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Robust H2 Control of the Nuclear Reactor Systems

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ABSTRACT

Robust control theory aims to analyze and design an accurate control system when the system has significant uncertainties. The goal is to synthesize a control law to maintain the system response and error signals to be within given tolerances despite the effect of the uncertainties on the system and to maintain the stability for all plant models in an expected band of uncertainty [1].

In this paper the design of a robust controller using the linear quadratic Gaussian, H2 optimal control and the robust tracking with disturbance rejection algorithms are represented where the fuel and coolant temperatures feedback are included.

Keywords

Robust control, kinetic equation, Linear Quadratic Gaussian, H2 optimal control, nuclear reactor.

1. INTRODUCTION

The problem is to find a well-defined optimal controller to be applied to the nuclear reactor system. As the actual nuclear reactor system equations shows that the system is nonlinear in its nature, so it is difficult to design a suitable controller for this system directly, instead of that the design is based on a linearized model of the plant to be controlled [2], then the obtained controller is applied to the actual system.

The design procedure goes through some simplifications such as linearization about an equilibrium point, lumped parameters approximations or time delay, etc. The result is an approximate plant or as referred often uncertain plant. These uncertainties are due to linearization of the nonlinear system, unmolded dynamics, sensor and actuator noise ,and undesired external disturbances, to overcome all of these uncertainties it is important to concern about how the controller will work with the actual plant and to make sure that the design objectives will be achieved, another important point to know is whether the controller takes care not only of the given uncertainties but also of uncertainties that will appear due to the component failures, changes in environmental conditions and, manufacturing tolerance [1].

Many approaches have been suggested and developed for the robust control problems such as conventional feedback, optimal H_∞ controller and robust gain scheduling controller [2] however there is no attention paid to the H2 optimal control, the solution of H_2 control problem is robust and optimal as it considers and deals with input and output disturbances.

A special case of the H2 robust control is LQG/LTR method which can be solved in two stages where the disturbance is considered as a white noise and affects the output Cx as in the following equations

$$\dot{x} = Ax + B_1 w + B_2, \tag{1}$$

$$z = C_1 x + D_{11} w + D_{12} u \tag{2}$$

$$y = C_2 x + D_{21} w + D_{22} u \tag{3}$$

where **z** and y are the output vectors while w is the disturbance and u is the control signal.

H2 optimal control aims to find a controller K which stabilizes the plant and minimizes the following cost function:

$$J_2(K) = \|F(G,K)\|_2^2 \tag{4}$$

Where $\|F(G,K)\|_2$ is the H2 norm, $G(s) = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix}$

is the augmented system and K is the desired controller [2].

The cost function $J_2(K) = \sum_{k=1}^{m} [\int_0^\infty z(t)^T z(t) dt : v = e\delta(t)$ is minimized by the statistical space feedback controller

$$u(t) = K_{opt} x(t) \tag{5}$$

Where

$$K_{opt} = -(D_{12}^T D_{12})^{-1} B_2^T P \tag{6}$$

And P is the solution of the following Riccati equation:

$$A^T P + PA - PB_2 (D_{12}^T D_{12})^{-1} B_2^T P + C_1^T C_1 = 0 \tag{7}$$

Hence H2 optimal estimation problem is equal to dual control problem where it is equivalent to state feedback control problem. The optimal controller consists of H2 optimal state estimator and H2 optimal state feedback of the estimated state which can be the same as Linear Quadratic Gaussian problem with loop transfer recovery (LQG/LTR) due to its effectiveness in accommodating plant uncertainty [3].

The procedure is a straight forward way beside it provides not only a desirable performance in normal of the controlled plant but also in fault accommodation.

The paper discusses the robustness of the H2 optimal control and LQG controller to improve the nuclear reactor power response and the temperature response then the results is compared to those obtained using robust tracking problem with disturbance rejection.

The paper is organized as follows: section 2 represents the nuclear reactor model (actual and nominal plants).Section3 introduce the H2 optimal control and LQG, robust tracking with disturbance rejection is represented in section 4. The results are found in section 5, and the main conclusions are summarized in section 6.

2. NUCLEAR REACTOR MODELING

The model used in this paper is the nominal Pressurized Water Reactor model (PWR-type) TMI nuclear power plant reactor and its kinetic equation with one delayed neutron group and temperature feedback.

The actual system equations can be summarized in the following equations [4]:

$$\frac{dn}{dt} = \frac{\delta\rho-\beta}{\wedge} n - \lambda c \quad (8)$$

$$\frac{dc}{dt} = \frac{\beta}{\wedge} n - \lambda c \quad (9)$$

where

n ≡ neutron density (n^3/cm)

c ≡ neutron precursor density (atom/cm^3)

λ ≡ effective precursor radioactive decay constant(s^{-1})

∧≡ effective prompt neutron lifetime(s)

β ≡ fraction of delayed fission neutrons

k ≡ k_{eff} ≡ effective neutron multiplication factor

δρ ≡ $\frac{k-1}{k}$ ≡ reactivity (Since k≈1.000, δρ≈k-1 ; at steady state k=1 , δρ = 0)

For computational purposes the normalized versions of equations (1) and (2) will be used so the normalized equations will be as follow:

$$\frac{dn_r}{dt} = \frac{\delta\rho-\beta}{\wedge} n_r + \frac{\beta}{\wedge} c_r \quad (10)$$

And

$$\frac{dc_r}{dt} = \lambda n_r - \lambda c_r \quad (11)$$

$n_0 \equiv$ $initial\ equilibrium\ (steady-state)\ neutron\ density,$

$c_0 \equiv initial\ equilibrium\ (steady-state) precursor\ density$

$n_r \equiv n/n_0$, neutron density relative to equilibrium density

$c_r \equiv c/c_0$, precursor density relative to initial equilibrium density

Reactor temperatures vary as a function of power generated and heat transfer and it affects the reaction chain so it has to be included in the normalized point-kinetic equations for accurate calculation of n_r.

Reactor temperatures can be expressed as following,

$$\frac{dT_f}{dt} = \frac{f_f\, P_{0a}}{\mu_c} n_r - \frac{\Omega}{\mu_f} T_f + \frac{\Omega}{2\mu_f} T_l + \frac{\Omega}{2\mu_f} T_e \quad (12)$$

$$\frac{dT_l}{dt} = \frac{(1-f_f)P_{0a}}{\mu_c} n_r + \frac{\Omega}{\mu_c} T_f - \frac{(2M+\Omega)}{2\mu_c} T_l + \frac{(2M-\Omega)}{2\mu_c} T_e, \quad (13)$$

$$\frac{d\delta\rho_r}{dt} = G_r Z_r \quad (14)$$

$$\delta\rho = \delta\rho_r + \alpha_f\left(T_f - T_{f0}\right) + \frac{\alpha_c(T_l - T_{l0})}{2} + \frac{\alpha_c(T_e - T_{e0})}{2} \quad (15)$$

The five states appear in the nominal model represents the relative reactor power (n_r), the relative precursor density (c_r), the average fuel temperature T_f, the average coolant temperature leaving the reactor T_l and the reactivity $\delta\rho_r$. The model is nonlinear because total reactivity $\delta\rho$ which is composed of the rod reactivity $\delta\rho_r$ and temperature feedback reactivity from equation (8) multiplies the reactor power state to determine the reactor power rate change [4].

The linearized system can be represented by the following state space equations,

$$\dot{x} = Ax + Bu\,,\ y = Cx + Du \quad (16)$$

Where,

$$x = \begin{bmatrix}\delta n_r \\ \delta c_r \\ \delta T_f \\ \delta T_c \\ \delta\rho_r\end{bmatrix} \quad y = [\delta n_r], \qquad u = [z_r]\,,$$

$$A = \begin{bmatrix} -\frac{\beta}{\nu} & \frac{\beta}{\nu} & n_{r0}\frac{\alpha_f}{\nu} & n_{r0}\frac{\alpha_c}{2\nu} & \frac{n_{r0}}{\nu} \\ \lambda & -\lambda & 0 & 0 & 0 \\ \frac{f_f P_{0a}}{\mu_f} & 0 & -\frac{\Omega}{\mu_f} & \frac{\Omega}{2\mu_f} & 0 \\ \frac{(1-f_f)P_{0a}}{\mu_c} & 0 & \frac{\Omega}{\mu_c} & -(2M+\Omega)/2\mu_c & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix}0\\0\\0\\0\\G_r\end{bmatrix}, \quad C = [1\ \ 0\ \ 0\ \ 0\ \ 0] \quad \text{and} \quad D = [0]$$

$\delta\rho_r$ ≡ Control rod reactivity,

δρ ≡ $\delta\rho_r$ for a system without temperature feedback,

z_r ≡ Control rod speed in units of fraction of core length per second,
G_r ≡ The reactivity worth of the control rod per unit length.

With z_r in units of fraction of core length per second, G_r is the total worth of the rod.

The simulation has been done by applying the controller to the nonlinear system while the linearized reactor model is used to design the suitable controller.

3. H2 AND LQG CONTROLLER DESIGN

As the control rod motion is used to regulate the power output to a demand power through a conventional output feedback reactor control where the state feedback gain Gc is considered as a single design variable so a proper selection must be done.

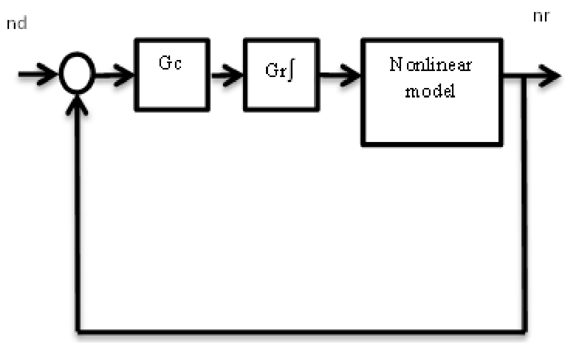

Figure 1 Conventional output feedback control

The previous design as shown in Figure 1 is a straight forward design [3] and it is effective to accomplish a limited control objectives however if the performance and robustness of the model have to be improved specially the temperature responses so it is better to use a state feedback control as in Figure 2 where there is an optimal state estimator and optimal state feedback and the solution of both parts can be calculated independently of each other.

A suitable H2 controller can be designed using equations from (1) to (5) to find

$$K(s) = \begin{bmatrix} A_k & B_k \\ C_k & D_k \end{bmatrix} \tag{17}$$

Where

$\dot{x}_k = A_k x_k + B_k y$ And $u = C_k x_k + D_k y$

However the closed loop A-matrix can be written in the following form [12]:

$$\begin{bmatrix} \dot{x} \\ \dot{x}_k \end{bmatrix} = \begin{bmatrix} A + B_2 D_k C_2 & B_2 C_k \\ B_k C_2 & A_k \end{bmatrix} \begin{bmatrix} x \\ x_k \end{bmatrix} \tag{18}$$

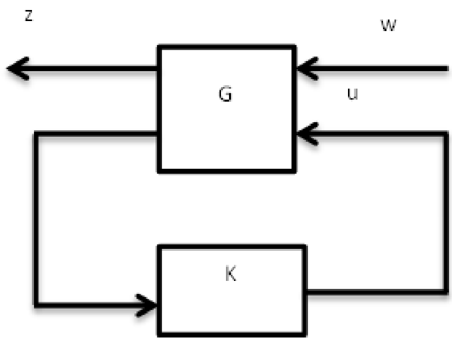

Figure 2 H2 controller design

When considering the disturbances are stochastic white noise processes with a Gaussian distribution then the problem will be known as the Linear Quadratic Gaussian (LQG) control problem which solves a well-defined optimal control problem.

This method is a model based compensator design with output feedback gain and it uses the Linear Quadratic Gaussian and optimal state feedback gain at the same time as shown in Figure 3 to achieve both performance and robustness at the output.

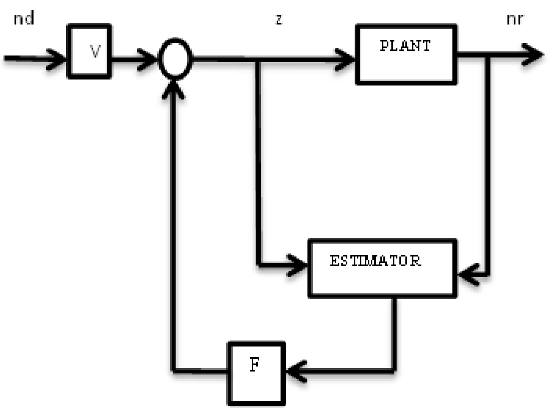

Figure 3 LQG controller design

The method can be applied through two basic steps; first step is to generate a target feedback loop (TFL) that meets the performance specifications without affecting the stability-robustness constraints. Second step is to use a straight forward way to design a special compensator K(s) so the performance of the feedback system approximates the performance of the target feedback loop [3]

The LQG design depends on if the uncertainties are at the input or the output if it is at the input so the first step is to design a target feedback loop using the linear quadratic regulator and then using the Kalman filter. On the other hand if the uncertainties are at the output so the first step is a filter design then the second step is a controller design [5].

For the nuclear reactor model to eliminate the steady state error an integral control action is provided by appending an integrator to the plant which will add a sixth state to the system, now the augmented system can be expressed as

$$A_p = \begin{bmatrix} A - G_c BC & G_c B \\ 0 & 0 \end{bmatrix} \quad B_p = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C_p = [C \quad 0]$$
$$D_p = D \tag{19}$$

Where,

G_c is the classical output feedback gain then the first step is to design the Kalman filter through the following algorithm.

Consider the following model

$$\dot{x} = Ax + Bu + Gw \tag{20}$$

$$y = cx \tag{21}$$

$$z = Hx + v \tag{22}$$

Where, w, v are zero mean Gaussian white noise processes at the input and output, z are the measurements available and y are the controlled plant outputs.

Consider that C=H so the Kalman filter equations for the state estimator, error and the gain are:

$$\dot{\hat{x}} = A\hat{x} + K_f[z - H\hat{x}] \tag{23}$$

$$\dot{e} = \left[A - K_f H\right]e + Gw - K_f v \tag{24}$$

$$K_f = PH^T R^{-1} \tag{25}$$

Where,

P is the error covariance matrix and can be calculated as the solution of the following Riccati equation:

$$\dot{P} = AP + PA^T + GQG^T - PH^T R^{-1} HP, P(0) = P_0 \tag{26}$$

The second step is to design an optimal feedback gain to minimize the following performance index

$$J = \int_0^\infty (qx^T Q_0 x + u^T R_0 u)dt \tag{27}$$

Where,

$Q_0=Q_0^T \geq 0$, $R_0=R_0^T>0$ and q is a scalar design parameter.

The control law can be written as

$$u = -K_0 x \quad with \quad K_0 = R_0^{-1} B^T P \tag{28}$$

And P is the solution of the following Riccati equation

$$0 = PA + A^T P + qC^T Q_0 C + PB^T R_0^{-1} BP \tag{29}$$

At the end the robust controller is designed by both K_f obtained from the filter design and K_0 obtained from the controller design so $G_{KF}(s)$ has the desired loop shape and $GK_0(s) \cong G_{KF}(s)$ where G is the plant and G_{KF} is the target feedback loop.

4. ROBUST TRACKING WITH DISTURBANCE REJECTION.

Another approach can be applied to the same system in another way is the robust tracking with rejection disturbance as the internal loop will use the output feedback gain while in the outer loop is a compensator designed using the Kalman filter and an integrator is added to integrate the error signal which is a feedback signal [6][7]. This algorithm is effective and it can be applied to the system as shown in Figure 4.

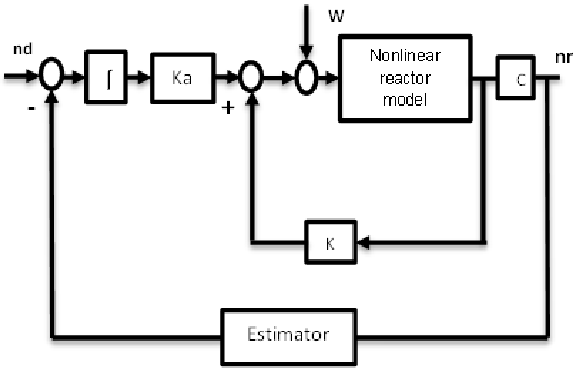

Figure 4 Robust controller with disturbance rejection

$$u = [K \;\; K_a]\begin{bmatrix} x \\ x_a \end{bmatrix} \tag{30}$$

$$Ap = \begin{bmatrix} A - BK & BK_a \\ -C & 0 \end{bmatrix} \tag{31}$$

$$B_p = [B_1 \quad B_2] \tag{32}$$

Where,

$$B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \text{and} \quad B_2 = \begin{bmatrix} B \\ 0 \end{bmatrix}$$

$$C_p = [C \quad 0] \tag{33}$$

$$D_p = 0 \tag{34}$$

The output will asymptotically track constant references and reject the constant disturbances, also using the integrator will yield robust reference tracking and disturbance rejection [6][10] where the characteristic equation of the original system is represented by the following equation :

$$G(s) = C(sI - A - BK)^{-1}B = \frac{N(s)}{D(s)} \tag{35}$$

And the characteristic equation of the augmented system is:

$$\bar{G}\,(s) = det\left(\begin{bmatrix} sI - A - BK & -BK_a \\ C & s \end{bmatrix}\right) \tag{36}$$

5. SIMULATION RSULTS

Simulation is done using the nonlinear plant model with fuel and coolant temperature feedback and step change of relative power from 1 (100%) up to 1.1 (110%). Simulation is done using Matlab Simulink and m-files [9]

5.1 H2 controller results:

Using Matlab to compute the optimal H2 controller.

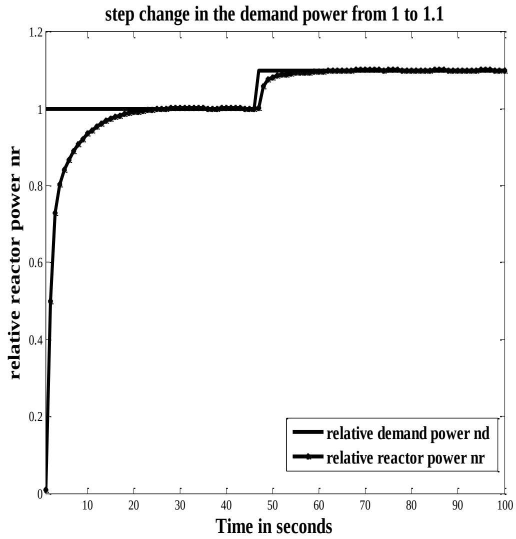

Figure 5 Step change in demand power from 1 to 1.1

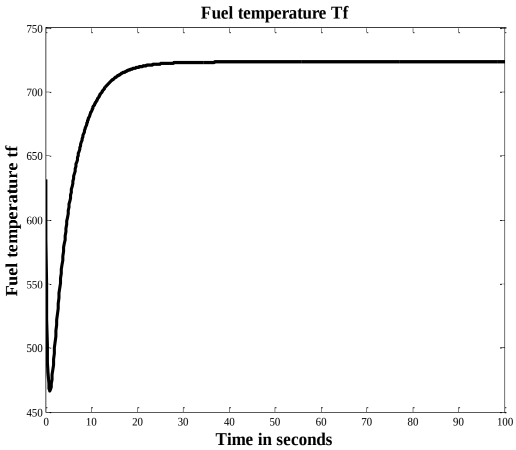

Figure 6 Fuel temperature for step change in the demand power from 1 to 1.1

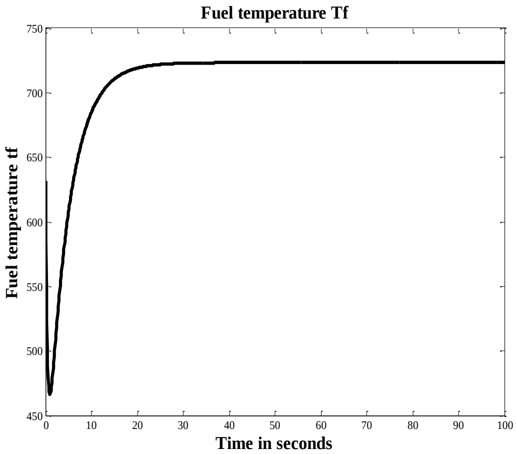

Figure 7 Outlet coolant temperature with step change in the demand power from 1 to 1.1

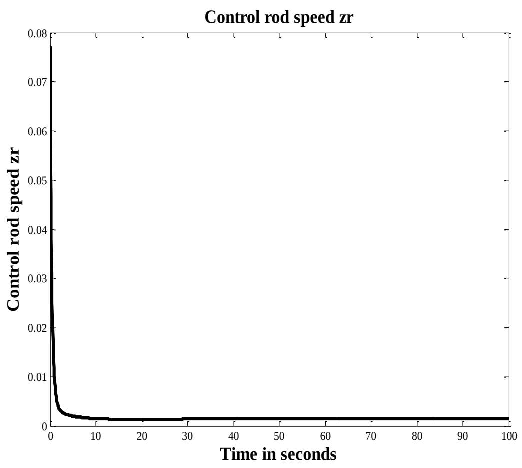

Figure 8 Control rod speed with step change in the demand power

5.2 LQG results

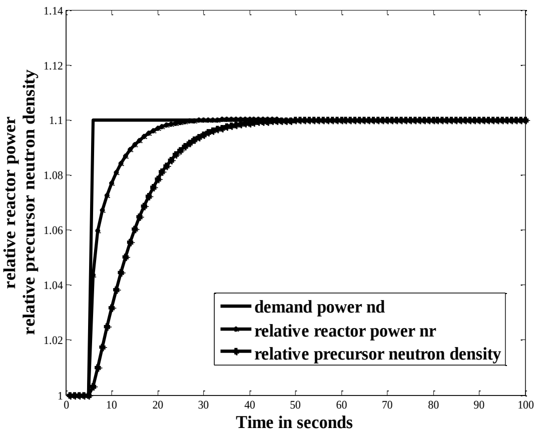

Figure 9 Relative reactor power (nr), relative precursor neutron density versus demand power with step change 0.1 to increase the power from 1 to 1.1.

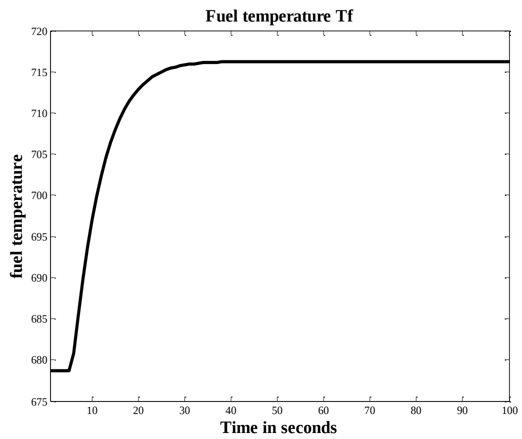

Figure 10 Fuel temperatures with step change in the demand power

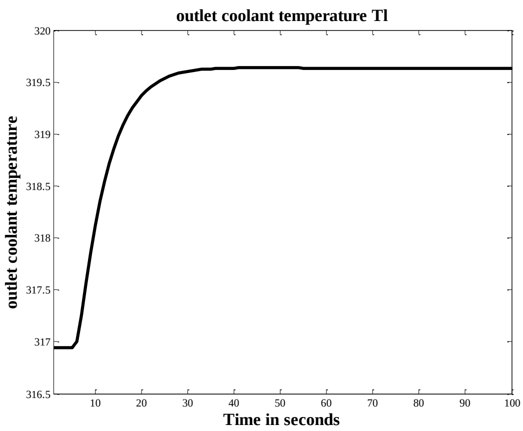

Figure 11 Outlet coolant temperatures with step change in the demand power.

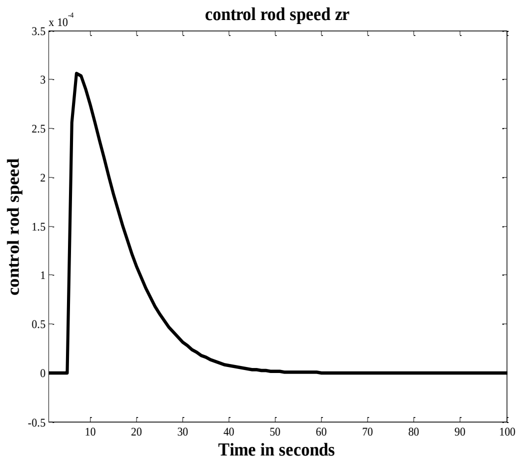

Figure 12 Control rod speed with step change in the demand power.

Two gains are calculated the first is the optimal control gain K0=[-0.00 -0.00 0 0 0.001 0.0031] and the second is Kalman gain K_f and it is equal to

K_f=[0.7109 0.1126 75.3311 3.5914 0.007 1.4142]

5.3 Robust tracking results

In this algorithm also two gains are calculated which are Kalman filter gain K_f and their values are:

K_f= [1.0799 0.0195 22.2980 -0.3681 0.0081 0] , and the optimal control gain obtained for the system and it equals K_0=[0 0 0 0 0.0016]

Where first step is to calculate the optimal control gain K_0 and then second step is to apply Kalman filter to the augmented plant and then the estimator gain K_f is obtained.

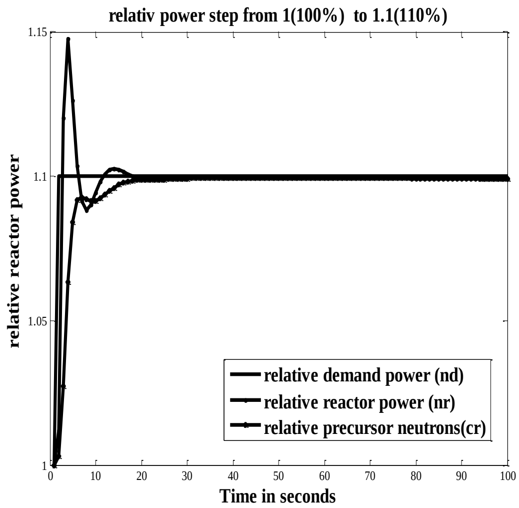

Figure 13 Relative reactor power (nr) ,relative precursor neutron density (cr) and the relative demand power (nd)

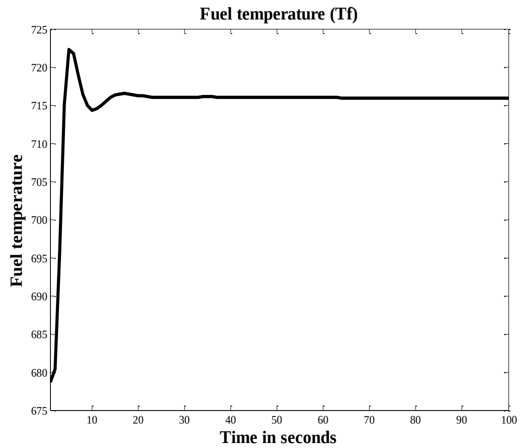

Figure 14 Fuel temperature with step change in the demand power.

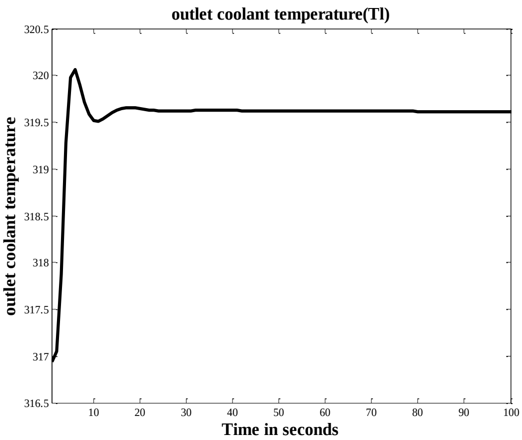

Figure 15 Outlet coolant temperature with step change in the demand power.

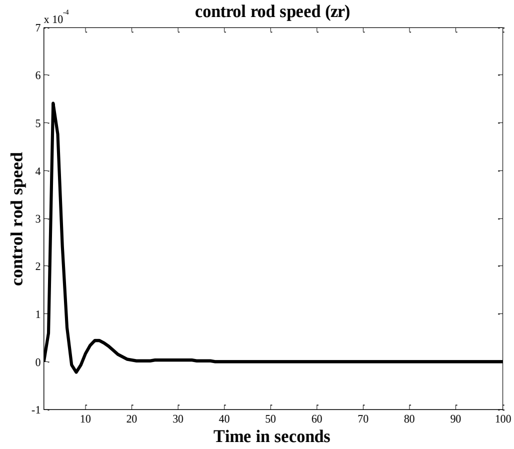

Figure 16 Control rod speed with step change in the demand power.

Table 1 comparison between used control techniques

Method	**Performance**	**Rising time**	**Steady state arrival time**
H2 Figure 5	Over damped without overshoot	5	30
LQG Figure 9	Over damped without overshoot	20 seconds	55 seconds
Robust Tracking Figure 13	Under damped with overshoot	10 seconds	40 seconds

In the simulation results firstly, the results using H2 optimal control, are shown as the demand output power response shown in Figure 5, corresponding reactor temperatures shown in Figure 6 and Figure 7 and the control rod speed shown in Figure 8.

Secondly, the results using LQG control are represented as the demand output power is shown in Figure 9, the reactor temperatures are shown Figure 10 and Figure 11 and the control rod speed is shown in Figure 12.

Finally the results of the robust tracking control are represented as the demand output power is shown in Figure 13, the reactor temperatures are shown in Figure 14 and Figure 15 and the control rod speed is shown in Figure 16.

6. CONCLUSION

The results obtained in the simulation part show that using H2 and LQG optimal control gives better results than robust tracking with disturbance rejection method as the response in both cases reaches its steady state in few seconds which achieves a good performance and robustness while in the case of using robust tracking control there is an overshoot that affects the system responses.

In all methods applied in this paper the disturbance is compensated effectively, as there is a temperature feedback taken into account it is observed that the temperatures responses are improved using H2 and LQG optimal control algorithms compared to that obtained using the robust tracking control algorithm.

The controllers design in all cases is based on linear model but the simulation is done using the nonlinear plant as linear simulation cannot provide an accurate idea about the system behavior.
The main advantage of using H2 and LQG controllers rather than other methods is that the temperature performance is improved while in the robust tracking method the responses suffer from an overshoot that is not desirable in the reactor systems [11].

In future work it is strongly recommended to work on improving the system responses using other control algorithms such as H∞ control and mixed H2/H∞ methods.

7. PARAMETERS FOR 5^{TH} ORDER NONLINEAR SIMULATION OF A PWR

β =0.0065	λ= 0.125 s^{-1}
Λ=0.0001 s	f_f =0.98
G_r=0.01 total rod reactivity	T_e =290 C
P_{0a}=2500 MW	μ_c =70.5 MW s/C
μ_f =26.3 MWs/c	M=92.8 MW/C
Ω =6.53 MW/C	α_f =-0.00005 reactivity /C

α_c =0.00001 reactivity/C

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