Energy based Detection Scheme for Orthogonal Frequency Division Multiplexing

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ABSTRACT
Orthogonal Frequency Division Multiplexing (OFDM) has been accepted as the modulation scheme of choice for the next generation high-speed wireless communication systems due to the advantages that it offers like high spectral efficiency, resistance to multipath fading and resistance to frequency selective fading. Moreover, it lends itself to simple channel equalization. Conventional single carrier systems do not provide such advantages and hence, OFDM would almost ubiquitously be used for high speed wireless data transmission. However, the main drawback of such systems over single carrier systems is that in the presence of noise, there is an increased computational complexity at the receiver end to decode the data. In this paper, a low complexity detection algorithm is proposed for OFDM systems. Maximum likelihood detection is taken as the baseline detection algorithm and the proposed algorithm is compared with ML detection algorithm. Comparison results are plotted and conclusions are drawn.

General Terms
Multicarrier communication systems, Orthogonal Frequency division multiplexing.

Keywords: OFDM, Computational complexity, symbol energy.

1. INTRODUCTION
Orthogonal Frequency Division Multiplexing (OFDM) is a special case of multicarrier transmission where a single data stream is transmitted over a number of lower rate subcarriers [1]. OFDM has been placed at the apex due to the advantages that it offers like resistance to frequency selective fading and resistance to multipath fading. In addition to robustness that OFDM offers in wireless channels, it also provides high spectral efficiency and lends itself to simple channel equalization. Owing to such advantages, OFDM has received widespread attention in the communications industry as evidenced by standards such as xDSL (digital subscriber lines), IEEE 802.11a/g for wireless Local Area Networks (WLANs) [2] and 3GPP Long Term Evolution (LTE) wireless cellular systems [3] among many others. However, in the presence of noise, symbol detection at the receiver becomes computationally complex when large numbers of subcarriers are employed. This would be the case while transmitting through wideband channels that have very low coherence bandwidth. Considerable research has gone into developing detection algorithms for OFDM systems. A brief overview of existing techniques is given below.

The paper is organized as follows. Section-II gives an introduction to OFDM and describes the modulation and demodulation process. Special emphasis is laid on the design of signal constellation since it is this design of constellation that facilitates the use of the proposed detection algorithm. The form of the received signal which is corrupted by noise is also given and the problem of detection is formulated. Section-III describes the proposed low complexity detection algorithm. Maximum likelihood detection is taken as the baseline detection algorithm and in section-IV, the proposed algorithm is compared with the baseline detection algorithm on the basis of computational complexity and bit error rate performance. Plot of SNR versus BER is obtained through simulation for both ML and proposed detection algorithms and the results are plotted. Section-V gives the summary and conclusions.

2. Problem Formulation
C = [F_1(t), F_2(t) ... F_N(t)] is the set of N orthogonal subcarriers used by the transmitter. N must be chosen such that the bandwidth of each subcarrier is lesser than the coherence bandwidth of the channel.


While all of the above references provide effective methods for symbol detection, most of them concentrate mainly on narrowband, impulsive and coloured noise. The noise offered by wireless channels, however, cannot be modelled as any of these and hence, the methods provided in [4-9] cannot be directly used when communicating over wireless channels. Alternate detection schemes are required for this purpose. This paper proposes a novel detection algorithm for OFDM systems in the presence of AWGN. The basic aim of the paper is to reduce the computational complexity at the receiver end for symbol detection. In the scheme that we propose, the receiver is required to perform no vector manipulations to detect data. The algorithm operates purely on scalar manipulations thereby reducing the computational load on the receiver for symbol detection.
S = {S₁, S₂ ... Sₘ} is the signal constellation used. M is determined by the required data rate. The set S must contain vectors from the vector space R³. Furthermore, the vectors in S must be chosen in such a way that no two vectors in S have the same energy. The rationale for these conditions will be clear in the subsequent sections. Having chosen the signal constellations as above, modulation and demodulation are done as follows:

### 2.1 Modulation

The data symbols to be transmitted are produced sequentially by the signal constellation. Let this data sequence be X₁, X₂, X₃ and so on. Each of these symbols is an N-tuple as per the choice of the set S. The above sequence of information symbols is modulated and transmitted as follows:

\[ T(t) = \sum_{k=0}^{∞} \sum_{j=1}^{N} \{X_{jk} \cdot i_{jk} \} F_{j}(t - kT_{s}) \]  
\[ \text{(1)} \]

### 2.2 Demodulation

The signal obtained at the output of the channel equalizer of the receiver, in the ideal case, is given by

\[ R(t) = T(t) + N(t) \]  
\[ \text{(2)} \]

where N(t) is the noise that was added in the channel which we assume to be AWGN. By correlating the received signal with \( F_{j}(t-kT_{s}) \), the receiver recovers the i-th entry of the N-tuple sent in the k-th time frame.

\[ \{R(t), F_{j}(t-kT_{s})\} = X_{jk}(i) + N_{jk}(i) \]  
\[ \text{(3)} \]

where

\[ N_{jk}(i) = \{N(t), F_{j}(t-kT_{s})\} \]  
\[ \text{(4)} \]

By doing the above operation for all i, the receiver demodulates the vector sent in the k-th time frame and by doing it for all k, the receiver demodulates the transmitted sequence of vectors. Let this demodulated vector sequence be Y₀, Y₁, Y₂ ..., where each Yᵢ is a vector in R³. This demodulated data is the noise corrupted version of the transmitted vector sequence. Mathematically, the received vector during k-th time frame, \( Y_{k} \) can be expressed in terms of transmitted vector at k-th time frame, \( X_{k} \) as

\[ Y_{k} = X_{k} + N_{k} \]  
\[ \text{(5)} \]

where \( N_{k} \) is a Gaussian random vector in R³ each entry of which is given by (4). Hence, the problem of detection can be formulated as follows. Recover the transmitted vector \( X_{k} \) from the received vector \( Y_{k} \) for all values of k. In theory, ML detection algorithm gives the best results in terms of bit error rate but this algorithm involves high computational complexity and hence may not be a practical solution for the problem of detection. The following section proposes a novel detection algorithm that reduces this computational complexity significantly thereby making it a practical solution for the problem of detection.

### 3. PROPOSED DETECTION ALGORITHM

The added noise vector \( N_{k} \) has its entries to be i.i.d Gaussian random variables with zero mean and variance \( V^{2} \). Let \( N_{\text{max}} \) be the value such that the probability that \( N_{\text{max}}(t) \) takes a value greater than \( N_{\text{max}} \) is \( P_{c} \). \( P_{c} \) is typically taken to be close to zero. Hence, with a high degree of confidence, \( N_{\text{max}} \) can be assumed to be the upper bound value taken by noise. The proposed algorithm is basically built upon the above assumption. The improbable case when noise takes a value greater than \( N_{\text{max}} \) is when an error occurs in detection. Error simulation for the proposed algorithm has been done and the results are provided in the next section.

Since each entry of the noise vector has an upper bound, the energy of the added vector also has an upper bound which happens when all the entries of the noise vector take their maximum value simultaneously. This upper bound is given by

\[ E_{\text{UB}} = [N(N_{\text{max}})]^{2} \]  
\[ \text{(6)} \]

Now, the signal constellation is chosen in such a way that no two symbols in the constellation have the same energy. Suppose that the minimum energy difference between any two symbols is \( E_{\text{min}} \). \( E_{\text{min}} \) is chosen according to the following inequality.

\[ E_{\text{min}} \geq 2E_{\text{UB}} \]  
\[ \text{(7)} \]

Suppose that the transmitted vector is \( S_{j} \) and its energy is \( E_{j} \). From the triangle inequality, we can conclude that the energy of received vector which is the sum of the transmitted vector and noise would be upper bounded by \( E_{j} + E_{\text{UB}} \) and lower bounded by \( E_{j} - E_{\text{UB}} \). Mathematically,

\[ E_{j} - E_{\text{UB}} \leq E_{j} \leq E_{j} + E_{\text{UB}} \]  
\[ \text{(8)} \]

where \( E_{j} \) is the energy of the received vector.

This detection algorithm involves only comparison of scalar quantities and doesn’t involve any tedious vector manipulations (except while computing the energy of the received vector) unlike ML detection algorithm which requires the receiver to do vector operations for detection. Hence, the proposed algorithm is computationally more efficient compared to ML detection algorithm. The next section proves this efficiency by providing relevant comparison results.

### 4. SIMULATION AND RESULTS

This section provides the comparison results of computational complexity and BER performance for proposed system and baseline system. Figure 1 is the plot of computational complexity versus the constellation size. The black colored line corresponds to ML detection algorithm and the red colored line corresponds to the proposed detection algorithm. The figure has been plotted for an OFDM system employing 16 subcarriers. The constellation size is varied from 2 to 1024 corresponding to a data rate of 1 bit to 10 bits per transmission. The graph has been plotted in logarithmic scale. The computational complexity has been measured as the total number of additions and multiplications.
required to detect each vector which is an element in $R^{16}$. The plot shows that the proposed algorithm comfortably outperforms ML detection algorithm in terms of computational complexity.

Figure 2 provides the comparison result for BER performances of ML detection algorithm and the proposed detection algorithm. The constellation for the system employing the proposed algorithm is chosen as per the conditions mentioned in the paper. The constellation for system employing ML algorithm is chosen from a 2-QAM constellation resulting in a total of 8 QAM symbols being transmitted per time frame. The constellation size for the proposed detection algorithm is chosen to be 256 so that the number of bits per transmission for the systems employing the two algorithms is the same. SNR is varied by scaling the symbols in the signal constellation in the case of proposed scheme while the constellation power is varied by scaling the QAM constellation in the case of ML detection scheme. The BER expression for ML detection algorithm is given in [10] and the graph is plotted in logarithmic scale. It is seen from the figure that for lower values of SNR, the proposed detection algorithm has a lower BER while for higher values of SNR, ML detection algorithm has a lower BER. Therefore, it can be concluded that the proposed scheme provides better performance than ML detection scheme in terms of both computational complexity and bit error rate for low values of SNR whereas the proposed algorithm sacrifices BER for computational complexity for higher values of SNR. Error control coding can be done at the transmitter before transmission to reduce the BER of the proposed scheme but it results in a reduction in data rate. This highlights the interplay between computational complexity, bit error rate and data rate for the proposed detection algorithm. The exact choice of parameters, therefore, would depend upon the system specifications like total available transmit power or the required BER.

5. SUMMARY AND CONCLUSIONS

In this paper, a low complexity detection algorithm which uses specially designed signal constellations was proposed for OFDM systems. The computational complexity of the proposed algorithm was compared with that of ML detection algorithm. The results prove that the proposed algorithm is computationally more efficient than ML detection algorithm.

This result also confirms to intuition since the proposed algorithm works purely on scalar operations which is not the case with ML detection algorithm. Finally, it is concluded that the proposed detection algorithm would be feasible in situations where ML algorithm wouldn’t.

6. REFERENCES