A Comparative Study of Gaussian Noise Removal Methodologies for Gray Scale Images

Israt Jahan Tulin
Lecturer of CSE Department
BGMEA University of Fashion and Technology
Uttara, Dhala-1230

ABSTRACT
Image filtering is a technique to preserve important signal elements such as edges, smoothing the details of the image to make images appear clear and sharpener. Among all the non linear concepts to suppress Gaussian noise the fuzzy logic based approaches are important as they are capable of reasoning with vague and uncertain information. In this study, have made comparative study with the existing noise reduction methods where the images contaminated with Gaussian noise and found the best result by using fuzzy image filter with the help of fuzzy rules which make use of membership functions. In this article, to perform fuzzy smoothing, fuzzy derivative concept is also applied. This method provides better input for further image processing techniques. It also increases the contrast of the images, fine details and sharpening the edges as well. This comparative study, is made by numerical measures and visual inspection.

General Terms
Image contaminated with Gaussian noise, Average filtering, Wiener filtering, Median filtering and Fuzzy filtering.

Keywords
Image filtering, noisy image and fuzzy techniques.

1. INTRODUCTION
This comparative study mainly aims at Gaussian Noise [1, 2] which is also good at removing other Noises like Impulsive [3, 4] and Multiplicative Noise [5]. Impulsive Noise consists of random occurrences of energy spikes having random amplitude and spectral content. Multiplicative noise is a type of signal-dependent noise where brighter areas of the images appear noisier. Gaussian noise [6] is a type of statistical noise in which the amplitude of the noise follows that of a Gaussian distribution. When Images are Acquired and Transmitted over channels they are often corrupted by Impulse Noise [7,8] due to faulty communications and Noisy channels .i.e. Noise characterized by transient short-duration disturbances distributed essentially uniformly over the useful pass band of a transmission system. The Main aim of an Image De-noising Algorithm is to reduce the Noise Level, while keeping its fundamental structure like corners and edges [9]. Most of the Algorithms mainly deals with fat-tailed Noise like Impulsive Noise [10,11]and not specifically designed for Gaussian noise or do not produce convincing results when applied to handle this type of noise. Impulse noise [7,8] consists of very large positive or negative spikes of short duration. A positive spike has a value much larger than those of background signals and appears like a bright spot on the image. On the other hand, a negative spike has a value much smaller than those of background signals and appears like a dark spot on the image. They both are easily detected by the eyes and degrade the image quality. Fat-tail distributed "impulsive" noise is sometimes called salt-and-pepper noise or spike noise. An image that is contaminated with salt-and-pepper noise will have dark pixels in bright regions and bright pixels in dark regions. Dead pixels, analog to digital converter errors, bit errors in transmission are the reasons for this type of noise. By using dark frame subtraction and by interpolating around dark/bright pixels, this noise can be eliminated in large part. This feasibility of these noise reduction methods is shown by experimental results.

2. GAUSSIAN NOISE
In many occasions, noise in digital images is found to be additive in nature with uniform power in the whole bandwidth with Gaussian probability distribution. Such a noise is referred to as additive white Gaussian noise (AWGN).The AWGN IS mathematically represented by, η

$$N_{AWGN}(t) = \eta_G(t)\sigma$$

$$f_{AWGN} = f(x, y) + \eta_{G(x, y)}$$

Where $\eta_G(t)$ is a random variable that has a Gaussian probability distribution .It is an additive noise that is characters by its variance σ^2.In (12),the noisy image is represented as a sum of the original uncorrupted image and the Gaussian distributed random noise η_G. When the variance of the random noise η G is very low, $\eta_{G(x, y)}$ is zero or very close to zero at many pixel locations. Under such circumstances, the noisy image f_{AWGN} is same or very close to the original image f(x, y) at many pixel locations(x, y). Gaussian noise is generated during film exposure and development of the image. In this study each pixel in the output image is computed as a function of one or several pixels in the original image, usually located near the location of the output pixel [12-13]. The best solution for noise reduction or filtering is to process the image by its individual pixels based upon the appearance of its immediate neighbor pixels [14]. Image filtering improves the quality of the image by the way of enhancement of edges of the images. So many techniques such as mean filter, median filter, fuzzy filter and some other denoising techniques have been developed to suppress Gaussian noise [15-18]. Among all other filters, fuzzy image filter is very effective [19] and it can manage vague and uncertainty information efficiently.

3. FILTERING TECHNIQUES

3.1 Average Filtering
The average (mean) filter [20] smoothes images data, the eliminating noise. Using the gray level values in a square or rectangular window surrounding on each individual pixel, this filter performs spatial filtering.

For example:

a1	a2	a3
a4	a5	a6
a7	a8	a9

Fig: 1 filter window

The average filter computes the sum of all pixels in the filter window and then divides the sum by the number of pixels in the filter window:

Filtered pixel = (a1 + a2 + a3 + a4…+ a9) / 9

NOTE: In order to filter pixels located near the edges of the image, edge pixel values are replicated to give sufficient data.

Such common names of average filtering are mean filtering, smoothing, average, box filtering etc.

Mean filtering, is an easy, intuitive and simple method to smooth images. It is often used to reduce noise in images by reducing the amount of intensity variation between one pixel and the next. The filter works as low-pass one. For any unpredictable element of an image, the basic idea behind filter is to take an average across its neighborhood. By replacing each pixel value in an image with the average value of its neighbor including itself, is the main idea of mean filtering. This has the effect by which pixel value is implemented and it is unrepresentative of their surroundings. Mean filtering is usually considered as a convolution filter. Like other convolutions it is based around a kernel. When calculating the mean, this kernel represents the shape and size of the neighborhood to be sampled. Although larger kernels (*e.g.* 5×5 squares) can be used for more severe smoothing, it is often used a 3×3 square kernel, as shown in Figure 2. (Note that in order to produce a similar but not identical effect as a single pass with a large kernel. a small kernel can be applied more than once.)

$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

Fig: 2 averaging kernel often used in mean filtering

Computing the straightforward convolution of an image with this kernel carries out the mean filtering process.

3.2 Wiener Filtering

The most important technique for removal of blur in images due to linear motion or unfocussed optics is the Wiener filter [21] from a signal processing standpoint, blurring due to linear motion in a photograph is the result of poor sampling. Each pixel in a digital representation of the photograph should represent the intensity of a single stationary point in front of the camera. Unfortunately, if the shutter speed is too slow and the camera is in motion, a given pixel will be an amalgam of intensities from points along the line of the camera's motion. This is a two-dimensional analogy to,

$$G(u, v)=F(u ,v).H(u, v)$$

Where F is the Fourier transform of an "ideal" version of a given image, and H is the blurring function. In this case H is a sin c function: if three pixels in a line contain into from the same point on an image, the digital image will seem to have been convolved with a three-point boxcar in time domain. Ideally one could reverse-engineer a Fest, or F estimate, if G and H are known. This technique is inverse filtering [21].

The Wiener Filter is a noise filter based on Fourier iteration. Its main advantages are the short computational time it takes to find a solution.

The goal of the Wiener filter is to filter out noise that has corrupted a signal. It is based on a statistical approach.

Typical filters are designed for a desired frequency response. However, the design of the Wiener filter takes a different approach. One is assumed to have knowledge of the spectral properties of the original signal and the noise, and one seeks the LTI filter whose output would come as close to the original signal as possible. Wiener filters are characterized by the following:

1 Assumption: signal and (additive) noise are stationary linear stochastic processes with known spectral characteristics or known autocorrelation and cross correlation.
2 Requirements: the filter must be physically realizable, i.e. casual (this requirement can be dropped, resulting in a non-casual solution).
3 Performance criterion: minimum mean-square error (MMSE).

This filter is frequently used in the process of de-convolution.

3.3 Median Filtering

The filter median [22] is a simple nonlinear operator. It replaces the middle pixel in the window with the median-value of its neighboring pixels. The moving window for the median filter was 7×7. The edge-preserving nature makes the filter useful in cases where edge blurring is undesirable [23].
In an image processing before performing higher-level processing steps such as edge detection, it is usually necessary to perform a high degree of noise reduction. The median filter is a non-linear digital filtering technique. It is often used to remove noise from images or other signals.

Median filtering is a common step in image processing. It is particularly useful to reduce speckle noise and salt and pepper noise. Its edge-preserving nature makes it useful in cases where edge blurring in undesirable.

Algorithm
The idea is to calculate the median of neighboring pixels' values. This can be done by repeating these steps for each pixel in the image.

- Store the neighboring pixels in an array. The neighboring pixels can be chosen by any kind of shape, for example a box or a cross. The array is called the window, and it should be odd sized.
- Sort the window in numerical order.
- Pick the median from the window as the pixel value.

Example
To demonstrate, the median filter will be applied to the following array with a window size of 3, repeating edge values:

X= [2 80 6 3]

y[1] = Median[2 2 80] = 2
y[2] = Median[2 80 6] = Median[2 6 80] = 6
y[3] = Median[80 6 3] =Median [3 3 6] = 3

So
y = [2 6 6 3]

Where y is the median filtered output of x.

4. FUZZY FILTERING TECHNIQUES

Fuzzy set theory [24, 25, 26] has been successfully applied to pattern recognition fields. It is suitable for dealing with problems containing high levels of uncertainty, to which class pattern recognition or image processing problems usually belong. Obviously, the recovery of heavily noise-corrupted images is a task with high uncertainty levels. The general idea behind the filter is to average a pixel using other pixel values from its neighborhood, but simultaneously to take care of important image structures such as edges. The main concern of the present filter is to distinguish between local variations due to noise and due to image structure. In order to accomplish this, we derive a value for each pixel. This value expresses the degree in which the derivative in a certain direction is small. Such a value is derived by applying fuzzy rule [27, 28, and 29] for each direction corresponding to the neighboring pixels of the processed pixel. The further construction of the filter is then based on the observation that a small fuzzy derivative [, 25, 26,] most likely is caused by noise, while a large fuzzy derivative most likely is caused by an edge in the image. Consequently, for each direction we will apply two fuzzy rules that take this observation into account (and thus distinguish between local variations due to noise and due to image structure), and that determine the contribution of the neighboring pixel values. The result of these rules (16 in total) is defuzzified and a "correction term" is obtained for the processed pixel value.

Two important features of the present filter are: first, the filter estimates a "derivative" in order to be less sensitive to local variations due to image structures such as edges; second, the membership functions are adapted accordingly to the noise level to perform smoothing operation.

Estimating derivatives and filtering are very complex as for filtering we need a good indication of the edges, while to find these edges we need filtering. To counter this problem, we start by looking for the edges. We try to provide a robust estimate by applying fuzzy rules.

Fuzzy image processing scheme is a collection of different fuzzy approaches to image processing [30]. It has three main stages. They are (i) Image fuzzification (ii) Membership modification (iii) Image defuzzification.

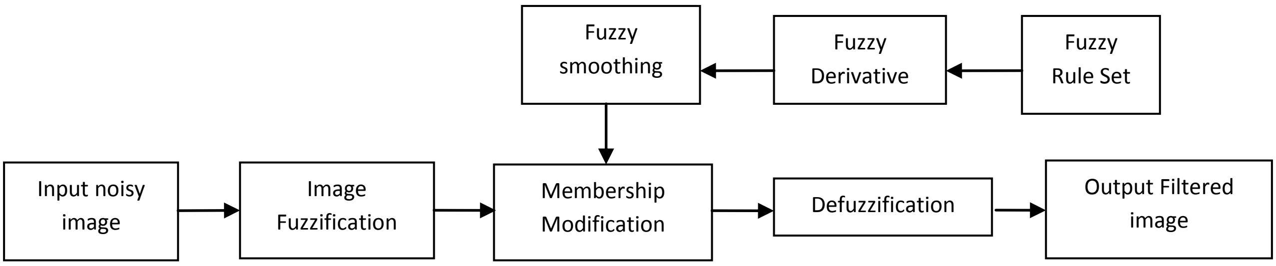

Fig 3: Basic structure of denoising image by using Fuzzy logic Algorithm.

4.1 Grouped mean fuzzy derivative

By applying this algorithm, the gray values of the neighboring pixels (n x n window) are stored in an array. It is then sorted in ascending or descending order. Then membership values are assigned to the neighborhood pixels. The neighborhood of a central pixel (x y) in 3 x 3 window of pixels is given by

PNW	PN	PNE
PW	PXY	PE
PSW	PS	PSE

Fig: 4 Pixels around a central pixel PXY

Now mean value of pixel values of PNW, PXY, PSE; PN, PXY, PS; PNE, PXY, PSW; PW, PXY, PE are calculated. After the calculation of the mean values for each group, grouped mean of all these groups is found. The pixel having the grouped mean, membership value '1' is assigned. The membership value '0' is assigned to the lowest and highest gray values. Now we consider only 2 x k + 1 pixel in the sorted pixels and they are the median gray values in the sorted list.

4.2 Noise Estimation

Consider a 3X3 neighborhood of a pixel (x, y). A simple derivative at the central pixel position (x, y) value in the direction D (D=NW, W, SW, S, SE, E, NE, N) is defined as the difference between the value of the central pixel (x, y) and its neighbor pixel in that direction D. This derivative value is denoted by Ñd (x, y).

For example, PN (x, y) = f(x, y-1)-f(x, y)

The principle of the fuzzy derivative [7, 9, 10] is based on the following observation. Consider an edge passing through the neighborhood of a pixel (x, y) in the direction SW, NE. The derivative value Ñnw (x, y) will be large, but also derivative values of neighboring pixels perpendicular to the edge's direction can expected to be large. For example, in NW - direction we can calculate the derivative values Ñnw (x, y), Ñnw (x-1, y+1) and Ñnw (x + 1, y- 1). The idea is to cancel out the effect of one derivative value which turns out to be high due to noise. Therefore, if two out of three derivative values are small, it is safe to assume that no edge is present in the considered direction. This observation will be taken into account when we formulate the fuzzy rule [20,21] to calculate the fuzzy derivative.

NW: (if Ñnw(x,y) is small and Ñnw(x-1,y+1) is small) or (if Ñnw(x,y) is small and Ñnw(x+1,y-1) small) or (if Ñnw(x1,y+1) is small or Ñnw(x+1,y-1) is small then Ñnwf(x,y) is small

4.3 Smoothing

To compute the correction term ∇for the processed pixel value, we use a pair of fuzzy rules for each direction. The idea behind the rules is the following: if no edge is assumed to be present in a certain direction, the (crisp) derivative value in that direction can and will be used to compute the correction term. The first part (edge assumption) can be realized by using the fuzzy derivative value, for the second part (filtering) we will have to distinguish between positive and negative values. For example, let us consider the direction NW. Using the values $\nabla^F_{NW}(x, y)$ and $\nabla_{NW}(x, y)$, we fire the following two rules, and compute their truthiness $\lambda^+_{NW}$and λ^-_{NW}:

λ^+_{NW}: if ∇^F_{NW} (x, y) is small and ∇_{NW} (x, y)is positive then C is positive

λ^-_{NW}: if ∇^F_{NW} (x, y) is small and ∇_{NW} (x, y)is negative then C is negative.

For the properties positive and negative, we also use linear membership functions. Again, we implement the AND-operator and OR-operator by respectively the minimum and maximum.

The final step in the computation of the fuzzy filter is the defuzzification. We are interested in obtaining a correction term∇, which can be added to the pixel value of location (x,y).Therefore, the truthiness of the rules λ^+_D and λ^-_D, D € dir(so for all directions) are aggregated by computing and rescaling the mean truthiness as follows:

$$\Delta = \frac{L}{8} \sum_{D \text{€} dir} (\lambda_D^+ - \lambda_D^-)$$

Where dir contains the directions and L represents the number of gray levels. So, each direction contributes to the correction term Δ.

The fuzzy rules for smoothing are as follows:

1. If a pixel is darker than neighboring pixels then make it brighter
2. If a pixel is brighter than neighboring pixels then make it darker.
3. Else leave it unchanged.

		(PN)N √	
	PNW √	PN	PNE √
(PW)W √	PW	P	PE
	PSW √	PS	PSE

Fig: 5 Estimating the noise

5. IMAGE QUALITY EVALUATION MATRIX

The filter performances are usually compared in term of PSNR and MSE. Larger values of PSNR and small values of mse indicate less noise power in an image.

Mean Square Error (MSE)

MSE is given by

$$MSE = \frac{1}{MN} \sum_{X=1}^{M} \sum_{y=1}^{N} [\mathrm{f(x,y)} - \mathrm{f'(x,y)}]^2$$

Where f(x,y) – M x N initial image

f(x,y) - Noised image

Peak Signal Noise Ratio (PSNR)

PSNR is defined as

PSNR = 10 $\log_{10}(255^2/\mathrm{MSE})$ dB.

If PSNR value is high or MSE value is low THEN image quality is better. Filtering techniques are as follows:

Filter	MSE	PSNR
Mean filter	382	20.05db
Median filter	245	22.07db
Wiener filter	125	25.02db
Proposed filtering technique	112	28.09db

Table 1: Comparison of the MSE'S and PSNR of restored images between different images

6. EXPERIMENT RESULTS

In this comparative study, different noise reduction techniques are tested on various standard grayscale images and found that fuzzy filter is the effective algorithms which gives the better results in terms of Visual Quality, high speed etc. To compare performance of the above approaches, these techniques have been repeated several times and the results evaluated using MSE and PSNR criteria.

(a) Original Image

(b) Corrupted Image by Gaussian Noise

(c) Mean Filter (PSNR: 20.05db)

(d) Median Filter (PSNR: 22.07db)

(e) Wiener Filter (PSNR: 25.02db)

(f) Fuzzy Filter (PSNR: 28.09db)

Fig 6: Restoration results of different filters in grayscale images

7. CONCLUSION

In this Paper using correction Term Method, we have introduced a new algorithm for de-noising images. The implementation of our fuzzy Filter [31, 32, 33, 34] in the image restoration process was discussed. All the techniques and operations provide an efficient working. The output image is enhanced according to the user's requirements. This Framework is useful to enhance images of PGM format. To support other system we can extend this framework. It can be extended into other fields such as archeology, Medical Image processing, Remote sensing etc. The existing system is available for reducing fat-tailed noise like impulse noise [35, 36]. Median filter [37, 38, 39] and Low-pass filters are in job at present. Median Filters [37, 38] mainly concentrate only on Impulsive Noise. Gaussian Noise is not specifically concentrated. It does not distinguish local variation due to noise and due to image structure proposed filter can clean an image completely of noise without making it blurry (Although that can produce an image that is very plasticy if depth level of the Tool is increased.).It presents a new technique for filtering narrow-tailed and medium narrow-tailed noise by a fuzzy filter [31, 32, 34]. In the future work, this proposed fuzzy filter will be focused on improving the system performance for both gray scale and color images.

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