International Journal of Computer Applications (0975 – 8887)
Volume 178 – No.4, November 2017

Robust H∞ Controller Design for the Nuclear Reactor Systems

Rehab M. Saeed
Nuclear Research Center
Egyptian Atomic Energy Authority

Gamal M. El Bayoumi
Aerospace Engineering Dept.
Cairo University

Ibrahim E. Zeidan
Computers and Systems Engineering Dept.
Zagazig University

ABSTRACT
As it is important to improve the response of the nuclear reactor power system, many approaches tried to find the best way to design the suitable robust controller .This paper introduces the solution of H∞ control problem of the nuclear reactor systems as a robust controller that achieves both the robustness and performance improvement.

Keywords
H∞, Robust control, nuclear reactor systems, robustness

1. INTRODUCTION
The first mission of the design of the H∞ controller is to make the system insensitive towards the externals disturbances .this means that it is important to make the output independent of the external disturbance as possible [1].

The solution of the H∞ problem can be formulated in many ways [2], one is the Glover-Doyle algorithm which is the classic formulation as it achieves the basic mixed performance and robustness objectives through solving a family of stabilizing controllers such that $F_l(P,K) \leq \gamma$

Where

P(s) represents the plant nominal transfer function, K(s) represents feedback controller and γ is the H∞ norm.

Another techniques are used for the design of the H∞ controllers such as two transfer function method and three transfer function method [2].

The properties of designing a controller using H∞ method can be summarized as that the stabilizing feedback low u(s) =K(s) y(s) minimizes the norm of the closed loop transfer function, and it is suitable for the weighted mixed sensitivity problem where H∞ controller always cancels the stable poles of the plant with its transmission zeroes so the unstable poles of the plant inside the specified bandwidth will be shifted to its mirror image once a H∞ feedback loop is closed, another property is that using suitable weighting functions will allow very precise frequency domain loop shaping [3].

Mixed weight H-infinity controllers [4] will provide a closed loop response of the system according to the design specifications such as model uncertainty, disturbance attenuation at high frequencies,….etc. The H∞ controllers are of high order this may lead to large control requirements, also additional frequency dependent weights are augmented to the system.

The selection of the additional frequency dependent weights depends on what stability and performance design specifications are required to be shown [5].

Conventionally, H∞ controller employs two transfer functions which divide a complex control problem into two separate sections, one deals with stability and the other deals with the performance. So the objective of designing H∞ controller is to find a controller **K**, which based on the information **v**, generates a control signal **u**, which compensates the influence of w on z and minimizes the closed loop norm **w** to **z**.

The paper is organized as follow, section 2 represents the nuclear reactor model (actual and nominal plants).Section3 introduce the H∞ optimal control while the simulation results are represented in section 4 and the conclusion is introduced in section 5.

2. NUCLEAR REACTOR MODELING
The model used in this paper is the nominal Pressurized Water Reactor model (PWR-type) TMI nuclear power plant reactor and its kinetic equation with one delayed neutron group and temperature feedback.

The actual system equations can be summarized in the following equations [6]:

$$\frac{dn}{dt} = \frac{\delta\rho-\beta}{\Lambda} n - \lambda c \tag{1}$$

$$\frac{dc}{dt} = \frac{\beta}{\Lambda} n - \lambda c \tag{2}$$

Where,

$n \equiv$ neutron density $\left(\frac{n}{cm}^{3}\right)$

$c \equiv$ neutron precursor density $\left(\frac{atom}{cm^{3}}\right)$

$\lambda \equiv$ effective precursor radioactive decay constant(s^{-1})

$\Lambda \equiv$ effective prompt neutron lifetime(s)

$\beta \equiv$ fraction of delayed fission neutrons

$k \equiv k_{eff} \equiv$ effective neutron multiplication factor

$\delta\rho \equiv \frac{k-1}{k} \equiv$ reactivity (Since k≈1.000, δρ ≈k-1 ; at steady state k=1 , $\delta\rho = 0$)

For computational purposes the normalized versions of equations (1) and (2) will be used so the normalized equations will be as follow:

$$\frac{dn_r}{dt} = \frac{\delta\rho-\beta}{\Lambda} n_r + \frac{\beta}{\Lambda} c_r \tag{3}$$

$$\frac{dc_r}{dt} = \lambda n_r - \lambda c_r \tag{4}$$

$n_0 \equiv$ initial equilibrium (steady − state)neutron density,

$c_0 \equiv$ initial equilibrium (steady state)precursor density

$n_r \equiv n/n_0$, neutron density relative to equilibrium density

$c_r \equiv c/c_0$, precursor density relative to initial equilibrium density

As the reactor temperatures vary as a function of power generated and heat transfer and it affects the reaction chain so it has to be included in the normalized point-kinetic equations for accurate calculation of the output power (n_r).

Reactor temperatures can be expressed as following,

$$\frac{dT_f}{dt} = \frac{f_f\ P_{0a}}{\mu_c} n_r - \frac{\Omega}{\mu_f} T_f + \frac{\Omega}{2\mu_f} T_l + \frac{\Omega}{2\mu_f} T_e \quad (5)$$

$$\frac{dT_l}{dt} = \frac{(1-f_f)P_{0a}}{\mu_c} n_r + \frac{\Omega}{\mu_c} T_f - \frac{(2M+\Omega)}{2\mu_c} T_l + \frac{(2M-\Omega)}{2\mu_c} T_e, \quad (6)$$

$$\frac{d\delta\rho_r}{dt} = G_r Z_r \quad (7)$$

$$\delta\rho = \delta\rho_r + \alpha_f(T_f - T_{f0}) + \frac{\alpha_c(T_l - T_{l0})}{2} + \frac{\alpha_c(T_e - T_{e0})}{2} \quad (8)$$

The described model has five states which appear in the nominal model. These Five states are the relative reactor power (n_r), the relative precursor density (c_r), the average fuel temperature T_f, the average coolant temperature leaving the reactor T_l and the reactivity $\delta\rho_r$ respectively. The model is nonlinear because total reactivity $\delta\rho$ which is composed of the rod reactivity $\delta\rho_r$ and temperature feedback reactivity from equation (8) multiplies the reactor power state to determine the reactor power rate change [6].

The linearized system can be represented by the following state space equations,

$$\dot{x} = Ax + Bu\ , y = Cx + Du \quad (9)$$

Where,

$$x = \begin{bmatrix} \delta n_r \\ \delta c_r \\ \delta T_f \\ \delta T_c \\ \delta\rho_r \end{bmatrix}, \qquad y = [\delta n_r], \qquad \text{and} \quad u = [z_r]$$

And

$$A = \begin{bmatrix} -\frac{\beta}{\nu} & \frac{\beta}{\nu} & n_{r0}\frac{\alpha_f}{\nu} & n_{r0}\frac{\alpha_c}{2\nu} & \frac{n_{r0}}{\nu} \\ \lambda & -\lambda & 0 & 0 & 0 \\ \frac{f_f P_{0a}}{\mu_f} & 0 & -\frac{\Omega}{\mu_f} & \frac{\Omega}{2\mu_f} & 0 \\ \frac{(1-f_f)P_{0a}}{\mu_c} & 0 & \frac{\Omega}{\mu_c} & -(2M+\Omega)/2\mu_c & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ G_r \end{bmatrix}, \quad C = [1 \quad 0 \quad 0 \quad 0 \quad 0] \quad \text{and} \quad D = [0]$$

$\delta\rho_r \equiv$ Control rod reactivity,

$\delta\rho \equiv \delta\rho_r$ for a system without temperature feedback,

$z_r \equiv$ Control rod speed in units of fraction of core length per second,
$G_r \equiv$ The reactivity worth of the control rod per unit length.

With z_r in units of fraction of core length per second and G_r is the total worth of the rod.

The simulation has been done by applying the controller to the nonlinear system while the linearized reactor model is used to design the suitable controller.

3. H∞ CONTROLLER DESIGN

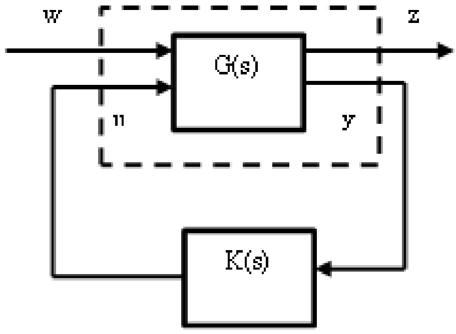

Figure 1 Design of H infinity controller

The model in figure (1) consists of **P(s)** and **K(s)**, where **P(s)** has the multi inputs of disturbances vector w that contains the system uncertainty and the measurement noise plus the control input **u** while the output is **y** and **z**.

As H_∞ controller design depends on solving two Riccati equations one for the state feedback control and the second for the estimation problem so the problem can be similar to Linear Quadratic Gaussian control (LQG). However there is a difference between H_∞ control and LQG control as the standard H_∞ optimal control problem is concerned with constructing a dynamic feedback controller u=K(s)y to minimize the H_∞ norm of the transfer function from w to z, [8].

Assume the state space equations of the system are in the following form

$$\dot{x} = Ax + B_1 w + B_2 u \quad (10)$$

$$z = C_1 x + D_{11} w + D_{12} u \quad (11)$$

$$y = C_2 x + D_{21} w + D_{22} u \quad (12)$$

And can be packed in G(s) as one matrix represents the system parameters where,

$$G(s) = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix} \quad (13)$$

Some important assumptions have to be done as [8]:

- The pair (A, B_2) are stabilizable and the pair (C_2,A) are detectable.
- $D_{11}=0$ and $D_{22}=0$ in order to simplify the solution.
- Dimension (x), dimension (w)=m_1 while dimension (u)=m_2, also dimension (z)=p_1 and dimension (y)=p_2, then the rank of $D_{12}=m_2$ and the rank of $D_{21}=p_2$. These assumptions will ensure that the controllers are proper
- Rank$\begin{bmatrix} A - jwI & B_2 \\ C_1 & D_{12} \end{bmatrix} = n + m_2$ for all frequencies.
- Rank $\begin{bmatrix} A - jwI & B_1 \\ C_2 & D_{21} \end{bmatrix} = n + p_2$ for all frequencies.

Now the new system will be describes as:

$$G(s) = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & 0 & D_{12} \\ C_2 & D_{21} & 0 \end{bmatrix} \quad (14)$$

The solution of this equation (equ 14) requires solving two Riccati equations one for the controller and the other for the observer and the control law is given by

$$u = -K_c\hat{x} \quad (15)$$

And the state estimator equation is

$$\dot{\hat{x}} = Ax + B_2u + B_1\hat{w} + Z_\infty K_e(y - \hat{y}) \quad (16)$$

Where

$$\hat{w} = \gamma^{-2}B_1^T X_\infty \hat{x}$$

$$\hat{y} = C_2\hat{x} + \gamma^{-2}D_{21}B_1^T X_\infty \hat{x}$$

And the controller gain Kc is

$$K_c = \widetilde{D}_{12}(B_2^T X_\infty + D_{12}^T C_1$$

Where,

$$\widetilde{D}_{12} = (D_{12}^T D_{12})^{-1}$$

And the estimator gain is $Z_\infty K_e$ instead of K_e where,

$K_e = y_\infty C_2^T + B_1 D_{21}^T)\widetilde{D}_{21}, \widetilde{D}_{21} = (D_{21}D_{21}^T)^{-1}$, $Z_\infty = (I - \gamma^{-2}Y_\infty X_\infty)^{-1}$ and the terms X_∞ and Y_∞ are the solution of the controller and estimator Riccati equations,

$$X_\infty = \mathrm{Ric}\begin{bmatrix} A - B_2\widetilde{D}_{12}D_{12}^T C_1 & -\gamma^{-2}B_1B_1^T - B_2\widetilde{D}_{12} \\ -\tilde{C}_1^T\tilde{C}_1 & -(A - B_2\widetilde{D}_{12}D_{12}^T)C_1 \end{bmatrix} \quad (17)$$

$$Y_\infty = \mathrm{Ric}\begin{bmatrix} (A - B_1D_{21}^T\widetilde{D}_{21}C_2)^T & -\gamma^{-2}C_1^TC_1 - C_2^T\widetilde{D}_{21}C_2 \\ -\widetilde{B}_1\widetilde{B}_1^T & -(A - B_1D_{21}^T\widetilde{D}_{21})C_2 \end{bmatrix} \quad (18)$$

With

$\widetilde{B}_1 = B_1(I - D_{21}\widetilde{D}_{21}D_{21}^T)$and $\tilde{C}_1 = B_1(I - D_{12}\widetilde{D}_{12}D_{12}^T)$

The above calculations are not easy to be done by hand but it is done using the Matlab subroutines.

When the H_∞ optimal control technique is applied to a plant an additional frequency dependent weights are to be augmented in the plant and these weights are selected to show particular stability and performance specifications.

The problem now becomes a mixed weight H-infinity controller design which provides a closed loop response of the system.

The added weight functions are W_s and W_t, and they have to be specified to meet the system specifications, where W_s is the performance weighting function which limits the magnitude of S the sensitivity function and W_t is the robustness weighting function to limit the magnitude of T the complementary sensitivity function.

4. SIMULATION AND RESULTS

The following values represent the parameters of the TMI pressurized water Reactor [6]

According to the assumptions taken into account the following parameters are used for the simulation using Matlab Toolbox [10].

β =0.0065	λ= 0.125 s^{-1}
Λ=0.0001 s	f_f =0.98
G_r=0.01 total rod reactivity	T_e =290 C
P_{0a}=2500 MW	μ_c =70.5 MW s/C
μ_f =26.3 MWs/c	M=92.8 MW/C
Ω =6.53 MW/C	α_f =-0.00005 reactivity /C
α_c =0.00001 reactivity/C	

$D_{11} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ and $D_{22} = 0$,

$D_{12} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $D_{21} = (0 \quad -1)$,

$B_1 = \begin{pmatrix} 0\,0\,0\,0\,1 \\ 0\,0\,0\,0\,0 \end{pmatrix}^T$, which represents the disturbance acts on the system,

$C_1 = \begin{pmatrix} 1\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0 \end{pmatrix}$ and $C_2 = (-1 \quad 0 \quad 0 \quad 0 \quad 0)$

The normalized system transfer function is:

$$G(s)=\frac{100s^3+173.6s^2+52.82s+4.085}{s^5+66.74s^4+151.8s^3+90.17s^2+7.8845s}$$

And the designed controller K has the following form

$$1{:}K(s)=\frac{-369.3s^4-5.395e04s^3-9.278e04s^2-2.824e04s-2185}{s^5+66.75s^4+152.5s^3+91.83s^2+8.845s+0.08435}$$

$$2{:}K(s)=\frac{-2.789e04s^4-1.861e06s^3-4.161e06s^2-2.411e06s-2.116e05}{s^5+66.75s^4+152.5s^3+91.83s^2+8.845s+0.08435}$$

And by applying the designed controller to the augmented plant the following results are obtained

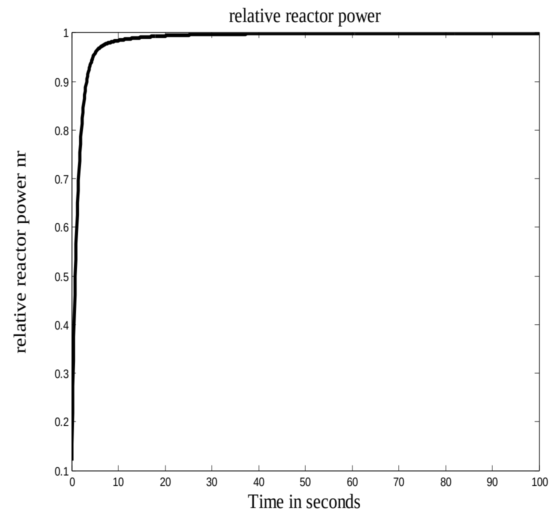

Figure 2 Relative reactor power

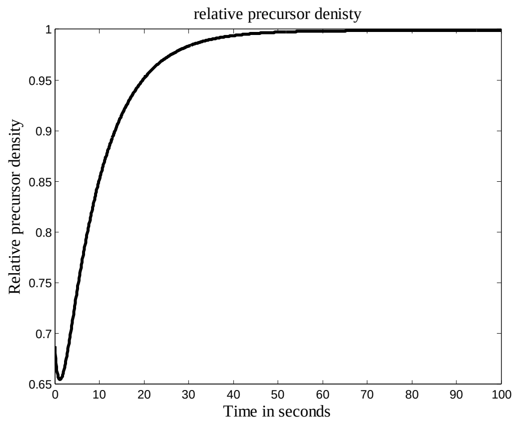

Figure 3 Relative precursor density

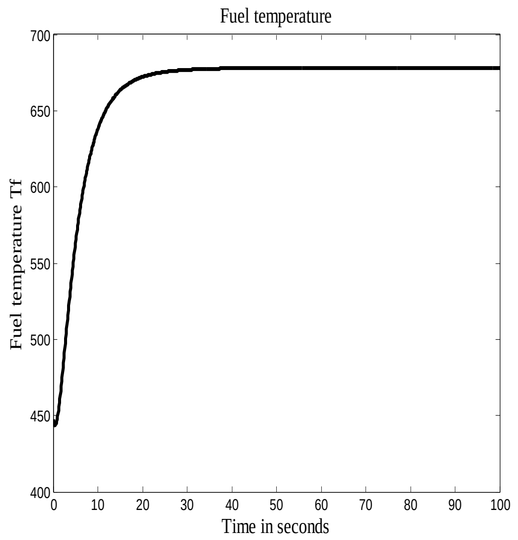

Figure 4 Fuel temperature

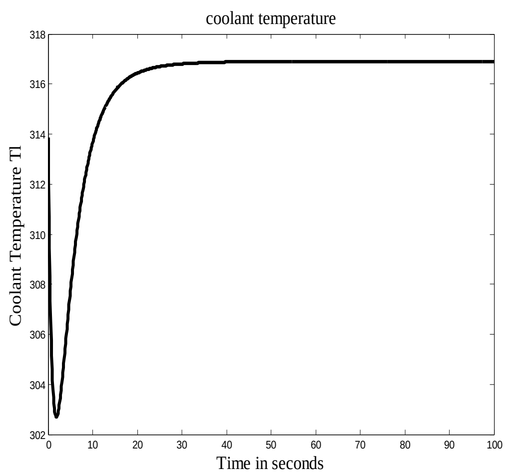

Figure 5 Coolant temperature

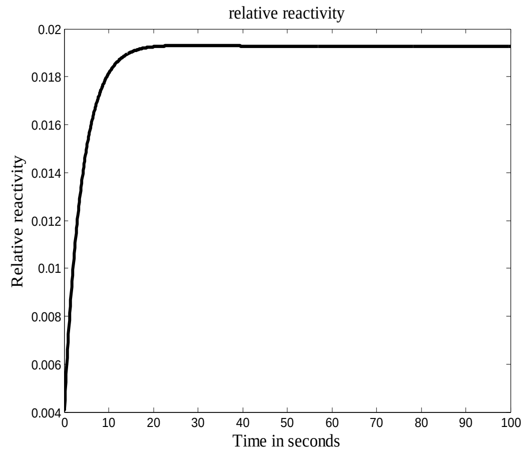

Figure 6 Relative reactivity

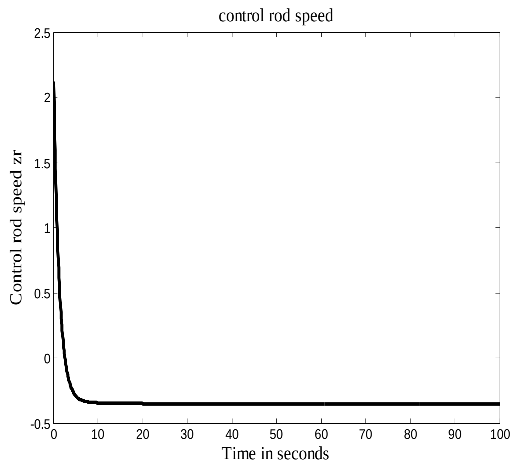

Figure 7 Control rod speed

5. CONCLUSION

Simulation results show that the time response of the output power and the control input as well. From these results it is observable that the system reaches its steady state in about 60 seconds and the maximum control rod speed does not exceed 2cm/sec which is a constraint for the control rod speed [9]. Also it is noticeable from the results that H-infinity controller achieves both robustness and good performance as it rejects the disturbances effectively.

Also the results show that the suggested control technique improves the fuel and temperature responses and the responses do not suffer from any overshoots.

Comparing the obtained results in this paper by the results obtained in H2 controller of the nuclear reactor [11] paper one can find that both of the techniques satisfy sufficient control requirements of the nuclear reactor.

It is recommended to apply the same technique to the reactor in the case of changing the power level by increasing or decreasing and also in case of low power (10%) to achieve a good control of the system in these cases.

6. REFERENCES

[1] J. Treunicht, May 15,2007, "H_∞ control"

[2] Ian R.Petersen,Valery A.Ugrinovski and Andrey v.Savkin,2000, "Robust Control design using H_∞ methods".

[3] Gerald a.Hartley,master thesis 1990,"Robust control design using H2 and H_∞ methods"

[4] Ankit Bansal,Veena Sharma,"Design and Analysis of Robust H-infinity Controller".

[5] Zames G,1981, "feedback of optimal sensitivity Model reference transformation ,Multiplicative semi-norms and approximate Inverse", IEEE Trans, Ac vol Ac-26.

[6] Robert M.Edwards, Kwange Y.Lee and Asok Ray ,1991 .Robust optimal control of nuclear reactors and power plants, University Park, Pennsylvania.

[7] Doyle J. C., Glover K., Khargonekar P. P and Francis B. A., 1988, "state space solution to standard H2 and H_∞ control problems", American control conference, Atlanta.

[8] Zhou K. and Khargonerkar P. P , 1988, "An algebraic Riccati Equation Approach to H_∞ optimization" systems and control letters.

[9] Yoon-Joon lee and Jung-In Choi, 1997, "Robust Controller Design for the Nuclear Reactor Power Control System", Journal of the Korean Nuclear Society.

[10] Richard Y. Chiang and Michael G. Safono, 1997,"Matlab Robust Control Toolbox".

[11] Rehab M.Saeed and Gamal.M. El-bayoumi, 2017, "Robust H2 control of the nuclear reactor",IJCA.