

Reliability and MTSF Evaluation of a Parallel-Series System using Weibull Failure Laws

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ABSTRACT

Here, the expressions for reliability and mean time to system failure (MTSF) of a parallel-series system of order (m, n) are derived by considering Weibull distribution for failure time of the components. The results of these measures are also obtained for a particular case of Weibull distribution i.e. for Rayleigh distribution. The behaviour of reliability and MTSF has been observed for arbitrary values of the number of components, number of subsystems, operating time of the components, shape parameter(β) and failure rate of the components. The analytical study of the measures has been confined only to the system of order (5,5). The results are shown numerically and graphically for arbitrary values of the different parameters.

Keywords

Parallel-Series System, MTSF, Reliability and Weibull Failure Laws

1. INTRODUCTION

Recently, the studies on operating systems with mixed mode structure have been reported by the scholars in the field of reliability theory. Elsyaed, A. (2012) developed reliability measures of a series-parallel system with some arbitrary distributions. Dao et al. (2014) obtained reliability of a multi-state series-parallel system with selective maintenance under different conditions. Also, the practical utility of mixed mode structures of the components in complex systems has been charised by the users due to their durability and fault free service characteristics. The researchers have proved that the arrangement of components in series is not better than that of parallel so far as reliability is concerned. Chauhan and Malik (2016) have analyzed series-parallel and parallel-series

3. SYSTEM DESCRIPTION

A Parallel-Series system of order (m, n) consisting 'm' identical parallel systems each of order 'n' arranged in series as shown in the Fig.1:

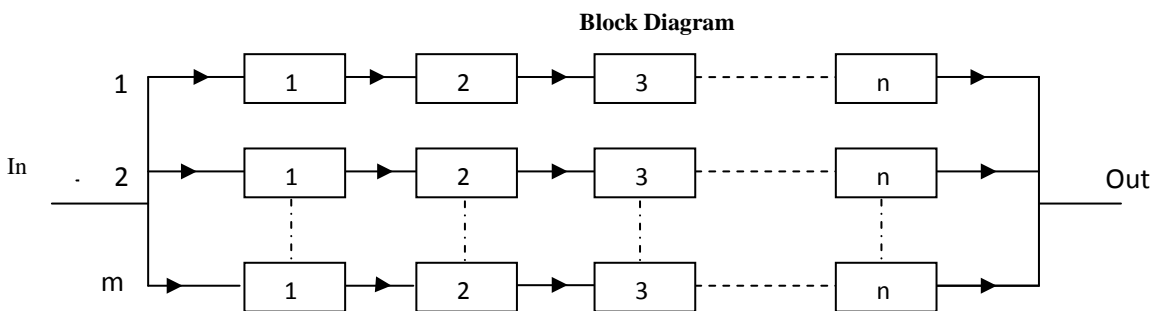


Fig.1: System with Parallel-Series Configuration of 'n' components.

The system reliability at time 't' is given by

$$= 1 - (1 - \prod_{i=1}^n R_i(t))^m \quad (1)$$

$$R_s(t) = 1 - \prod_{j=1}^m (1 - \prod_{i=1}^n R_i(t))$$

systems by considering exponential distribution for failure time of the components. They concluded that series-parallel structure of the components is better than that of parallel-series structure in terms of reliability. The reliability and MTSF of a series and parallel systems of order 'n' of identical components has been evaluated by Chauhan and Malik (2016 & 2017) with Weibull failure laws. It is revealed that parallel system is more reliable to use as compare to series system of identical components. Thus, it becomes necessary to examine the reliability measures a parallel system of 'm' subsystems each having 'n' non- identical components connected in series.

Thus, the purpose of the present study is to derive the expressions for reliability and mean time to system failure (MTSF) of a parallel-series system of order (m, n) with Weibull failure laws. The expressions for these measures are also obtained for a particular case i.e. for the Rayleigh failure laws. Giving arbitrary values to the parameters like number of components, number of subsystems, operating time of the component and failure rates of the components, the values of reliability and MTSF are evaluated. The behaviour of these measures is examined graphically with respect to the parameters. The analytical study of the measures has been confined only to the system of order (5, 5).

2. NOTATIONS

$R(t)$ = Reliability of the system, $R_i(t)$ = Reliability of the i^{th} component, $h(t)$ = Instantaneous failure rate of the system, $h_i(t)$ = Instantaneous failure rate of i^{th} component, λ = Constant failure rate, T = Life time of the system, T_i = Life time of the i^{th} component, m = Number of Subsystems, n = Number of Components connecting in Series.

The mean time to system failure is given by

$$MTSF = \int_0^\infty R_s(t) dt = \int_0^\infty [1 - (1 - \prod_{i=1}^n R_i(t))^m] dt \quad (2)$$

4. RELIABILITY MEASURES OF A PARALLEL-SERIES SYSTEM WITH WEIBULL DISTRIBUTION

Suppose, failure rate of all the components are governed by weibull failure laws i.e. $h_i(t) = \lambda_i t^{\beta_i}$

Then, the components reliability is given by

$$R_i(t) = e^{-\int_0^t h_i(u) du} = e^{-\int_0^t \lambda_i u^{\beta_i} du} = e^{-\lambda_i \frac{t^{\beta_i+1}}{\beta_i+1}}$$

Therefore, the system reliability is given by

$$R_s(t) = 1 - \left(1 - \prod_{i=1}^n R_i(t)\right)^m = 1 - \left(1 - \prod_{i=1}^n e^{-\lambda_i \frac{t^{\beta_i+1}}{\beta_i+1}}\right)^m$$

$$\text{And, } MTSF = \int_0^\infty R_s(t) dt = \int_0^\infty \left[1 - \left(1 - \prod_{i=1}^n e^{-\lambda_i \frac{t^{\beta_i+1}}{\beta_i+1}}\right)^m\right] dt$$

For identical components, we can have $h_i(t) = \lambda t^\beta$

The component reliability is given by $R_i(t) = e^{-\lambda \frac{t^{\beta+1}}{\beta+1}}$

Then the system reliability is given by

$$R_s(t) = 1 - \left(1 - \prod_{i=1}^n e^{-\lambda \frac{t^{\beta+1}}{\beta+1}}\right)^m = 1 - \left(1 - e^{-n\lambda \frac{t^{\beta+1}}{\beta+1}}\right)^m$$

$$= \binom{m}{1} e^{-n\lambda \frac{t^{\beta+1}}{\beta+1}} - \binom{m}{2} e^{-2n\lambda \frac{t^{\beta+1}}{\beta+1}} + \binom{m}{3} e^{-3n\lambda \frac{t^{\beta+1}}{\beta+1}} - \dots + (-1)^m e^{-mn\lambda \frac{t^{\beta+1}}{\beta+1}}$$

$$MTSF = \int_0^\infty \left[\binom{m}{1} e^{-n\lambda \frac{t^{\beta+1}}{\beta+1}} - \binom{m}{2} e^{-2n\lambda \frac{t^{\beta+1}}{\beta+1}} + \binom{m}{3} e^{-3n\lambda \frac{t^{\beta+1}}{\beta+1}} - \dots + (-1)^m e^{-mn\lambda \frac{t^{\beta+1}}{\beta+1}} \right] dt$$

$$\begin{aligned} MTSF &= \binom{m}{1} \frac{\Gamma \frac{1}{\beta+1}}{[n\lambda(\beta+1)^\beta]^{\frac{1}{\beta+1}}} \\ &- \binom{m}{2} \frac{\Gamma \frac{1}{\beta+1}}{[2n\lambda(\beta+1)^\beta]^{\frac{1}{\beta+1}}} + \dots + (-1)^m \frac{\Gamma \frac{1}{\beta+1}}{[mn\lambda(\beta+1)^\beta]^{\frac{1}{\beta+1}}} \\ &= \frac{\Gamma \frac{1}{\beta+1}}{[n\lambda(\beta+1)^\beta]^{\frac{1}{\beta+1}}} \left[\binom{m}{1} - \binom{m}{2} \frac{1}{2^{\frac{1}{\beta+1}}} + \binom{m}{3} \frac{1}{3^{\frac{1}{\beta+1}}} - \dots \dots - 1m1m1\beta+1 \right] \end{aligned}$$

5. RELIABILITY MEASURES FOR ARBITRARY VALUES OF THE PARAMETERS

Reliability and mean time to system failure (MTSF) of the system has been obtained for arbitrary values of the parameters associated with number of components (n), failure rate (λ), operating time of the component (t) and shape parameter (β). The results are shown numerically and graphically as:

TABLE1: Reliability Vs No. of Components (n) and Subsystems (m)

m	n	Reliability				
		$\lambda=0.01, \beta=0.1, t=10$	$\lambda=0.02, \beta=0.1, t=10$	$\lambda=0.03, \beta=0.1, t=10$	$\lambda=0.04, \beta=0.1, t=10$	$\lambda=0.05, \beta=0.1, t=10$
1	1	0.891859	0.795412	0.709395	0.63268	0.56426
	2	0.795412	0.63268	0.503241	0.400284	0.31839
	3	0.709395	0.503241	0.356996	0.253251	0.17965
	4	0.63268	0.400284	0.253251	0.160227	0.10137
	5	0.564261	0.31839	0.179655	0.101372	0.0572
2	1	0.988305	0.958144	0.915549	0.865076	0.81013
	2	0.958144	0.865076	0.75323	0.64034	0.53540
	3	0.915549	0.75323	0.586546	0.442366	0.32703
	4	0.865076	0.64034	0.442366	0.294781	0.19246
	5	0.810131	0.535408	0.327034	0.192468	0.11112
3	1	0.998735	0.991437	0.975458	0.95044	0.91726
	2	0.991437	0.95044	0.877415	0.784306	0.68333
	3	0.975458	0.877415	0.734148	0.583588	0.44793
	4	0.95044	0.784306	0.583588	0.407776	0.27433
	5	0.917267	0.68333	0.447936	0.27433	0.16193
4	1	0.999863	0.998248	0.992868	0.981795	0.96395
	2	0.998248	0.981795	0.939105	0.870645	0.78415

	3	0.992868	0.939105	0.829056	0.689045	0.54711	
	4	0.981795	0.870645	0.689045	0.502666	0.34789	
	5	0.96395	0.784154	0.547117	0.347893	0.20990	
	5	1	0.999985	0.999642	0.997927	0.993313	0.98429
		2	0.999642	0.993313	0.96975	0.922424	0.85287
3		0.997927	0.96975	0.890082	0.767794	0.62848	
4		0.993313	0.922424	0.767794	0.582353	0.41399	
5		0.984292	0.852877	0.62848	0.413998	0.25510	

Fig.2: Reliability Vs No. of Components (n) and Subsystems (m)

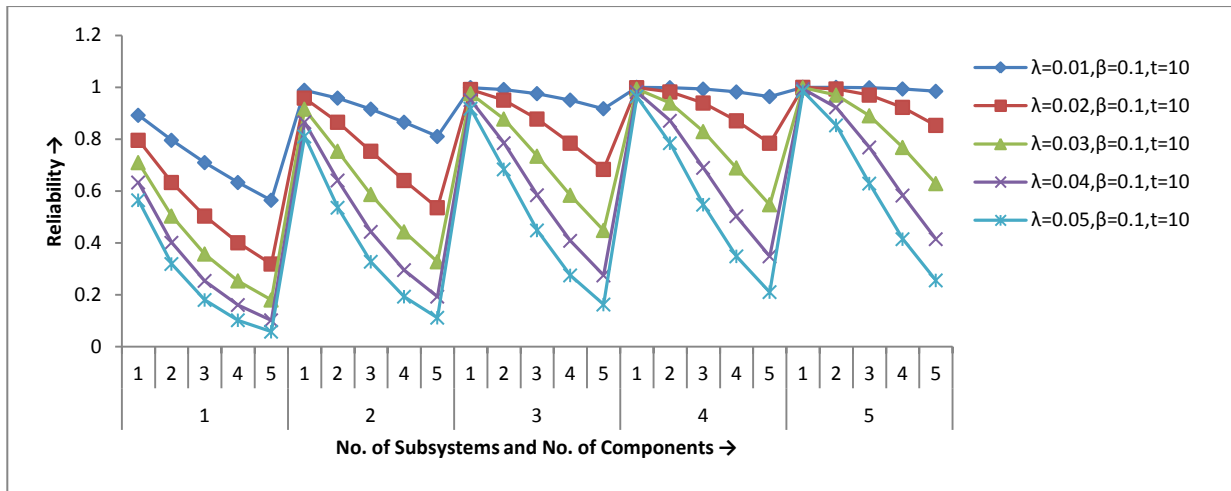


TABLE2: MTSF Vs No. of Components (n) and Subsystems (m)

m	n	MTSF				
		$\lambda=0.01, \beta=0.1, t=10$	$\lambda=0.02, \beta=0.1, t=10$	$\lambda=0.03, \beta=0.1, t=10$	$\lambda=0.04, \beta=0.1, t=10$	$\lambda=0.05, \beta=0.1, t=10$
1	1	69.23057	36.8667	25.50065	19.63228	16.02768
	2	36.8667	19.63228	13.57962	10.45459	8.53507
	3	25.50065	13.57962	9.39301	7.23143	5.9037
	4	19.63228	10.45459	7.23143	5.56728	4.5451
	5	16.02768	8.53507	5.9037	4.5451	3.71059
2	1	101.5944	54.10113	37.42169	28.80996	23.52029
	2	54.10113	28.80996	19.92782	15.3419	12.52504
	3	37.42169	19.92782	13.78405	10.61197	8.66355
	4	28.80996	15.3419	10.61197	8.16988	6.66984
	5	23.52029	12.52504	8.66355	6.66984	5.44522
3	1	122.5923	65.2829	45.15611	34.76449	28.38153
	2	65.2829	34.76449	24.04655	18.5128	15.11375
	3	45.15611	24.04655	16.63297	12.80528	10.45416
	4	34.76449	18.5128	12.80528	9.85845	8.04838
	5	28.38153	15.11375	10.45416	8.04838	6.57065
4	1	138.0924	73.53705	50.86549	39.15999	31.96999
	2	73.53705	39.15999	27.08692	20.8535	17.02468

	3	50.86549	27.08692	18.73599	14.42434	11.77595	
	4	39.15999	20.8535	14.42434	11.10492	9.06599	
	5	31.96999	17.02468	11.77595	9.06599	7.40142	
	5	1	150.3587	80.06909	55.3837	42.63844	34.80978
		2	80.06909	42.63844	29.49296	22.70584	18.53692
3		55.3837	29.49296	20.40024	15.7056	12.82197	
4		42.63844	22.70584	15.7056	12.09133	9.87129	
5		34.80978	18.53692	12.82197	9.87129	8.05887	

Fig.3: MTSF Vs No. of Components (n) and Subsystems (m)

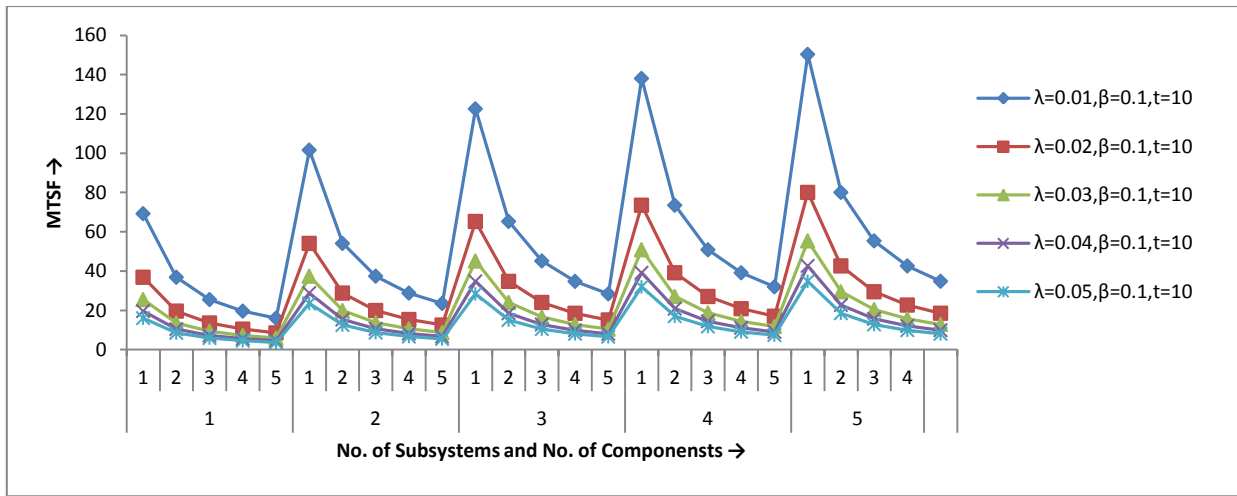


TABLE3: Reliability Vs No. of Components (n) and Subsystems (m)

m	n	Reliability				
		$\beta=0.1,$ $\lambda=0.01, t=10$	$\beta=0.2,$ $\lambda=0.01, t=10$	$\beta=0.3,$ $\lambda=0.01, t=10$	$\beta=0.4,$ $\lambda=0.01, t=10$	$\beta=0.5,$ $\lambda=0.01, t=10$
1	1	0.891859	0.876276	0.857716	0.835754	0.809921
	2	0.795412	0.767859	0.735678	0.698485	0.655972
	3	0.709395	0.672856	0.631003	0.583762	0.531286
	4	0.63268	0.589608	0.541221	0.487882	0.430299
	5	0.564261	0.516659	0.464214	0.407749	0.348509
2	1	0.988305	0.984692	0.979755	0.973023	0.96387
	2	0.958144	0.946111	0.930134	0.909089	0.881645
	3	0.915549	0.892977	0.863841	0.826746	0.780307
	4	0.865076	0.831578	0.789522	0.737735	0.675441
	5	0.810131	0.766381	0.712934	0.649239	0.575559
3	1	0.998735	0.998106	0.99712	0.995569	0.993132
	2	0.991437	0.98749	0.981533	0.972589	0.959283
	3	0.975458	0.964988	0.949758	0.927885	0.897027
	4	0.95044	0.930881	0.903437	0.865689	0.815099
	5	0.917267	0.887083	0.846194	0.792262	0.72348
4	1	0.999863	0.999766	0.99959	0.999272	0.998695
	2	0.998248	0.997096	0.995119	0.991735	0.985992

	3	0.992868	0.988546	0.981461	0.969983	0.951735
	4	0.981795	0.971634	0.955699	0.931217	0.894662
	5	0.96395	0.945422	0.917593	0.876967	0.81985
5	1	0.999985	0.999971	0.999942	0.99988	0.999752
	2	0.999642	0.999326	0.99871	0.997508	0.995181
	3	0.997927	0.996253	0.993159	0.987506	0.977377
	4	0.993313	0.988359	0.979676	0.964775	0.939989
	5	0.984292	0.97362	0.955848	0.927134	0.882634

Fig.4: Reliability Vs No. of Components (n) and Subsystems (m)

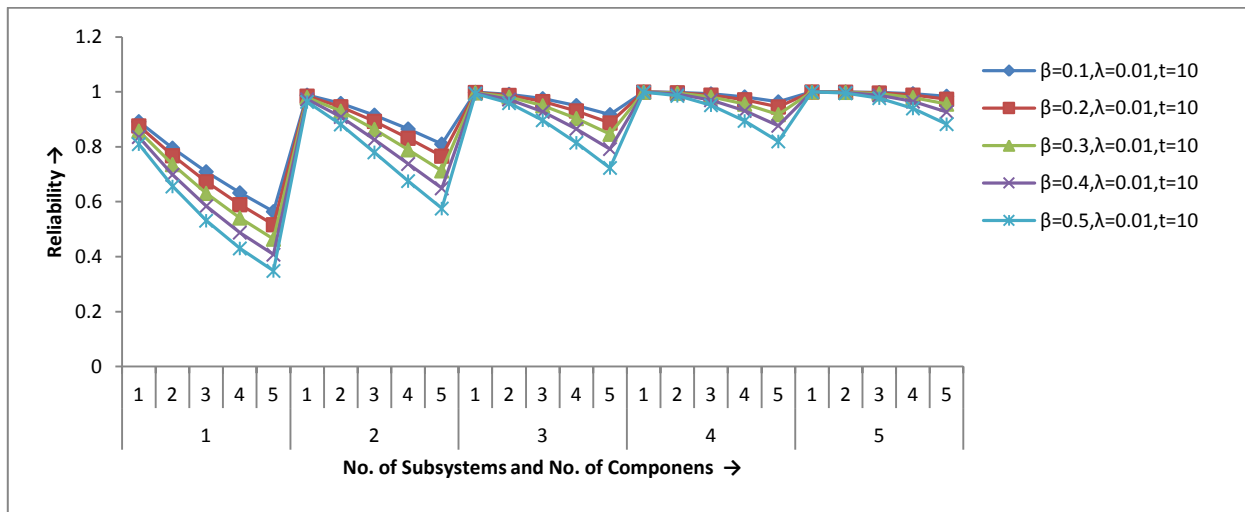


TABLE4: MTSF Vs No. of Components (n) and Subsystems (m)

m	n	MTSF				
		$\beta=0.1, \lambda=0.01, t=10$	$\beta=0.2, \lambda=0.01, t=10$	$\beta=0.3, \lambda=0.01, t=10$	$\beta=0.4, \lambda=0.01, t=10$	$\beta=0.5, \lambda=0.01, t=10$
1	1	69.23057	50.82565	39.04661	31.09362	25.48514
	2	36.8667	28.52493	22.90983	18.95177	16.05463
	3	25.50065	20.34613	16.77131	14.18634	12.25198
	4	19.63228	20.34613	13.44189	11.55123	10.11378
	5	16.02768	13.29254	11.32176	9.84934	8.71579
2	1	101.5944	73.12637	55.18339	43.23546	34.91564
	2	54.10113	41.04079	32.37777	26.35231	21.99548
	3	37.42169	29.27339	23.70239	19.72601	16.78569
	4	28.80996	23.03336	18.99701	16.06191	13.85628
	5	23.52029	19.12489	16.00071	13.69543	11.94098
3	1	122.5923	87.24829	65.18166	50.61187	40.5435
	2	65.2829	48.96645	38.24405	30.84828	25.5408
	3	45.15611	34.92657	27.99684	23.09147	19.49128
	4	34.76449	27.48149	22.43894	18.80224	16.0897
	5	28.38153	22.81823	18.89976	16.03202	13.86568
4	1	138.0924	97.52847	72.37083	55.85796	44.50689
	2	73.53705	54.736	42.46216	34.04581	28.03758

	3	50.86549	39.04185	31.08474	25.48497	21.39668	
	4	39.15999	30.71954	24.91383	20.75115	17.66257	
	5	31.96999	25.50683	20.98429	17.69379	15.22114	
	5	1	150.3587	105.5874	77.96021	59.90692	47.54602
		2	80.06909	59.25894	45.74161	36.51368	29.95212
3		55.3837	42.26795	33.48549	27.3323	22.85774	
4		42.63844	33.25796	26.83799	22.25534	18.86865	
5		34.80978	27.6145	22.60496	18.97635	16.26051	

Fig.5: MTSF Vs No. of Components (n) and Subsystems (m)

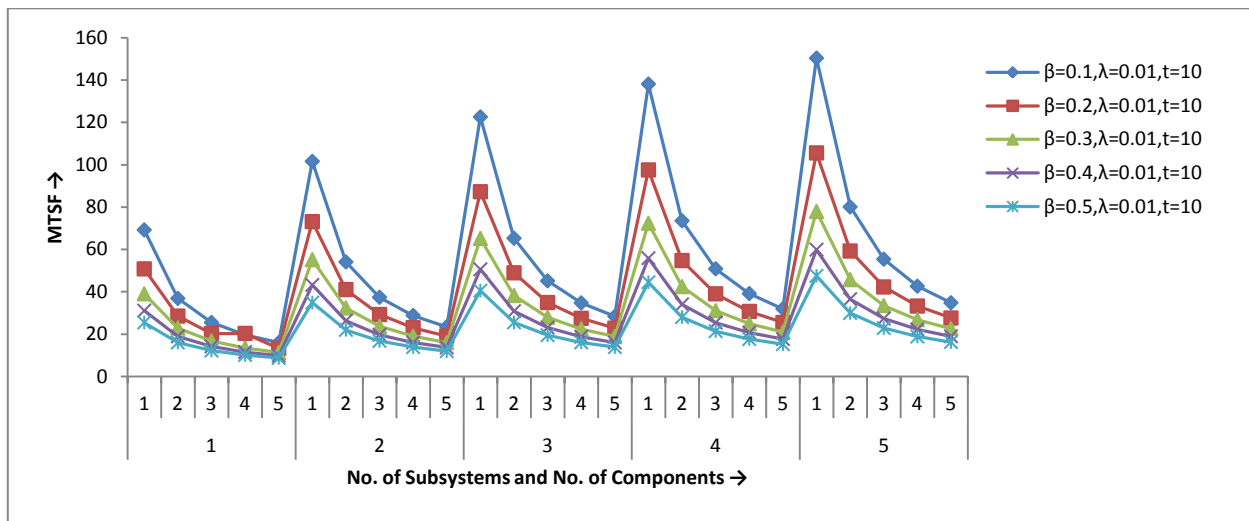
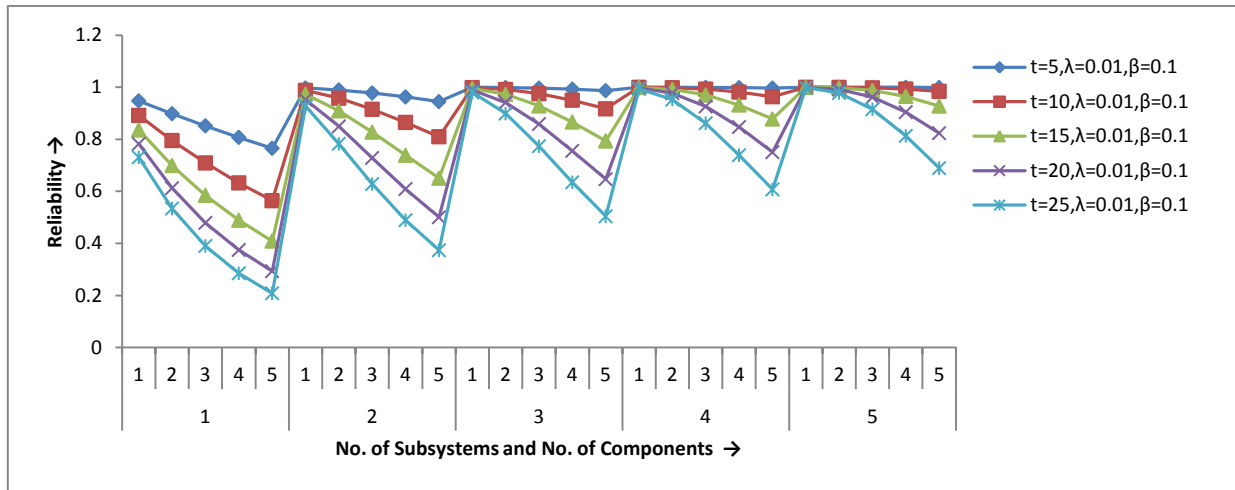


TABLE5: Reliability Vs No. of Components (n) and Subsystems (m)

m	n	Reliability				
		t=5, $\lambda=0.01, \beta=0.1$	t=10, $\lambda=0.01, \beta=0.1$	t=15, $\lambda=0.01, \beta=0.1$	t=20, $\lambda=0.01, \beta=0.1$	t=25, $\lambda=0.01, \beta=0.1$
1	1	0.948009	0.891859	0.836294	0.782451	0.73083
	2	0.89872	0.795412	0.699387	0.612229	0.534112
	3	0.851994	0.709395	0.584893	0.479039	0.390345
	4	0.807698	0.63268	0.489142	0.374825	0.285276
	5	0.765705	0.564261	0.409067	0.293282	0.208488
2	1	0.997297	0.988305	0.9732	0.952672	0.927547
	2	0.989742	0.958144	0.909632	0.849634	0.782948
	3	0.978094	0.915549	0.827686	0.7286	0.628321
	4	0.96302	0.865076	0.739025	0.609156	0.489169
	5	0.945106	0.810131	0.650798	0.50055	0.373508
3	1	0.999859	0.998735	0.995613	0.989704	0.980498
	2	0.998961	0.991437	0.972834	0.941692	0.898878
	3	0.996758	0.975458	0.928471	0.858611	0.773404
	4	0.992889	0.95044	0.866679	0.755654	0.634897
	5	0.987139	0.917267	0.793645	0.647029	0.504124
4	1	0.999993	0.999863	0.999282	0.99776	0.994751
	2	0.999895	0.998248	0.991834	0.97739	0.952889

	3	0.99952	0.992868	0.970308	0.926342	0.861854
	4	0.998632	0.981795	0.931892	0.847241	0.739052
	5	0.996987	0.96395	0.878058	0.750549	0.607508
5	1	0.999996	0.999985	0.999882	0.999513	0.998587
	2	0.999989	0.999642	0.997545	0.991233	0.978051
	3	0.999929	0.997927	0.987675	0.961627	0.915779
	4	0.999737	0.993313	0.965206	0.904499	0.813494
	5	0.999294	0.984292	0.92794	0.823709	0.689338

Fig.6: Reliability Vs No. of Components (n) and Subsystems (m)



6. RELIABILITY MEASURES FOR A SPECIAL CASE (RAYLEIGH DISTRIBUTION) OF WEIBULL DISTRIBUTION

The Rayleigh distribution has extensively been used in life testing experiments, reliability analysis, communication engineering, clinical studies and applied statistics. This distribution is a special case of Weibull distribution with the shape parameter $\beta=1$.

When components are governed by Rayleigh failure laws, the component reliability is given by

$$R_i(t) = e^{-\int_0^t h_i(u)du} = e^{-\int_0^t \lambda_i u du} = e^{-\frac{\lambda_i t^2}{2}}$$

where $h_i(t) = \lambda_i t$

Therefore, the system reliability is given by

$$R_s(t) = 1 - [1 - \prod_{i=1}^n R_i(t)]^m$$

$$= 1 - \left[1 - e^{-\frac{(\lambda_1 + \lambda_2 + \lambda_3 + \dots + \lambda_n)t^2}{2}} \right]^m = 1 - \left[1 - e^{-\frac{\sum_{i=1}^n \lambda_i t^2}{2}} \right]^m$$

$$= 1 - \left[1 - \binom{m}{1} e^{-\frac{\sum_{i=1}^n \lambda_i t^2}{2}} + \binom{m}{2} e^{-\frac{\sum_{i=1}^n \lambda_i t^2}{2}} - \binom{m}{3} e^{-\frac{\sum_{i=1}^n \lambda_i t^2}{2}} + \dots + (-1)^m e^{-\frac{\sum_{i=1}^n \lambda_i t^2}{2}} \right]$$

MTSF =

$$\binom{m}{1} \sqrt{\frac{\pi}{2 \sum_{i=1}^n \lambda_i}} - \binom{m}{2} \sqrt{\frac{\pi}{2 \sum_{i=1}^n \lambda_i}} + \dots - (-1)^m \sqrt{\frac{\pi}{2 \sum_{i=1}^n \lambda_i}}$$

For identical component we can have $\lambda_i t = \lambda t$

The system reliability is given by

$$R_s(t) = 1 - \left[1 - e^{-\frac{n\lambda t^2}{2}} \right]^m = \binom{m}{1} e^{-\frac{n\lambda t^2}{2}} - \binom{m}{2} e^{-\frac{2n\lambda t^2}{2}} + \binom{m}{3} e^{-\frac{3n\lambda t^2}{2}} - \dots - (-1)^m e^{-\frac{mn\lambda t^2}{2}}$$

$$MTSF = \binom{m}{1} \sqrt{\frac{\pi}{2n\lambda}} - \binom{m}{2} \sqrt{\frac{\pi}{4n\lambda}} + \binom{m}{3} \sqrt{\frac{\pi}{6n\lambda}} - \dots - (-1)^m \sqrt{\frac{\pi}{2mn\lambda}}$$

$$= \sqrt{\frac{\pi}{2n\lambda}} \left[\binom{m}{1} - \binom{m}{2} \sqrt{\frac{1}{2}} + \binom{m}{3} \sqrt{\frac{1}{3}} + \dots + (-1)^m \sqrt{\frac{1}{m}} \right]$$

7. RELIABILITY MEASURES FOR ARBITRARY VALUES OF THE PARAMETERS

Reliability and mean time to system failure (MTSF) of the system has been obtained for arbitrary values of the parameters associated with number of components (n), failure rate (λ) and operating time of the components (t). The results are shown numerically and graphically:

TABLE6: Reliability Vs No. of Components (n) and Subsystems (m)

m	n	Reliability				
		$\lambda=0.01,t=10$	$\lambda=0.02,t=10$	$\lambda=0.03,t=10$	$\lambda=0.04,t=10$	$\lambda=0.05,t=10$
1	1	0.606531	0.367879	0.22313	0.135335	0.082085
	2	0.367879	0.135335	0.049787	0.018316	0.006738
	3	0.22313	0.049787	0.011109	0.002479	0.000553
	4	0.135335	0.018316	0.002479	0.000335	0.000045
	5	0.082085	0.006738	0.000553	0.000045	0.00000037
2	1	0.845182	0.600424	0.396473	0.252355	0.157432
	2	0.600424	0.252355	0.097095	0.036296	0.01343
	3	0.396473	0.097095	0.022095	0.004951	0.001106
	4	0.252355	0.036296	0.004951	0.000671	0.0000907
	5	0.157432	0.01343	0.001106	0.0000907	0.00000074
3	1	0.939084	0.74742	0.531138	0.353538	0.226594
	2	0.74742	0.353538	0.142048	0.053947	0.020078
	3	0.531138	0.142048	0.032958	0.007418	0.001658
	4	0.353538	0.053947	0.007418	0.001006	0.000136
	5	0.226594	0.020078	0.001658	0.000136	0.00001117
4	1	0.990569	0.899075	0.717029	0.516676	0.348353
	2	0.899075	0.516676	0.225352	0.088284	0.033239
	3	0.717029	0.225352	0.054325	0.012332	0.002762
	4	0.516676	0.088284	0.012332	0.001676	0.000227
	5	0.348353	0.033239	0.002762	0.000227	0.0000186
5	1	0.990569	0.899075	0.717029	0.516676	0.348353
	2	0.899075	0.516676	0.225352	0.088284	0.033239
	3	0.717029	0.225352	0.054325	0.012332	0.002762
	4	0.516676	0.088284	0.012332	0.001676	0.000227
	5	0.348353	0.033239	0.002762	0.000227	0.00002063

Fig.7: Reliability Vs No. of Components (n) and Subsystems (m)

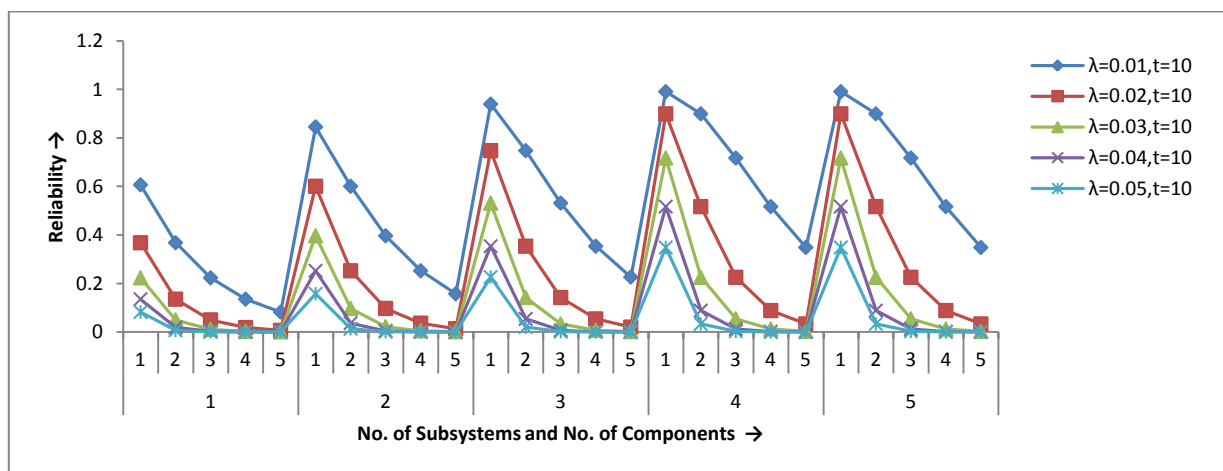


TABLE7: MTSF Vs No. of Components (n) and Subsystems (m)

m	n	MTSF				
		$\lambda=0.01,t=10$	$\lambda=0.02,t=10$	$\lambda=0.03,t=10$	$\lambda=0.04,t=10$	$\lambda=0.05,t=10$
1	1	12.53314	8.86227	7.23601	6.26657	5.60499
	2	8.86227	6.26657	5.11663	4.43113	3.96333
	3	7.23601	5.11663	4.17771	3.61801	3.23604
	4	6.26657	4.43113	3.61801	3.13329	2.8025
	5	5.60499	3.96333	3.23604	2.8025	2.50663
2	1	16.20401	11.45797	9.35539	8.10201	7.24666
	2	11.45797	8.10201	6.61526	5.72898	5.12416
	3	9.35539	6.61526	5.40134	4.6777	4.18386
	4	8.10201	5.72898	4.6777	4.051	3.62333
	5	7.24666	5.12416	4.18386	3.62333	3.2408
3	1	18.24863	12.90373	10.53585	9.12431	8.16103
	2	12.90373	9.12431	7.44997	6.45186	5.77072
	3	10.53585	7.44997	6.08288	5.26793	4.71178
	4	9.12431	6.45186	5.26793	4.56216	4.08052
	5	8.16103	5.77072	4.71178	4.08052	3.64973
4	1	19.63643	13.88505	11.3371	9.81821	8.78168
	2	13.88505	9.81821	8.01654	6.94253	6.20958
	3	11.3371	8.01654	6.54548	5.66855	5.0701
	4	9.81821	6.94253	5.66855	4.90911	4.39084
	5	8.78168	6.20958	5.0701	4.39084	3.92729
5	1	20.67528	14.61963	11.93688	10.33764	9.24627
	2	14.61963	10.33764	8.44065	7.30981	6.5381
	3	11.93688	8.44065	6.89176	5.96844	5.33833
	4	10.33764	7.30981	5.96844	5.16882	4.62313
	5	9.24627	6.5381	5.33833	4.62313	4.13506

Fig.8: MTSF Vs No. of Components (n) and Subsystems (m)

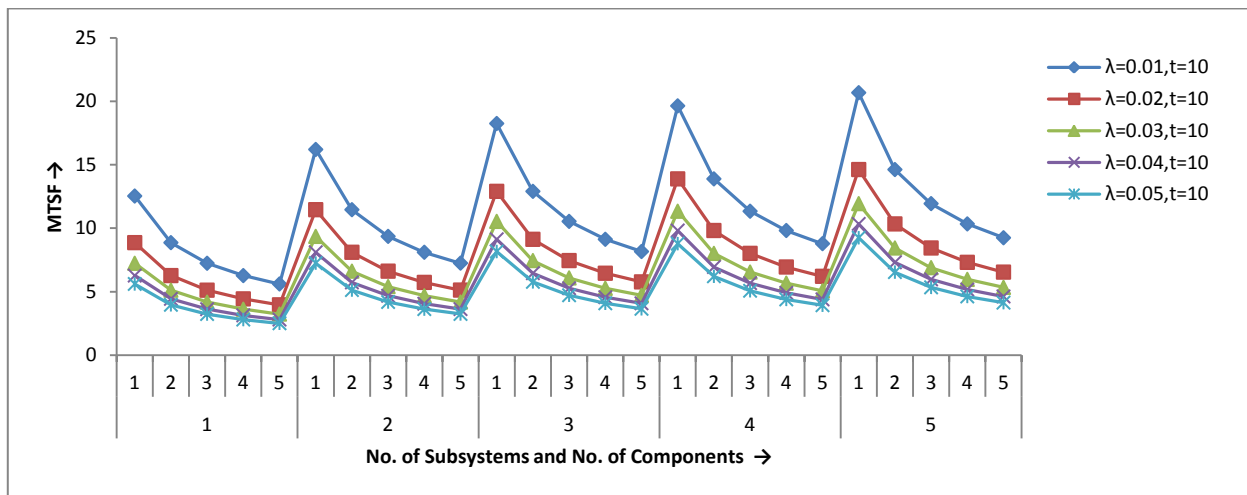
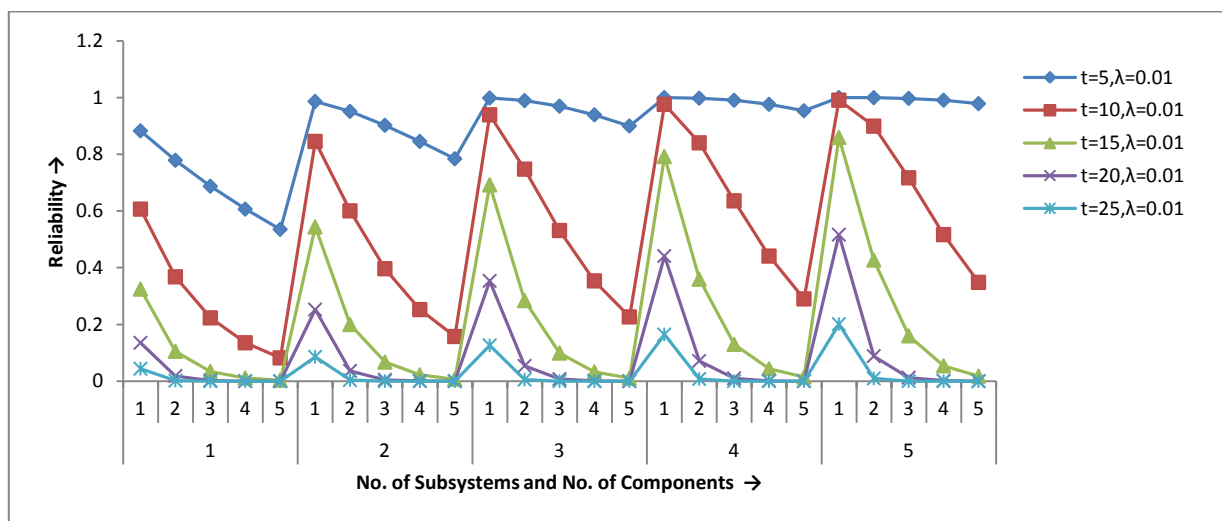


TABLE8: Reliability Vs No. of Components (n) and Subsystems (m)

m	n	Reliability				
		t=5,λ=0.01	t=10,λ=0.01	t=15,λ=0.01	t=20,λ=0.01	t=25,λ=0.01
1	1	0.882497	0.606531	0.324652	0.135335	0.043937
	2	0.778801	0.367879	0.105399	0.018316	0.00193
	3	0.687289	0.22313	0.034218	0.002479	0.0000848
	4	0.606531	0.135335	0.011109	0.000335	0.00000037
	5	0.535261	0.082085	0.003607	0.000045	0.000000016
2	1	0.986193	0.845182	0.543906	0.252355	0.085943
	2	0.951071	0.600424	0.199689	0.036296	0.003857
	3	0.902212	0.396473	0.067265	0.004951	0.00017
	4	0.845182	0.252355	0.022095	0.000671	0.00000074
	5	0.784018	0.157432	0.0072	0.0000907	0.000000032
3	1	0.998378	0.939084	0.691978	0.353538	0.126104
	2	0.989177	0.74742	0.284042	0.053947	0.00578
	3	0.969421	0.531138	0.099182	0.007418	0.000254
	4	0.939084	0.353538	0.032958	0.001006	0.0000011
	5	0.899625	0.226594	0.010781	0.000136	0.000000049
4	1	0.999809	0.976031	0.791978	0.441027	0.164501
	2	0.997606	0.840339	0.359503	0.071274	0.007699
	3	0.990438	0.635755	0.130006	0.009878	0.000339
	4	0.976031	0.441027	0.043701	0.001341	0.0000014
	5	0.953352	0.290079	0.014348	0.000182	0.000000065
5	1	0.999978	0.990569	0.859513	0.516676	0.20121
	2	0.99947	0.899075	0.427011	0.088284	0.009615
	3	0.99701	0.717029	0.159776	0.012332	0.000424
	4	0.990569	0.516676	0.054325	0.001676	0.0000018
	5	0.978321	0.348353	0.017903	0.000227	0.000000081

Fig.9: Reliability Vs No. of Components (n) and Subsystems (m)



8. DISCUSSION AND CONCLUSION

Here we discuss a parallel-series system having 'm' subsystems connected in parallel and each subsystem has 'n' components connected in series. By considering the Weibull failure laws, we examine the effect of number of components, failure rates of the components, shape parameter and operating time of the components on reliability and mean time to system failure. Here, we observed that reliability and MTSF goes on decreasing with the increase of number of components (connected in series), failure rate and operating time of the components while their values increase with the increase of number of subsystems (connected in parallel). The effect of these parameters has also been observed for a particular case of Weibull distribution i.e. for Rayleigh failure laws. It is concluded that the parallel-series system will have more reliability if failure rate of the components follows Weibull distribution rather than that of Rayleigh distribution. Further, reliability of a parallel-series system goes on decreasing with the increases of operating time irrespective of distribution follows by failure time of components and subsystems. For Weibull failure laws, the reliability and mean time to system failure (MTSF) keep on decreasing with the increase the shape parameter. Hence, it is concluded that least number of components and more subsystems should be used in a parallel-series system for enhancing the performance.

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