3, A) = (F_2,A) \tilde{\cap} ((F_3, A) \tilde{\cap} 1_X)$

$= (F_2,A) \tilde{\cap} [(F_3, A) \tilde{\cap} ((F_1, A) \tilde{\cup} (F_2,A))]$ [since, $(F_1, A) \tilde{\cup} (F_2,A) = 1_X$, by theorem 3.17]

$= (F_2,A) \tilde{\cap} [((F_3, A) \tilde{\cap} (F_1, A)) \tilde{\cup} ((F_3, A) \tilde{\cap} (F_2,A))]$

$= ((F_2,A) \tilde{\cap} (F_3, A) \tilde{\cap} (F_1, A)) \tilde{\cup} ((F_2,A) \tilde{\cap} (F_3, A) \tilde{\cap} (F_2,A))$

$= ((F_1, A) \tilde{\cap} (F_2,A)) \tilde{\cup} ((F_3, A) \tilde{\cap} (F_2,A))$ [as $(F_1, A) \tilde{\cap} (F_2,A) \tilde{\subset} (F_3,A)$]

$= ((F_1, A) \tilde{\cup} (F_3, A)) \tilde{\cap} (F_2,A)$

$= \tilde{X} \tilde{\cap} (F_2,A)$ [since, $(F_1, A) \tilde{\cup} (F_3, A) = \tilde{X}$, by theorem 3.17]

$= (F_2,A)$

which implies $(F_2,A) \tilde{\subset} (F_3, A)$. So by definition of maximal soft δ-open sets & by the fact that $(F_3,A)= \tilde{X}$, it follows that $(F_2,A)=(F_3,A)$.

Theorem 2.25: Let (F_1,A), (F_2, A) and (F_3, A) be three non-empty proper maximal soft δ-open sets in X such that $(F_1,A) \neq (F_2,A) \neq (F_3,A)$, then $(F_1,A) \tilde{\cap} (F_2,A) \tilde{\not\subset} (F_1,A) \tilde{\cap} (F_3,A))$.

Proof : If possible let, $(F_1,A) \tilde{\cap} (F_2,A) \tilde{\subset} (F_1, A) \tilde{\cap} (F_3,A))$.
$\Rightarrow$ $[(F_1,A) \tilde{\cap} (F_2,A)] \tilde{\cup} [(F_2, A) \tilde{\cap} (F_3,A))] \tilde{\subset} [(F_1,A) \tilde{\cap} (F_3,A)] \tilde{\cup} [(F_2,A) \tilde{\cap} (F_3,A)]$.
$\Rightarrow$ $((F_1,A) \tilde{\cup} (F_3, A)) \tilde{\cap} (F_2,A) \tilde{\subset} ((F_1,A) \tilde{\cup} (F_2,A)) \tilde{\cap} (F_3,A)$. Since by hypothesis, $(F_1,A) \neq (F_2, A)$ & $(F_2, A) \neq (F_3, A)$, so, by theorem 3.17, we must have, $(F_1,A) \tilde{\cup} (F_3, A)= \tilde{X}$ and $(F_1,A) \tilde{\cup} (F_2, A) = \tilde{X}$. So, $1_X \wedge (F_2, A) \tilde{\subset} \tilde{X} \tilde{\cap} (F_3, A)$ so that $(F_2, A) \tilde{\subset} (F_3, A) \Rightarrow (F_2, A)=(F_3, A)$ or $(F_3, A)= \tilde{X}$ (since, (F_2, A) is maximal soft δ-open set). But by hypothesis, $(F_3, A) \neq \tilde{X}$. So $(F_2,A) = (F_3, A)$, which contradicts the hypothesis that $(F_2, A) \neq (F_3, A)$. Hence, $(F_1,A) \tilde{\cap} (F_2,A) \tilde{\not\subset} (F_1, A) \tilde{\cap} (F_3,A))$.

4. CONCLUSION

The soft set theory proposed by Molodtsov offers a general mathematical tool for dealing with uncertain or vague objects. It is shown that soft sets are special type of information system known as single valued information system. In this work, the concept of maximal soft δ-open sets and minimal soft δ-closed sets which are fundamental results for further research on soft topological spaces are introduced and aimed in investigating the properties of these new notions of open sets with example, counter examples and some of their fundamental results are also established. I hope that the findings in this paper will help researcher enhance and promote the further study on soft topological spaces to carry out a general framework for their applications in separation axioms, connectedness, compactness etc. and also in practical life.

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