Airflow Process Across Vertical Vents Induced by Stack-Driven Effect with an Opposing Flow in one of the Upper Openings

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ABSTRACT
Natural ventilation of building provides improvement of internal comfort and air quality conditions leading to a significant reduction of cooling energy consumption. Design of natural ventilation systems for many types of building is based on buoyancy forces. However, external wind flow can have significant effects on buoyancy-driven natural ventilation. Airflow process across vertical vents induced by stack-driven effect with an opposing flow in one of the openings was presented. Equations governing air flow are utilized; models of fluid and thermo-dynamical problems have been described. Boussinesq assumption is invoked and analytic techniques are employed to obtain the possible solutions of model equation. Various parameters on air flow process were used to see the effect of changes of parameters to the overall flow distributions, and ascertain the best one for optimal natural ventilation.

Keywords  
Effective velocity distribution, Airflow Rate, Mass - transfer Rate, Effective Temperature Distribution

1. INTRODUCTION
Generally, airflow distributions in buildings are considered to be as a result of the knowledge of the exact air supply to a building. Knowledge of the exact air supply to a building is necessary to determine its thermal performance and the concentration of the indoor pollutants. The exchange of air can be achieved either by mechanical means (Mechanical ventilation) or through the large opening of the building envelope (Natural ventilation). Exchanges between external ambience and interior space of buildings caused by flows that are driven by wind or by temperature differences are the foundation of natural ventilation process. Of course natural ventilation is being pursued by humans, who are increasingly spending more time indoors, to extend the possibilities of living in uncongenial or squally conditions etc. The improvements of the quality of the interior space both in its attractiveness, spaciousness, luminosity, and more importantly its proper natural ventilation are major concerns for designers of modern structures. Air flow modeling gives Architects and Engineers the luxury to consider several design options in the minimum amount of time. As a result, the final design is not based on a tentative approach, but is a result of a professional design process considering several options and selecting the optimal solution. Air flow models are used to simulate the rates of incoming and outgoing air flows for a domain with known leakages under given weather and shielding conditions. Air flow models can be divided into two main categories, single-zone models and multi-zone models.

Single-zone models assume that the structure can be described by a single, well mixed zone. Many attempts to investigate this phenomenon have been made by some researchers. [1] Investigated the case of multiple openings in a wall and obtained an expression for the rate of flow of air in terms of temperature differences using Bernoulli’s equation for ideal flow, and did not consider the case of a single opening. A great deal of attention has been given in the studies of natural convection to problems of air flow process in buildings. The study of natural convection and their applications to practical situations, ranging from heating equipment to the cooling of turbine blades was conducted by [2]. [3] Described a simple basic theory of natural convection across openings in vertical partitions and generalized to include both heat and mass-transfer in a single-sided ventilated domain. [4] Shows the interaction of the wind with building by considering the wind-driven flow only. Many also attempted to investigate this phenomenon studied both displacement ventilation (where the interior is stratified) by [5], and the mixing ventilation (where the interior has uniform temperature) by [6]. [7] described a convective heat and mass-transfer through large openings, which plays an important role in the thermal behavior of domains and they also reviewed the model which describe the Phenomena due to gravitational and boundary layer pumping flows proposed up to 1992. [8] Developed a model that combined the ideas of displacement ventilation from [9] and with a hydraulic exchange by [10]. [11] Presented a consistent pressure-based formulation for natural ventilation of single-zone and multi-zone buildings with multiple openings. [12] considered building having two openings at different vertical level on opposite walls, the height of the two openings are relatively small, and the areas of the top and bottom openings are $A_L$ and $A_B$ respectively. There is an indoor source of heat $E$, the wind force can assist or oppose the thermal buoyancy force, when the indoor temperature is uniform and specified the ventilation flow rate driven by stack force alone. [13] Described a simple macro-scale mathematical model for the prediction of a heat recovery factor. The model is based on the steady-state one dimensional convection-diffusion equation, and provides a simple analytical solution for the calculation of heat recovery. [14] Considered single-sided natural ventilation, by using a computational fluid dynamics (CFD) model, together with analytical and empirical models. The CFD model was applied to determine the effects of buoyancy, wind, or their combination on ventilation rates and indoor conditions. [15] Considered natural ventilation in a full-scale building induced by combined wind and buoyancy forces. The overall objectives were to verify and validate a CFD model for the naturally ventilated buildings, collect high quality full-scale experimental data for CFD validation and
formulate guidelines for modeling natural ventilation in design practice and a steady envelope flow model were applied to calculate mean ventilation rates. [16] Examined the combination of both natural ventilation methods. A test room of single-sided ventilation was equipped with a vertical vent. Ventilation rate through the openings was evaluated based on the air flow velocity measured at the surface area of the openings. The vertical vent was kept closed during the first run of the experiments then the same experiments repeated where the vent was in use. Based on the experimental results, the effects of the vertical vent on the ventilation rate were clarified and a model was suggested based on combination for the two ventilation methods. [17] Described air flow rate across a vertical opening induced by a thermal source in a room, various parameters were used in designing natural ventilation. Analytical expressions derived from simple hydraulic model were reviewed. [18] considered the civil building with inside heating source, using the technique of CFD (computational fluid dynamics), changing the area of air inlet, and simulating the fluid field of the thermal natural ventilation, this paper presents the natural ventilation rate and the temperature difference between inlet air and exhaust air of different conditions, analyses the temperature field and velocity field in different conditions, gives the variation trends of thermal stratification height and Effective thermal coefficient with different inlet areas. The simulation results can provide theory foundation for natural ventilation systems design. [19] Considered wind-driven cross ventilation in building with small openings. [20] A simple mathematical model of stack ventilation flows in multi-compartment buildings is developed with a view to providing an intuitive understanding of the physical processes governing the movement of air and heat through naturally ventilated buildings. [21] described a model predictive control (MPC) strategies for buildings with mixed-mode cooling (window opening position, fan assist, and night cooling schedule) and demonstrates their potential performance bounds in terms of energy savings within thermal comfort constraints, in comparison with standard heuristic rules used in current practice.

The main objective of this paper is to analytically investigate airflow process inside a rectangular domain with three openings on vertical walls. This is the novel approach which we believe will lead to better understanding of the phenomenon and help in optimizing the designers for better natural ventilation.

2. DOMAIN DESCRIPTION

The domain considered is un-stratified cross ventilated rectangular domain with three-openings. In which the domain has two upper and one lower rectangular opening. Two upper openings have the same area of 0.7m × 1.0m while; for the lower opening is 0.7m × 2.0m. Dimension of the domain is 5.3m × 3.6m × 2.8m hight, with air as the connecting fluid. The domain envelop is separated from one another by a vertical rectangular openings of height y and width l, which is illustrated in Figure 1. The density in the domain is maintained at ρ with temperature at θ, and pressure at P.

3. RESEARCH METHODOLOGY

Flow chart of research methodology showing the process of obtaining results for airflow process in the domain is illustrated in Figure 2.

![Flow chart of research methodology showing the process of obtaining results for airflow process in the domain](image)

4. MODEL FORMULATION

Airflow process across vertical vents induced by stack-driven effect with an opposing flow in one of the upper openings is considered (see Figure 3). The flow is steady that depends on...
the height of the opening on the vertical walls. Airflow is assumed to be at low speed so that it will behaves like incompressible fluid. Internal heat source is negligible. Boussinesq assumption is invoked. Navier-stokes equations with appropriate boundary conditions described the problem. The model equations are written in a dimensionless form and solved analytically by means of variation of parameters.

Fig 3: Schematic diagram of airflow process inside unstratified cross ventilated rectangular domain with three opening

Governing Equations are: Navier-Stokes equations are simplified by the above mentioned assumptions. In which Continuity Equation is satisfied identically. The convection motion is then driven by the stack-driven effect.

\[ \frac{\partial u}{\partial y} = g \beta \Delta \theta + v \frac{\partial^2 u}{\partial y^2} \]  \(1\)

\[ \frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2} \]  \(2\)

Assuming there is an opposing flow from one side of the upper opening, the indirect velocity of air is given by, \(v_0\) and effective velocity and temp. Distributions are \(u = u_{eff}\), \(\theta = \theta_{eff}\).

The resulting governing Equations in Equation (1), (2), can be re-written as,

\[ \frac{\partial u_{eff}}{\partial y} = g \beta \Delta \theta + u_{eff} \frac{\partial^2 u_{eff}}{\partial y^2} \]  \(3\)

\[ \frac{\partial \theta_{eff}}{\partial y} = \alpha \frac{\partial^2 \theta_{eff}}{\partial y^2} \]  \(4\)

Scaling \(y\) with \(y' L\), effective velocity \(u_{eff}\) with \(\frac{u_{eff} g \beta \Delta \theta}{\alpha_{eff}}\), \(v_0\) with \(\frac{v_0}{\alpha_{eff}}\), and introducing \(\theta\) with \(\theta_{eff} \left( \alpha \Delta \theta \right) + \theta_0\) the dimensionless governing equations are

\[ -C \frac{\partial u_{eff}}{\partial y} = pr \frac{\partial^2 u_{eff}}{\partial y^2} + \theta_{eff} \]  \(5\)

\[ -C \frac{\partial \theta_{eff}}{\partial y} = \frac{\partial^2 \theta_{eff}}{\partial y^2} \]  \(6\)

Where, \(C = -v_0 Pr\)

With dimensionless initial/boundary conditions as;

\[ 0 \leq y \leq 2, \theta_w > 0, \theta_r = 0, \theta_{w2} > \theta_{w1}, u_{eff}(0) = 0, u_{eff}(1) = 0 \]  \(7\)

\[ 0 \leq y \leq 2, \theta_w > 0, \theta_r = 0, \theta_{w2} > \theta_{w1}, \theta_{eff}(0) = 0 \]  \(8\)

In which subscripts \(w1, w2, eff\) denotes vertical wall 1, wall 2, effective and \(\theta_0\) is the effective thermal coefficient.

5. SOLUTION OF THE MODEL EQUATIONS
5.1 Effective Temperature Distribution
The solution of Equation (6), yields to,

\[ \theta_{eff}(y) = C_1 e^{-cy} \]  \(9\)

The two constant \(C_1\) and \(C_2\) can be determined by prescribing the boundary conditions in Equation (8) for the effective temperature field, thus obtaining:

\[ C_1 = \frac{1 - \theta_0 (1 - e^{-cy})}{1 - e^{-cy}} \]

\[ C_2 = \frac{-1}{1 - e^{-cy}} \]  \(10\)

Equation (9), together with the boundary conditions (8), yields to

\[ \theta_{eff}(y) = \frac{1 - \theta_0 (1 - e^{-cy}) - e^{-cy}}{1 - e^{-cy}} \]  \(11\)

Putting \(C = -v_0 Pr\), into Equation (11), one obtain the effective temperature distribution as,

\[ \theta_{eff}(y) = \frac{1 - \theta_0 (1 - e^{-Pry}) - e^{-Pry}}{1 - e^{-Pry}} \]  \(12\)

5.2 Effective Velocity Distribution
Putting Equation (12) into (5),

\[ \frac{d^2 u_{eff}}{dy^2} + \frac{C}{Pr} \frac{d u_{eff}}{dy} = - \frac{1}{Pr} \left[ \frac{1 - \theta_0 (1 - e^{-Pry}) - e^{-Pry}}{1 - e^{-Pry}} \right] \]  \(13\)

Recall that, \(C = -v_0 Pr\)

Equation (13), can be re-written as,

\[ \frac{d^2 u_{eff}}{dy^2} + \frac{C}{Pr} \frac{d u_{eff}}{dy} = 0 \]  \(14\)

Concerning Equation (14), one can start solving it from the homogeneous part:

\[ \frac{d^2 u_{eff}}{dy^2} + \frac{C}{Pr} \frac{d u_{eff}}{dy} = 0 \]  \(15\)

Two independent solutions of Equation (5) are, \(u_c, u_p\)

\[ u_c = C_3 u_1 + C_4 u_2 \]

In which,

\[ u_c = C_3 + C_4 e^{-\frac{y}{Pr}} \]  \(16\)

\[ u_p = V_1 u_1 + V_2 u_2 \]

In which,

\[ u_p = V_1 + V_2 e^{-\frac{y}{Pr}} \]  \(17\)

Where, \(u_1 = 1, u_2 = e^{-\frac{y}{Pr}}\)

By employing the variation of parameters methods, one can write Worskin of this solution as
\[ W(u_1,u_2) = -\frac{c}{Fr}e^{-\frac{c}{Fr}} \]

(18)

Let,

\[ R = -\frac{1}{Fr} \left( 1 - \theta_0 (1 - e^{-c}) - e^{-c} \right) \]

(19)

\[ V_1 = \int \frac{-Ru_1}{w(u_1,u_2)} dy \]

\[ V_2 = \int \frac{Ru_2}{w(u_1,u_2)} dy \]

(20)

Equation (17), can be written as,

\[ \theta_0 (1 - e^{-c}) + Pr (1 - e^{-c}) - (1 - e^{-c}) \]

The general solution of Equation (5) will be expected as,

\[ c + Cy + Pr (1 - Pr) + \theta_0 (1 - e^{-c}) - e^{-c} \]

(21)

(22)

Putting Equation (14), (20), into (21), one obtains

\[ \theta_0 (1 - e^{-c})y - e^{-c}y \]

Where,

\[ V_1 = \frac{1}{c(1 - e^{-c})} \left( \theta_0 (1 - e^{-c})y - e^{-c}y \right) \]

\[ V_2 = \frac{p c y}{c(1 - e^{-c})} \left( 1 - \theta_0 (1 - e^{-c}) - e^{-c} \right) \]

The two constant \( C_3, C_4 \) can be determined by prescribing the boundary conditions (7) for the effective velocity field, thus obtaining:

\[ C_3 = \frac{1}{F(1 - e^{-c})} \left( 1 + \frac{Pr}{Fr} \right) (c - Fr (1 - e^{-c})) \]

\[ C_4 = -\frac{1}{F(1 - e^{-c})} \left( 1 + \frac{Pr}{Fr} \right) (c - Fr (1 - e^{-c})) \]

(24)

Where,

\[ A = C^2, B = 1 - e^{-c}, D = \frac{c}{Fr}, E = 1 - Pr, F = AB \]

Therefore, the general solution of Equation (5), can be written as,

\[ u_{eff}(y) = C \left[ H (e^{0.5Pr} - e^{0}) - e^{0.5Pr} \right] + K (1 - e^{0.5Pr}) \]

(25)

\[ m(y) = \rho \int Q(y)dy \]

(30)

By integrating Equation (28), one obtain the mass transfer rate as,

\[ m(y) = \frac{\rho A_{Dp\cdot Cd} e}{(Fr)^2(1 - e^{-c})} \left( 1 + \frac{Pr}{Fr} \right) (e^{0.5Pr} - e^{0}) \left( 1 + \frac{Pr}{Fr} \right) \]

(32)
6. ASYMPTOTIC BEHAVIOR AND DISCUSSION OF THE RESULTS

In this section the main features of the solutions found in the previous section (5) will be discussed. This is done in order to be able to assess the effect of changes of parameters to the overall flow distributions, while keeping other operating conditions and parameters fixed. And ascertain the best one for optimal natural ventilation.

In Figure 4 and 5, plots of the dimensionless effective velocity field $u_{\text{eff}}(y)$ versus $\theta_0$ are reported for $0 \leq y \leq 2$. As $\theta_0$ increase, the velocity across the openings increases, and as $Pr$ increase, the velocity across the openings increases. Therefore, It is found that the best value for optimal natural ventilation is at $\theta_0 = 0.5$ and $Pr = 0.712$.

Figure 6, 7 and 8, plots of the dimensionless volumetric airflow rate $Q(y)$ versus $\theta_0$, $Pr$ and $c_d$ are reported for $0 \leq y \leq 2$. As $\theta_0$ increase, the volumetric airflow rate decreases, and as $Pr$ increase, the volumetric airflow rate decreases, and as $c_d$ increase, the volumetric airflow rate also increases. Therefore, It is found that the best value for optimal natural ventilation is at $\theta_0 = 0.1$, $Pr = 0.710$ and $c_d = 0.75$. 

![Effect of changes of effective thermal coefficient to effective velocity distribution](image1)

![Effect of changes of Prandtl number to volumetric airflow rate](image2)

![Effect of changes of effective thermal coefficient to volumetric airflow rate](image3)

![Effect of changes of Prandtl number to effective velocity distribution](image4)

![Effect of changes of effective thermal coefficient to effective velocity distribution](image5)
Figure 9, 10 and 11, plots of the dimensionless mass-transfer rate $m(y)$ versus $c_d$, $Pr$ and $\theta_0$ are reported for $0 \leq y \leq 2$. As $c_d$ increase, the mass-transfer rate also increases, as $Pr$ increase, the mass-transfer rate decreases, and as $\theta_0$ increase, the mass-transfer rate decreases. Therefore, It is found that the best value for optimal natural ventilation is at $c_d = 0.75$, $Pr = 0.710$ and $\theta_0 = 0.1$.

Figure 12 and 13, plots of the dimensionless effective temperature field $\theta_{eff}(y)$ versus $\theta_0$ and $Pr$ are reported for $0 \leq y \leq 2$. As $\theta_0$ increase, the temperature distribution across the openings is decreases, and as $Pr$ increase, the temperature distribution across the openings is decreases. It is found that the best value for optimal natural ventilation is at $\theta_0 = 0.1$, and $Pr = 0.710$. therefore, its sh
7. CONCLUSION
A study of airflow process across vertical vents induced by stack-driven effect with an opposing flow in one of the openings was presented. Some parameters and factors such as, area, height, size, and position of the openings are also described, which were believed to have significant effects on natural ventilation process in buildings. Fluid and thermodynamical models were developed; analytical techniques were employed to obtain the possible solutions of; (1) Effective velocity distribution (2) Effective temperature distribution and (3) volumetric airflow, and mass-transfer rates. Various parameters on air flow process were used to see the effect of changes of parameters to the overall flow distributions, and ascertain the best one for optimal natural ventilation. Therefore, expected objectives in the paper are achieved.

The paper lead to the following conclusions:
The orientation of the domain has a strong effect to the airflow around it.
The temperature distributions and induced natural airflow rate depend highly on area of the openings.
The temperature distribution increases due to the decrease in effective thermal coefficient. Therefore, the temperature in the domain was within comfortable conditions for the lower value of effective thermal coefficient.
The rate of flux that is coming in is moving in a higher rate so that at time goes on there will be equilibrium of temperature between ambient and the interior. At the flow level $Q_{\text{min}} = 0$, while, at the neutral height level $Q_{\text{max}} = 0.0073$, $m_{\text{max}} = 0.0119$.
The greater vertical distance between the openings, and the greater temperature difference between the inside and the outside, the stronger is the effect of the buoyancy.

Our models are only valid for cross-ventilated domains with openings at the same height.
To date there are few validated models for natural ventilation process on the basis of a theoretical approach. Lastly, we believe that our findings will help in developing a better understanding of natural ventilation process and help Mathematicians to gain more insights into the phenomenon and therefore come up with better models.

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9. BIBLIOGRAPHY
9.1 Nomenclature
$C_1, C_2, C_3, C_4$ Coefficients;
$A_{\text{OP}}$ Total area of the openings;
$C_d$ Discharge coefficient;
g Acceleration due to gravity;
$P$ Pressure;
$u_{\text{eff}}$ Effective velocity distribution;
$u_{\text{eff}}^*$ Dimensionless effective velocity distribution;
m Mass – transfer rate;
$Q$ Volumetric airflow rate;
$W$ Wroskian;
$u_c$ Complimentary solution;
$u_p$ Particular solution;
$V_1, V_2, u_1, u_2$ Functions on $y$;
$v_0$ Velocity of the opposing flow in one of the upper opening;
$v_0^*$ Dimensionless velocity of the opposing flow in one of the upper opening;
$Y_{m_d}$ Neutral height;
9.1.1 Greek Symbols
\( \theta_{\text{eff}} \)  Effective temperature distribution;
\( \theta^*_{\text{eff}} \)  Dimensionless effective temperature distribution;
\( \theta_{w1}, \theta_{w2} \)  Temperature on wall1 and wall2;
\( \alpha \)  Thermal diffusitivity;
\( \alpha_{\text{eff}} \)  Effective thermal diffusitivity;
\( \beta \)  Thermal conductivity;
\( v \)  Kinematic viscosity;
\( u_{\text{eff}} \)  Effective kinematic viscosity;
\( \theta_a, \theta_r \)  Ambient, and room temperatures;
\( \Delta \theta \)  Temperature difference;
\( \theta_0 \)  Effective thermal coefficient;
\( \rho \)  Density of the air.

9.1.2 Subscripts
\( \text{eff} \)  Effective;
\( w1, w2 \)  Wall1 and wall2;
\( a, r \)  Ambient, room;
\( m_d \)  Middle of the openings;
\( OP \)  Opening;

9.1.3 Superscripts
\( T \)  Total;

10. REFERENCES
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