

Hybridization of Soft Expert Set with AHP in Decision Making

M. Saeed
Department of Mathematics
University of Management and
Technology
Lahore. Pakistan

Sahrish Mahmood
Department of Mathematics
University of Management and
Technology
Lahore. Pakistan

Hina Taufeeq
Department of Mathematics
University of Management and
Technology
Lahore. Pakistan

ABSTRACT

In this note a typical problem of selecting manager is presented where soft expert set is indecisive. Because of this indecisiveness, hybridization of soft expert set with AHP technique has been developed, and solution is achieved. Furthermore, the effectiveness of result is verified by applying TOPSIS technique.

Keywords

Hybridization, Indecision, Soft Expert Set, Agree soft expert set, Disagree soft expert set, TOPSIS, RTD Matrix, CETD Matrix, AHP Technique

1. INTRODUCTION

Molodtsov[13] (1999) introduced soft set theory as a new mathematical tool to overcome on the uncertainties while modeling the issues in Medical Sciences, Computer sciences, Social Sciences, Engineering, Physics and Economics. In addition to this, Maji et al [11] also introduced an application of soft sets in decision making problems and comprehend the fundamental philosophy of soft set theory.

Over the past few years these methods are widely being used for several problems. Numerous researchers had done this study before and they created models too, to solve issues in decision making. Irony with the work is this that most of the models can tackle only one opinion or with one expert. And this creates hurdles for the user while obtaining data from the questionnaire.

Furthermore, Alkhalaleh and Salleh [17] stated soft expert set first then introduced a model for the users, by using this user can know the opinion of the experts without any operations.

Feng Kong And hongyan Liu [4] they aimed to find out the key factors that affect success in E-commerce using Fuzzy AHP, and give an evaluation method for E-commerce in order to help researchers and managers to determine the drawbacks and opportunities.

Jana Krejci et al [8] highlighted the necessity of applying the concept of constrained fuzzy arithmetic instead of the concept of standard fuzzy arithmetic in a fuzzy extension of Analytic Hierarchy Process (AHP). Zulqarnain and Saeed [20] worked on interval valued fuzzy soft matrix (IVFSM) in decision making. They combined IVFSS with IVFS and soft sets, which is the consequence of Yang et al [19] Sabir Hussain[15] worked on soft expert topological spaces, Shabir, M and Naz, M[16] on soft topological spaces, Wand Y.J and Lee[18] worked on generalized TOPSIS for fuzzy multiple criteria group decision making. Gao [5-6], Ramik [14], Dodd [3], Hwang [7] and Linkov [10] works further on decision making problems.

Mainly the paper emphasizes is to attain a verdict for a everyday life problem by using soft expert set and the efficiency of the same issue is demonstrated with the help of Combined Effect Time Dependent Data (CETD) Matrix and it is checked for stability via Analytic Hierarchy Process (AHP). Finally, the results are verified by TOPSIS technique.

2. PRELIMINARIES

2.1 Definition: (Soft Expert Set)

A pair (F, A) is called a soft expert set over U , where F is a mapping given by

$$F: A \rightarrow P(U)$$

Where $P(U)$ denotes the power set of U .

2.2 Definition: (Agree-Soft expert Set)

An agree soft expert set $(F, A)_1$ over U is a soft expert subset of (F, A) is defined as

$$(F, A)_1 = \{F_1 e : e \in E * X * 1\}$$

2.3 Definition: (Disagree-Soft Expert Set)

An disagree soft expert set $(F, A)_0$ over U is a soft expert subset of (F, A) is defined as

$$(F, A)_0 = \{F_0(e) : e \in E * X * 0\}$$

2.4 Definition: (AHP)

The Analytic Hierarchy Process (AHP) is a multi-criteria decision making method allows decision makers to model a complex problem in a hierarchical structure, showing the relationships of the goal, objectives (criteria), and alternatives.

3. DESCRIPTION OF PROPOSED METHODOLOGY

The proposed methodology requires making a decision for selection of the Manager. This was initially done by applying Soft Expert Set on the opinions of experts. Steps of Soft

Expert set are as follows:

- Assigning weights to parameters
- Construction of Agree Matrix
- Construction of Average Time Dependent (ATD) Matrix
- Construction of Refined Time Dependent(RTD) Matrix
- Construction of CETD Matrix
- Sum of column Matrix
- Observation of outcome and arrive at result.

The implementation of Soft Expert Set resulted in multiple decisions instead of one. Arriving at a situation where two persons out of four were ineligible. So to select between two

remaining persons, the AHP technique is used. Steps of AHP are as follows [12]:

- Make a pair wise comparison matrix for criteria. In this study, a three point scale has been used as previous studies. because it has been demonstrated that most responders do not use more than three judgments (equal, more, much more) and lay users reported puzzled at a moment of using a more complex scale.
- Determine the relative normalized weight (W_j) of each criteria/sub-criterion by normalizing the geometric mean of the rows in the comparison matrix:

$$GM_j = \prod_{i=1}^N a_{ij}^{1/N}$$

$$W_j = \frac{GM_j}{\sum_{j=1}^N GM_j}$$

4. AN IMPLEMENTATION OF SOFT EXPERT SET

In this section, the implementation of soft expert set in the problem of decision making is presented. The problem that is to be considered is as follows:

Assume that a company wants to hire Manager. There are four nominees, who, from the universe $U = \{s_1, s_2, s_3, s_4\}$. Let $X = \{p, q, r\}$ be a set of expert members. These experts consider a set of parameters $E = \{e_1, e_2, e_3\}$ where the parameter $i, (i = 1, 2, 3)$ stands for the Language Test, Professional Test and Academic Record respectively.

4.1 Decision Making Team

The expert team was formed by:

- p = Administrative head of the entire company (with 15 years of experience)
- q = Head of HRM department (with 10 years of experience)
- r = Managing Director (MD) under whose supervision the managers have to work to achieve certain goals. (with 5 years of experience)

4.2 Description of Criteria

- $e_1 = \text{Language Test}$ is described as the command of a person over language. The person is supposed to be exceptionally fluent in required linguistics.
- $e_2 = \text{Professional test}$ is described as the assessment of professional skills that a person has acquired. How well he/she adjusts with the environment as well as copes up with the situation of stress.
- $e_3 = \text{Academic Record}$ is the compiled form of the person's educational achievements i.e. acquired degrees, distinctions/grades (percentage, cpga etc)

Assume:

$$(F, Z) = \{((e_1, p, 1), \{s_1, s_2\}), ((e_1, q, 1), \{s_3, s_4\}),$$

$$((e_1, r, 1), \{s_1, s_2, s_3\}),$$

$$((e_2, p, 1), \{s_3, s_4\}), ((e_2, q, 1), \{s_1, s_2\}),$$

$$((e_2, r, 1), \{s_4\}),$$

$$((e_3, p, 1), \{s_1, s_3\}), ((e_3, q, 1), \{s_1, s_2, s_4\}),$$

$$((e_3, r, 1), \{s_1, s_4\}),$$

$$((e_1, p, 0), \{s_3, s_4\}), ((e_2, q, 0), \{s_1, s_2\}),$$

$$((e_1, r, 0), \{s_4\}),$$

$$((e_2, p, 0), \{s_1, s_2\}), ((e_2, q, 0), \{s_3, s_4\}),$$

$$((e_2, r, 0), \{s_1, s_2, s_3\}),$$

$$((e_3, p, 0), \{s_2, s_4\}), ((e_3, q, 0), \{s_3\}),$$

$$((e_3, r, 0), \{s_2, s_3\})\}$$

This states that the decision makers p, q & r choose $\{s_1, s_2\}, \{s_3, s_4\}$ & $\{s_1, s_2, s_3\}$ respectively by judging $\{s_1, s_2, s_3, s_4\}$ on the criteria e_1 i.e. Language test.

It can also be said that decision makers p, q & r did not choose $\{s_3, s_4\}, \{s_1, s_2\}$ & $\{s_4\}$ respectively on the criteria e_1 i.e. Language test.

1 shows the eligibility criteria of decision maker where as 0 shows ineligibility criteria.

Now, in order to make a decision, i.e. to choose one person out of others for Manager Post, the parameters are given some weight, say

$$e_1 = 0.5, e_2 = 0.7, e_3 = 1$$

The result can be made by the use of agree matrix as follows:

$$\begin{pmatrix} 2 & 2 & 2 & 1 \\ 1 & 1 & 1 & 2 \\ 3 & 1 & 1 & 2 \end{pmatrix}$$

This matrix is made with the problem described in (F, Z). The Average Time Dependent Data (ATD) Matrix obtained is

$$\begin{pmatrix} 4 & 4 & 4 & 2 \\ 1.43 & 1.43 & 1.43 & 2.86 \\ 3 & 1 & 1 & 2 \end{pmatrix}$$

To obtain the Refined Time Dependent Data (RTD) Matrix [2] we use.

$$\text{if } a_{ij} \leq (\mu_j - \alpha * \sigma_j) \text{ Then } e_{ij} = -1$$

$$\text{if } a_{ij} \in (\mu_j - \alpha * \sigma_j, \mu_j + \alpha * \sigma_j) \text{ Then } e_{ij} = 0$$

$$\text{if } a_{ij} \geq (\mu_j + \alpha * \sigma_j) \text{ Then } e_{ij} = +1$$

With values of α as follows 0.25, 0.50, 1 the following RTD's were obtained.

For $\alpha = 0.25$

$$\begin{pmatrix} 1 & 1 & 1 & -1 \\ -1 & -1 & -1 & 1 \\ 0 & -1 & -1 & -1 \end{pmatrix}$$

For $\alpha = 0.50$

$$\begin{pmatrix} 1 & 1 & 1 & -1 \\ -1 & -1 & -1 & 1 \\ 0 & -1 & -1 & 0 \end{pmatrix}$$

For $\alpha = 0.75$

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & -1 & -1 & 0 \end{pmatrix}$$

The CETD matrix formed with the above obtained RTD is

$$\begin{pmatrix} 3 & 3 & 3 & -2 \\ -3 & -2 & -2 & 3 \\ 0 & -3 & -3 & -1 \end{pmatrix}$$

The sum of above column matrix is(0 -2 -2 0). This matrix gives the outcome that persons at s_2 and s_3 positions cannot be opted for the manager post.

If, considered the same problem with different weights of parameter say

$$e_1 = 0.6, e_2 = 0.4, e_3 = 1$$

Then, the agree matrix will be same but the Average Time Dependand Data (RTD) Matrix obtained is

$$\begin{pmatrix} 3.33 & 3.33 & 3.33 & 1.67 \\ 2.5 & 2.5 & 2.5 & 5 \\ 3 & 1 & 1 & 2 \end{pmatrix}$$

The column matrix is(0 0 0 -2). (This is done just for the purpose of comparison between answer of two weights selected) This matrix gives the result that person at s_4 position cannot be opted for the manager post. Now, check for selection of the persons at s_1, s_2 and s_3 positions by applying AHP.

Here it can be briefly said that soft expert set did not choose any person, rather it eliminated one person in this particular case. To make a choice further AHP technique is applied.

5. HYBRIDIZATION WITH AHP TECHNIQUE

AHP is scalable. AHP is usually used to reduce the complexities of decision making in a certain and definite way. This method support (or else is applicable) for both single person also to make a group decision. This method does not necessarily need the presence of any specialist. [1]

The AHP is a powerful and flexible Multi Criteria Decision Making tool for dealing with complex problems where both qualitative and quantitative aspects are taken into consideration. [9]

The problem of selecting a best Manager is applied again with this technique to verify the results obtained from soft expert set. The persons are compared using the pair wise comparison scale for AHP preference. The following tables are framed with Saaty's scale for each of the parameter.

Table 1. Comparison Table For e_1

Parameter e_1	s_1	s_2	s_3	s_4
s_1	1	1	1	2
s_2	1	1	1	2
s_3	1	1	1	2
s_4	1/2	1/2	1/2	1

Table 2. Comparison Table For e_2

Parameter e_2	s_1	s_2	s_3	s_4
s_1	1	1	1	1/2
s_2	1	1	1	1/2
s_3	1	1	1	1/2
s_4	2	2	2	1

Table 3. Comparison Table For e_3

Parameter e_3	s_1	s_2	s_3	s_4
s_1	1	3	3	2
s_2	1/3	1	1	1/2
s_3	1/3	1	1	1/2
s_4	1/2	2	2	1

The above tables are used to find the Eigen vector A_{ij} by dividing the value using the column sum.

Table 4. Average table for parameter e_1

Parameter e_1	s_1	s_2	s_3	s_4	Average A_{ij}
s_1	0.2857	0.2857	0.2857	0.2857	0.2857
s_2	0.2857	0.2857	0.2857	0.2857	0.2857
s_3	0.2857	0.2857	0.2857	0.2857	0.2857
s_4	0.1428	0.1428	0.1428	0.1428	0.1428

Above table is the average table for parameter e_1 , which is obtained from comparison table of e_1 . This table will be used to find Eigen values for e_1 .

Table 5. Average table for parameter e_2

Parameter e_2	s_1	s_2	s_3	s_4	Average A_{ij}
s_1	0.2	0.2	0.2	0.2	0.2
s_2	0.2	0.2	0.2	0.2	0.2
s_3	0.2	0.2	0.2	0.2	0.2
s_4	0.4	0.4	0.4	0.4	0.4

Above table is the average table for parameter e_2 , which is obtained from comparison table of e_2 . This table will be used to find Eigen values for e_2 .

Table 6. Average table for parameter e_3

Parameter e_3	s_1	s_2	s_3	s_4	Average A_{ij}
s_1	0.4615	0.4285	0.4285	0.5	0.4546
s_2	0.1538	0.1428	0.1428	0.125	0.1411
s_3	0.1538	0.1428	0.1428	0.125	0.1411
s_4	0.23076	0.2857	0.2857	0.25	0.26304

Above table is the average table for parameter e_3 , which is obtained from comparison table of e_3 . This table will be used to find Eigen values for e_3 .

The corresponding Eigen values are:

- For $e_1(1.1428, 1.1428, 1.1428, 0.5714)$
- For $e_2(0.8, 0.8, 0.8, 1.6)$
- For $e_3(1.827, 0.561, 0.565, 1.195)$

The Eigen values suggest that s_1 is best choice for the manager post.

6. VERIFICATION WITH TOPSIS

Parameter table of AHP method is used:

Table 7. For e_1

Parameter e_3	s_1	s_2	s_3	s_4	Average A_{ij}
s_1	0.2857	0.2857	0.2857	0.2857	0.2857
s_2	0.2857	0.2857	0.2857	0.2857	0.2857
s_3	0.2857	0.2857	0.2857	0.2857	0.2857
s_4	0.1428	0.1428	0.1428	0.1428	0.1428

Here another technique is applied, known as TOPSIS. This method is applied on AHP just for verification of the results. In TOPSIS average is used as weights. The TOPSIS method includes following steps:

6.1 Step 1

Table 8. Calculate $(\sum x^2_{ij})^{\frac{1}{2}}$ for each column

s_i^2	s_1	s_2	s_3	s_4
s_1	0.08162	0.08162	0.08162	0.08162
s_2	0.08162	0.08162	0.08162	0.08162
s_3	0.08162	0.08162	0.08162	0.08162
s_4	0.02039	0.02039	0.02039	0.02039
$(\sum x^2_{ij})$	0.26525	0.26525	0.26525	0.26525
$(\sum x^2_{ij})^{\frac{1}{2}}$	0.51502	0.51502	0.51502	0.51502

6.2 Step 2

Table 9. Divide each column by $(\sum x^2_{ij})^{\frac{1}{2}}$ to get r_{ij}

	s_1	s_2	s_3	s_4
s_1	0.5547	0.5547	0.5547	0.5547
s_2	0.5547	0.5547	0.5547	0.5547
s_3	0.5547	0.5547	0.5547	0.5547
s_4	0.2773	0.2773	0.2773	0.2773

6.3 Step 3

Table 10. Multiply each column by w_{ij} to get v_{ij}

	s_1	s_2	s_3	s_4
s_1	0.15848	0.15848	0.15848	0.0792
s_2	0.15848	0.15848	0.15848	0.0792
s_3	0.15848	0.15848	0.15848	0.0792
s_4	0.07922	0.07922	0.07922	0.0396

6.4 Step 4

Determine Ideal Solution A^* from the step 3

$$A^* = \{0.15848, 0.15848, 0.15848, 0.0792\}$$

6.5 Step 5

Determine Ideal Solution A' from the step 3

$$A' = \{0.07922, 0.07922, 0.07922, 0.0396\}$$

6.6 Step 6

Calculate separation from the ideal Solution

$$A^* = \{0.15848, 0.15848, 0.15848, 0.0792\}$$

$$S^* = [\sum (v_{jj}^* - v_{ij})^2]^{\frac{1}{2}}$$

For each row

Table 11. Separation from ideal solution

	s_1	s_2	s_3	s_4
s_1	0	0	0	0
s_2	0	0	0	0
s_3	0	0	0	0
s_4	$(0.07922 - 0.15848)^2$	$(0.07922 - 0.15848)^2$	$(0.07922 - 0.15848)^2$	$(-0.0396)^2$

6.7 Step 7

Table 12. Calculate separation from the ideal S_i^*

	$\sum (v_{jj}^* - v_{ij})^2$	$S_i^* = \left[\sum (v_{jj}^* - v_{ij})^2 \right]^{\frac{1}{2}}$
s_1	0	0
s_2	0	0
s_3	0	0
s_4	0.020415	0.142881

6.8 Step 8

Calculate separation from the negative ideal solution

$$A' = \{0.07922, 0.07922, 0.07922, 0.0396\}$$

$$S_i' = \left[\sum (v_{jj}^* - v_{ij})^2 \right]^{\frac{1}{2}}$$

Table 13. For each row

	s_1	s_2	s_3	s_4
s_1	0.0062821	0.0062821	0.0062821	0.00156816
s_2	0.0062821	0.0062821	0.0062821	0.00156816
s_3	0.0062821	0.0062821	0.0062821	0.00156816
s_4	0	0	0	0

6.9 Step 9

Table 14. Calculate separation from the ideal S_i'

	$\sum (v_{jj}^* - v_{ij})^2$	$S_i' = \left[\sum (v_{jj}^* - v_{ij})^2 \right]^{\frac{1}{2}}$
s_1	0.02041	0.142879
s_2	0.02041	0.142879
s_3	0.02041	0.142879
s_4	0	0

6.10 Step 10

Calculate the relative closeness to the ideal solution C_i^*

$$C_i^* = \frac{S_i'}{S_i^* + S_i'}$$

Table 15. Relative closeness to ideal solution

	$C_i^* = \frac{S_i'}{S_i^* + S_i'}$	C_i^*
s_1	$\frac{0.142879}{0.142879}$	1
s_2	1	1
s_3	1	1
s_4	1	0

The results show that s_1, s_2, s_3 can be opted for the post.

- Similarly, we applied the steps on parameter e_2
- And the results show that s_4 was most suitable person.
- & for parameter e_3 , s_1 was suitable for the post.
- Since s_1 has satisfied the criteria in two parameters
- Hence s_1 will be selected in TOPSIS method.

7. CONCLUSION

In this paper, decision making problem was analyzed using soft expert set, through which results were indecisive. Consequently decision could not be made, and then AHP technique was applied to avoid uncertainty. Finally, the results were verified by TOPSIS method. When soft expert set is hybridized with AHP it reduces cost. TOPSIS technique verifies that it does not affect the final outcome. In future hybridization of soft expert set with other methods can be done, and their results can be further analyzed.

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