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Adaptive Neural Network Control for a Class of MIMO Uncertain Pure-Feedback Nonlinear Systems

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ABSTRACT

In this paper, robust adaptive neural network control is investigated for a class of multi-input-multi-output (MIMO) pure-feedback nonlinear system with unknown nonlinearities. The unknown nonlinearities could be come from unmodeled dynamics, modeling errors, or nonlinear time-varying uncertainties. Based on the backstepping design technique and the universal approximation property of the neural network (NN), robust adaptive control is synthesized by employing a single NN to approximate the lumped uncertain nonlinearities. The proposed control can eliminate the circularity problem completely, and guarantees semiglobal uniform ultimate boundedness (SGUUB) of all the signals in the closed-loop and convergence of the tracking error to an arbitrarily small residual set.

General Terms

Algorithms, Nonlinear Control Theory

Keywords

Adaptive control, neural network control, multi-input/multi-output (MIMO) nonlinear systems, backstepping

1. INTRODUCTION

Controlling nonaffine nonlinear systems becomes an important and challenging topic within the control systems community because, in practice, many engineering plants including continuous stirred-tank reactor systems [1], vibrating systems [2], active magnetic bearing systems [3] etc, are difficult to be exactly described in affine forms even though the modeling errors are neglected. On the other hand, due to the nonaffine nonlinearity, controller design for affine systems cannot be simply extended into nonaffine systems. Remarkable results in this area have been obtained, including adaptive neural network (NN) control [4, 5], adaptive fuzzy control [6], and backstepping control by incorporating the adaptive NNs control method[7].

For the problem of controlling these systems, one of the main difficulties comes from the nonaffine nonlinearity which is an implicit function of the control input u. The reason is that it is quite difficult to make controller design directly for the system even if the nonlinearity is known. To deal with this difficulty, adaptive NN control methods were proposed in [8, 9] via utilizing implicit function theory to achieve the control objectives. However, the control methods were limited by the lack of the rigorous theorem analysis due to the difficulties in the theorem analysis. By combining the Implicit Function Theorem and the Mean Value Theorem, which are not usually associated with NN control theory, rigorous stability proof of the closed-loop systems was first presented for the adaptive NN control in [1]. Using the idea of feedback linearization techniques, approximated linearizing feedback NN control methods were proposed for a class of nonaffine nonlinear systems in [10]. In the method, by adding and subtracting a pseudo-control signal into the plant model, adaptive NN control design is realized via designing the pseudo-control signal which is required to be invertible with respect to u. When there is no information about the systems, [6] solved the problem by adding and subtracting gu directly instead of the pseudo-control signal as said in [11], where g is a positive constant. In this paper, robust adaptive neural network control is presented for a class of MIMO pure-feedback nonlinear system with unknown nonlinearities by employing a single NN to approximate the lumped uncertain nonlinearities. The proposed control can eliminate the circularity problem completely, and guarantees SGUUB of all the signals in the closed-loop and convergence of the tracking error to an arbitrarily small residual set.

The rest of the paper is organized as follows. In what follows, some notations are introduced which will be used throughout the paper. Section II describes the problem formulation and introduces the approximation property of the Gaussian radial basis function (RBF) network as preliminary. Section III presents robust adaptive control design, and its main results are shown in Section IV. Section V illustrates the effectiveness of the proposed control problem through an example; Section VI concludes the paper.

Throughout the paper, the following notations are used.

—R denotes the field of real numbers, R^+ denotes the field of positive real numbers, R^n denotes the field of the real n-vectors.

—$|\cdot|$ denotes the absolute value of a scalar, $\|\cdot\|$ denotes the norm of a vector/matrix.

—$A \triangleq B$ means that B is defined as A.

—j, i_j, m_j and n are integer indices, and i_j, m_j are presented the i_jth and m_jth components of the corresponding items in the jth subsystem, respectively.

—$\hat{W}_j$ denote the estimates of neural weights W_j, and $\tilde{W}_j \triangleq \hat{W}_j - W_j$ denote the errors between $\hat{W}_j$ and W_j.
—$\bar{y}_{d_j}^{(i_j)} = [y_{d_j}^{(0)}, y_{d_j}^{(1)}, \cdots, y_{d_j}^{(i_j)}]^T \in R^{i_j+1}$ with $y_{d_j}^{(0)} \triangleq y_{d_j}$.

2. PROBLEM STATEMENT AND PRELIMINARIES

2.1 Problem Statement

Consider the following MIMO nonlinear system with each subsystem in the perturbed pure-feedback form

$$S_j : \begin{cases} \dot{x}_{j,i_j} = f_{j,i_j}(\bar{x}_{j,i_j}, x_{j,i_j+1}) + \Delta_{j,i_j}(X,t) \\ \dot{x}_{j,m_j} = f_{j,m_j}(X, \bar{u}_j) + \Delta_{j,m_j}(X, \bar{u}_j, t) \\ y_j = x_{j,1}, i_j = 1, 2, \cdots, m_j - 1, j = 1, 2, \cdots, n \end{cases} \quad (1)$$

with
x_{j,i_j} $\in R$, the i_jth state variable of S_j;
$\bar{x}_{j,i_j}$ $[x_{j,1}, x_{j,2}, \cdots, x_{j,i_j}]^T \in R^{i_j}$;
X $[\bar{x}_{1,m_1}^T, \bar{x}_{2,m_2}^T, \cdots, \bar{x}_{n,m_n}^T] \in R^{\Sigma_{i=1}^n m_i}$;
u_j $\in R$, the input of S_j;
$\bar{u}_j$ $[u_1, u_2, \cdots, u_j]^T \in R^j$, the vector of input;
y_j $\in R$, the output of S_j;
f_{j,i_j} the unknown smooth nonlinear functions.
Unlike most recent results, assume that f_{j,i_j} and f_{j,m_j} are unknown smooth implicit functions with respect to x_{j,i_j+1} and u_j, respectively; and $\Delta_{j,i_j}, j = 1, 2, \cdots, n, i_j = 1, 2, \cdots, m_j$, are unknown uncertainties [12], which could be due to many factors including measurement noise, modeling errors, external disturbances, or changes due to time variations, etc.

The control objective is to design a robust adaptive control for system S_j such that, given a desired trajectory $y_{d_j}(t) \in R$, it and its derivatives $y_{d_j}^{(k)}(t), k = 1, 2, \ldots, m_j$, being bounded, the output y_j tracks the desired trajectory y_{d_j} and all the signals in the closed-loop system remain semiglobally uniformly ultimately bounded (SGUUB).

ASSUMPTION 1. *There exist positive functions $\psi_{j,i_j}(\bar{x}_{j,i_j+1}), i_j = 1, 2, \ldots, m_j - 1$, and $\psi_{j,m_j}(X, \bar{u}_j)$, such that*

$$|\Delta_{j,i_j}(X,t)| \le \psi_{j,i_j}(\bar{x}_{j,i_j+1}), \forall (X,t) \in R^{n_m} \times R^+,$$

$$|\Delta_{j,m_j}(X, \bar{u}_j, t)| \le \psi_{j,m_j}(X, \bar{u}_j), \forall (X, \bar{u}_j, t) \in R^{n_m} \times R^j \times R^+,$$

where the functions $\psi_{j,i_j}(\bar{x}_{j,i_j+1})$ and $\psi_{j,m_j}(X, \bar{u}_j)$ might be unknown implicit functions with respect to x_{j,i_j+1} and u_j, respectively.

Define functions $h_{j,i_j}(\cdot), g_{j,i_j}(\cdot), i_j = 1, 2, \ldots, m_j$ as

$$h_{j,i_j}(\cdot) = f_{j,i_j}(\cdot) + \psi_{j,i_j}(\cdot), i_j = 1, 2, \ldots, m_j, \quad (2)$$

$$g_{j,i_j}(\bar{x}_{j,i_j+1}) = \frac{\partial h_{j,i_j}(\bar{x}_{j,i_j+1})}{\partial x_{j,i_j+1}}, i_j = 1, 2, \ldots, m_j - 1, \quad (3)$$

$$g_{j,m_j}(X, \bar{u}_j) = \frac{\partial h_{j,m_j}(X, \bar{u}_j)}{\partial u_j}. \quad (4)$$

ASSUMPTION 2. *The signs of $g_{j,i_j}(\cdot)$ are known, and there exist constants $0 < \underline{g}_{j,i_j} \le \bar{g}_{j,i_j} < \infty, i_j = 1, 2, \ldots, m_j$, such that*

$$\underline{g}_{j,i_j} \le |g_{j,i_j}(\cdot)| \le \bar{g}_{j,i_j}, \forall \bar{x}_{j,i_j+1} \in R^{i_j+1}, i_j = 1, 2, \ldots, m_j - 1,$$

$$\underline{g}_{j,m_j} \le |g_{j,m_j}(\cdot)| \le \bar{g}_{j,m_j}, \forall (X, \bar{u}_j) \in R^{n_m} \times R^j.$$

REMARK 1. *Compared with the assumptions made for systems in [12, 13, 14, 15], $|\Delta_i(\bar{x}_n, t)| \le p_i^* \phi_i(\bar{x}_i)$, where p_i^* is unknown positive constant and $\phi_i(\bar{x}_i)$ are known nonnegative smooth function, Assumption 1 is in a general form since it is not needed to know the expressions of the uncertainties Δ_{j,i_j}. Assumption 2 is introduced to guarantee the controllability of the system (1). From Assumption 2, we know that $g_{j,i_j}(\cdot), i_j = 1, 2, \ldots, m_j$ are strictly either positive or negative definite. Without lose of generality, it is assumed that $\underline{g}_{j,i_j} \le g_{j,i_j}(\cdot) \le \bar{g}_{j,i_j}$.*

REMARK 2. *Compared with the MIMO system in [17], where the interconnection terms are only limited in the functions f_{j,m_j}, system S_j described by (1) possesses the full interconnection structure since the uncertainties Δ_{j,m_j} containing all the system states emerge in each subsystem, which is the extension of SISO nonlinear system in the perturbed strict-feedback form studied in [16]. In practice, many engineering plants fall into this category, such as continuous stirred-tank reactor systems [1], vibrating systems [2] and active magnetic bearing systems [3], etc.*

2.2 Preliminaries

In this paper, RBF network [18] is introduced to approximate the continuous lumped unknown nonlinearities due to its good capabilities in function approximation. Before introducing our control design method, let us firstly recall the approximation property of the Gaussian RBF network. The Gaussian RBF network takes the form $W^T S(z)$, where $z \in \Omega_z \subset R^q$ is the input vector, $W \in R^l$ is the weight vector, $l > 1$ is the number of NN nodes, $S(\cdot) \in R^l$ is a basis function vector, i.e., $S(z) = [s_1(z), s_2(z), \cdots, s_l(z)]^T$, with $s_i(z)$ being Gausian functions, which have the following form

$$s_i(z) = \exp\left[\frac{-(z-\mu_i)^T(z-\mu_i)}{\eta^2}\right], i = 1, 2, \cdots, l, \quad (5)$$

where $\mu_i = [\mu_{i1}, \mu_{i2}, \cdot, \mu_{iq}]^T$ is the center of the receptive field, η is the width of the Gaussian functions.

According to the approximation property of the RBF network [19, 20], given any continuous real-valued function $\alpha^* : \Omega_z \to R$, where $\Omega_z \subset R^q$, there exist ideal weights W^* such that α^* can be approximated by an ideal Gaussian RBF network $W^{*T} S(z)$, i.e.,

$$\alpha^*(z) = W^{*T} S(z) + \varepsilon(z), \forall z \in \Omega_z \quad (6)$$

where $\varepsilon(z)$ is the NN approximation error. For simplicity of presentation, denote $\varepsilon(z)$ as ε.

ASSUMPTION 3 [19, 20]. *On the compact set $\Omega_z \subset R^q$, There exists ideal constant weights W^* such that $|\varepsilon| \le \varepsilon^*$ and $\|W^*\| \le w$, where constants $\varepsilon^* > 0$ and $w > 0$.*

LEMMA 1 [21]. *On the compact set $\Omega_z \subset R^q$, there exists a positive constant c_s such that $\|S(z)\| \le c_s$.*

3. ROBUST CONTROL DESIGN

In this section, robust adaptive control is developed using the backstepping design technique. The design procedure consists of (j, m_j) steps. At step $(j, i_j)(i_j = 1, 2, \ldots, m_j - 1)$, we give a virtual robust feedback control law to stabilize the (j, i_j)th subsystem, and let the virtual control law enter into the next step of the design process. At step (j, m_j), robust adaptive control is synthesized with employment of a single NN which is used to approximate the lumped unknown nonlinearities.

Step $j,1$: Define the tracking error variable $z_{j,1}$ as

$$z_{j,1} = x_{j,1} - y_{d_j}. \quad (7)$$

Its derivative is

$$\dot{z}_{j,1} = f_{j,1}(\bar{x}_{j,1}, x_{j,2}) + \Delta_{j,1}(X,t) - \dot{y}_{d_j}, \quad (8)$$

where $x_{j,2}$ is taken as a virtual control input to stabilize the $(j,1)$th subsystem.

To design the virtual input $x_{j,2}$, construct a Lyapunov function candidate $V_{j,1}$ as

$$V_{j,1}(z_{j,1}) = \frac{1}{2}z_{j,1}^2. \quad (9)$$

Based on Assumption 1, the time derivative of $V_{j,1}$ along the trajectory of (8) is

$$\begin{aligned}\dot{V}_{j,1} &= z_{j,1}[f_{j,1}(\bar{x}_{j,1}, x_{j,2}) + \Delta_{j,1}(X,t) - \dot{y}_{d_j}] \\ &\leq z_{j,1}[h_{j,1}(\bar{x}_{j,1}, x_{j,2}) + \nu_{j,1}], \quad (10)\end{aligned}$$

where $h_{j,1}(\cdot)$ is defined in (2), and $\nu_{j,1} = -\dot{y}_{d_j}$.

Combining Assumption 2 and the fact that $\partial[\nu_{j,1} + \frac{3}{2}c_{j,1}z_{j,1}]/\partial x_{j,2} = 0$, where $c_{j,1}$ is a positive constant to be specified later, we have $\partial[h_{j,1}(\bar{x}_{j,1}, x_{j,2}) + \nu_{j,1} + \frac{3}{2}c_{j,1}z_{j,1}]/\partial x_{j,2} > \underline{g}_{j,1} > 0$. Using the implicit function theorem, there exists a smooth function $x_{j,2}^* = \alpha_{j,1}^*(\bar{x}_{j,1}, y_{d_j}, \dot{y}_{d_j})$ such that $h_{j,1}(\bar{x}_{j,1}, \alpha_{j,1}^*) + \nu_{j,1} = -\frac{3}{2}c_{j,1}z_{j,1}$. Using the mean value theorem, there exists a constant $\lambda_{j,1} \in (0,1)$ such that

$$h_{j,1}(\bar{x}_{j,1}, x_{j,2}) = h_{j,1}(\bar{x}_{j,1}, \alpha_{j,1}^*) + g_{\lambda_{j,1}}(x_{j,2} - \alpha_{j,1}^*), \quad (11)$$

where $g_{\lambda_{j,1}} = g_{j,1}(\bar{x}_{j,1}, x_{\lambda_{j,1}}), x_{\lambda_{j,1}} = \lambda_{j,1}x_{j,2} + (1-\lambda_{j,1})\alpha_{j,1}^*$. From (10) and (11), we have

$$\begin{aligned}\dot{V}_{j,1} &\leq z_{j,1}[h_{j,1}(\bar{x}_{j,1}, x_{j,2}) + \nu_{j,1}] \\ &= z_{j,1}[-\frac{3}{2}c_{j,1}z_{j,1} + h_{j,1}(\bar{x}_{j,1}, x_{j,2}) - h_{j,1}(\bar{x}_{j,1}, \alpha_{j,1}^*)] \\ &= z_{j,1}[-\frac{3}{2}c_{j,1}z_{j,1} + g_{\lambda_{j,1}}(x_{j,2} - \alpha_{j,1}^*)]. \quad (12)\end{aligned}$$

Define a new variable $z_{j,2}$ as

$$z_{j,2} = x_{j,2} - \alpha_{j,1}^*. \quad (13)$$

Combining Assumption 2 and the following inequality

$$g_{\lambda_{j,1}} z_{j,1} z_{j,2} \leq \frac{c_{j,1}}{2}z_{j,1}^2 + \frac{\bar{g}_{\lambda_{j,1}}^2}{2c_{j,1}}z_{j,2}^2, \quad (14)$$

Then, (12) becomes

$$\dot{V}_{j,1} \leq -c_{j,1}z_{j,1}^2 + \frac{\bar{g}_{\lambda_{j,1}}^2}{2c_{j,1}}z_{j,2}^2. \quad (15)$$

Step $j, i_j (2 \leq i_j \leq m_j - 1)$: Define a new variable z_{j,i_j} as

$$z_{j,i_j} = x_{j,i_j} - \alpha_{j,i_j-1}^*, \quad (16)$$

where α_{j,i_j-1}^* is a smooth function of $\bar{x}_{j,i_j-1}$ and $\bar{y}_{d_j}^{(i_j-1)}$, i.e., $\alpha_{j,i_j-1}^* = \alpha_{j,i_j-1}^*(\bar{x}_{j,i_j-1}, \bar{y}_{d_j}^{(i_j-1)})$. The derivative of z_{j,i_j} is

$$\dot{z}_{j,i_j} = f_{j,i_j}(\bar{x}_{j,i_j+1}) + \Delta_{j,i_j}(X,t) - \dot{\alpha}_{j,i_j-1}^*, 2 \leq i_j \leq m_j - 1, \quad (17)$$

where x_{j,i_j+1} is taken as a virtual control input to stabilize the (j, i_j)th subsystem.

Construct a Lyapunov function candidate V_{j,i_j} as

$$V_{j,i_j} = \frac{1}{2}z_{j,i_j}^2. \quad (18)$$

Using Assumption 1, the time derivative of V_{i_j} along the trajectory of (17) is

$$\begin{aligned}\dot{V}_{j,i_j} &= z_{j,i_j}[f_{j,i_j}(\bar{x}_{j,i_j+1}) + \Delta_{j,i_j}(X,t) - \dot{\alpha}_{j,i_j-1}^*] \\ &\leq z_{j,i_j}[h_{j,i_j}(\bar{x}_{j,i_j+1}) + \nu_{j,i_j}], \quad (19)\end{aligned}$$

where $\nu_{j,i_j} = \Sigma_{k=1}^{i_j-1}|\partial\alpha_{j,i_j-1}^*/\partial x_{j,k}|h_{j,k}(\bar{x}_{j,k+1}) + \Sigma_{k=0}^{i_j-1}|\partial\alpha_{j,i_j-1}^*/\partial y_{d_j}^{(k)}|y_{d_j}^{(k+1)}$.

From (16) and (19), clearly that z_{j,i_j} and ν_{j,i_j} are not functions of x_{j,i_j+1}, thus we have $\partial[\nu_{j,i_j} + c_{j,i_j}z_{j,i_j}]/\partial x_{j,i_j+1} = 0$, where constant $c_{j,i_j} = c_{j,1} + (\bar{g}_{\lambda_{j,i_j-1}}^2/2c_{j,1})$. Combining Assumption 2, we obtain that $\partial[h_{j,i_j}(\bar{x}_{j,i_j+1}) + \nu_{j,i_j} + c_{j,i_j}z_{j,i_j}]/\partial x_{j,i_j+1} > \underline{g}_{j,i_j} > 0, \forall(\bar{x}_{j,i_j+1}) \in R^{i_j+1}$. Using the implicit function theorem, there exists a smooth function $x_{j,i_j+1}^* = \alpha_{j,i_j}^*(\bar{x}_{j,i_j}, \bar{y}_{d_j}^{(i_j)})$ such that $h_{j,i_j}(\bar{x}_{j,i_j}, \alpha_{j,i_j}^*) + \nu_{j,i_j} = -c_{j,i_j}z_{j,i_j}$. By employing the mean value theorem, there exists a constant $\lambda_{j,i_j} \in (0,1)$ such that

$$h_{j,i_j}(\bar{x}_{j,i_j+1}) = h_{j,i_j}(\bar{x}_{j,i_j}, \alpha_{j,i_j}^*) + g_{\lambda_{j,i_j}}(x_{j,i_j+1} - \alpha_{j,i_j}^*), \quad (20)$$

where $g_{\lambda_{j,i_j}} = g_{j,i_j}(\bar{x}_{j,i_j}, x_{\lambda_{j,i_j}}), x_{\lambda_{j,i_j}} = \lambda_{j,i_j}x_{j,i_j+1} + (1 - \lambda_{j,i_j})\alpha_{j,i_j}^*$.

Combining Assumption 2 and the inequality

$$g_{\lambda_{j,i_j}} z_{j,i_j} z_{j,i_j+1} \leq \frac{c_{j,1}}{2}z_{j,i_j}^2 + \frac{\bar{g}_{\lambda_{j,i_j}}^2}{2c_{j,1}}z_{j,i_j+1}^2, \quad (21)$$

then, (19) becomes

$$\begin{aligned}\dot{V}_{j,i_j} &\leq z_{j,i_j}[-c_{j,i_j}z_{j,i_j} + h_{j,i_j}(\bar{x}_{j,i_j}, x_{j,i_j+1}) \quad (22) \\ &- h_{j,i_j}(\bar{x}_{j,i_j}, \alpha_{j,i_j}^*)] \\ &= z_{j,i_j}[-c_{j,i_j}z_{j,i_j} + g_{\lambda_{j,i_j}}(x_{j,i_j+1} - \alpha_{j,i_j}^*)] \\ &\leq -\left(\frac{c_{j,1}}{2} + \frac{\bar{g}_{\lambda_{j,i_j-1}}^2}{2c_{j,1}}\right)z_{j,i_j}^2 + \frac{\bar{g}_{\lambda_{j,i_j}}^2}{2c_{j,1}}z_{j,i_j+1}^2. \quad (23)\end{aligned}$$

where $z_{j,i_j+1} = x_{j,i_j+1} - \alpha_{j,i_j}^*$.

Step j, m_j: Define a new variable z_{j,m_j} as

$$z_{j,m_j} = x_{j,m_j} - \alpha_{j,m_j-1}^*, \quad (24)$$

where $\alpha_{j,m_j-1}^* = \alpha_{j,m_j-1}^*(\bar{x}_{j,m_j-1}, \bar{y}_{d_j}^{(m_j-1)})$. The derivative of z_{j,m_j} is

$$\dot{z}_{j,m_j} = f_{j,m_j}(X, \bar{u}_j) + \Delta_{j,m_j}(X, \bar{u}_j, t) - \dot{\alpha}_{j,m_j-1}^*. \quad (25)$$

Taking the following Lyapunov function candidate V_{j,m_j} as

$$V_{j,m_j} = \frac{1}{2}z_{j,m_j}^2. \quad (26)$$

Using Assumption 1, the time derivative of V_{j,m_j} along the trajectory of (25) is

$$\begin{aligned}\dot{V}_{j,m_j} &= z_{j,m_j}[f_{j,m_j}(X,\bar{u}_j) + \Delta_{j,m_j}(X,\bar{u}_j,t) - \dot{\alpha}^*_{j,m_j-1}] \\ &\leq z_{j,m_j}[h_{j,m_j}(X,\bar{u}_j) + \nu_{j,m_j}]. \quad (27)\end{aligned}$$

where $\nu_{j,m_j} = \Sigma_{k=1}^{j,m_j-1}|\partial\alpha^*_{j,m_j-1}/\partial x_{j,k}|h_{j,k}(\bar{x}_{j,k+1}) + \Sigma_{k=0}^{j,m_j-1}|\partial\alpha^*_{j,m_j-1}/\partial y_{d_j}^{(k)}|y_{d_j}^{(k+1)}$.

Similarly, since z_{j,m_j} and ν_{j,m_j} are not functions of u_j, we have $\partial[\nu_{j,m_j} + c_{j,m_j}z_{j,m_j}]/\partial u_j = 0$, where constant $c_{j,m_j} = \frac{1}{2}c_{j,1} + (\bar{g}^2_{\lambda_{j,m_j-1}}/c_{j,1}) + [1 + (2c^2_{sj}/\sigma_j)]\bar{g}^2_{\lambda_{j,m_j-1}}$, with positive constants c_{sj} and σ_j to be specified later. Combining Assumption 2, we obtain the result that $\partial[h_n(X,\bar{u}_j) + \nu_{j,m_j} + c_{j,m_j}z_{j,m_j}]/\partial u_j > \underline{g}_{j,m_j} > 0$. Thereby, there exists a smooth function $u^*_j = \alpha^*_{j,m_j}(X,\nu_{j,m_j})$ such that $h_{j,m_j}(X,\alpha^*_{j,m_j}) + \nu_{j,m_j} = -c_{j,m_j}z_{j,m_j}$. By employing the mean value theorem, there exists a constant $\lambda_{j,m_j} \in (0,1)$ such that

$$h_{j,m_j}(X,\bar{u}_j) = f_{j,m_j}(X,\alpha^*_{j,m_j}) + g_{\lambda_{j,m_j}}(u_j - \alpha^*_{j,m_j}), \quad (28)$$

where $g_{\lambda_{j,m_j}} = g_{j,m_j}(X,x_{\lambda_{j,m_j}}), x_{\lambda_{j,m_j}} = \lambda_{j,m_j}u_j + (1-\lambda_{j,m_j})\alpha^*_{j,m_j}$.

From (27) and (28), we have

$$\begin{aligned}\dot{V}_{j,m_j} &\leq z_{j,m_j}[-c_{j,m_j}z_{j,m_j} + h_{j,m_j}(X,\bar{u}_j) - h_{j,m_j}(X,\alpha^*_{j,m_j})] \\ &= -c_{j,m_j}z^2_{j,m_j} + g_{\lambda_{j,m_j}}z_{j,m_j}(u_j - \alpha^*_{j,m_j}). \quad (29)\end{aligned}$$

In the ideal case, that all nonlinearities of system (1) are known exactly, a possible controller take the form

$$u_j = \alpha^*_{j,m_j}. \quad (30)$$

Substituting (30) into (29) and Taking the Lyapunov function candidate $V_{\bar{z}_j}$ as

$$V_{\bar{z}_j} = \sum_{k=1}^{m_j} V_{j,k}, \quad (31)$$

the time derivative of $V_{\bar{z}_j}$ is

$$\begin{aligned}\dot{V}_{\bar{z}_j} &= -\frac{c_{j,1}}{2}\sum_{k=1}^{m_j} z^2_{j,k} - \frac{c_{j,1}}{2}z^2_{j,1} \\ &- [\frac{\bar{g}^2_{\lambda_{j,m_j-1}}}{c_{j,1}} + (1 + \frac{2c^2_{sj}}{\sigma_j})\bar{g}^2_{\lambda_{j,m_j-1}}]z^2_{j,m_j}. \quad (32)\end{aligned}$$

Therefore, $V_{\bar{z}_j}$ are Lyapunov functions and $\bar{z}_{j,m_j} \to 0$ as $t \to \infty$.

4. ROBUST ADAPTIVE CONTROLLER AND STABILITY ANALYSIS

In the case that no exact knowledge for the system nonlinearities is available, a Gaussian RBF neural network $W_j^T S_j(X,\bar{u}_j)$ can be applied for the approximations of α^*_{j,m_j}, i.e.,

$$\alpha^*_{j,m_j} = W_j^{*T} S_j(X,\bar{u}_{j-1}) + \varepsilon_j, \quad (33)$$

where W^*_j denotes the ideal constant weights, and $|\varepsilon_j| \leq \varepsilon^*_j$ is the approximation error with constant $\varepsilon^*_j > 0$.

Choosing the practical control law u_j as

$$u_j = \hat{W}_j^T S_j(X,\bar{u}_{j-1}), \quad (34)$$

where $\hat{W}_j$ are the estimates to W^*_j, and are updated the NN weights driven by

$$\dot{\hat{W}}_j = \Gamma_j[z_{j,1}S_j(X,\bar{u}_{j-1}) - \sigma_j\hat{W}_j], \quad (35)$$

where $\Gamma_j = \Gamma_j^T > 0$, and $\sigma_j > 0$ are positive constant design parameters, the introduction of which can be improved the controller robustness in the presence of the NN approximation errors.

THEOREM 2. *Given the nonaffine nonlinear system (1) satisfying Assumption 1- 2, controller (34), and the adaptive law (35), then for any bounded initial conditions,*

1) *all the signals in the closed-loop system remain bounded, and the states X and the neural weight estimates $\hat{W}_j^T$ eventually converge to the compact set*

$$\Omega_s \triangleq \{X,\hat{W}_j | V_j < \frac{\varrho_j}{c_{j,1}}\} \quad (36)$$

2) *The output tracking error $y_j - y_{d_j}$ converges to a small neighborhood around zero by appropriately choosing design parameters.*

PROOF. 1) Consider the Lyapunov function candidate of the closed-loop system

$$V_j = V_{\bar{z}_j} + \tilde{W}_j^T\Gamma_j^{-1}\tilde{W}_j. \quad (37)$$

Its derivatives is

$$\begin{aligned}\dot{V}_j &\leq -\frac{1}{2}c_{j,1}\sum_{k=1}^{m_j-1} z^2_{j,k} - \frac{c_{j,1}}{2}z^2_{j,1} - c_{j,m_j}z^2_{j,m_j} \\ &+ g_{\lambda_{j,m_j}}z_{j,m_j}[\tilde{W}_j^T S_j(X,\bar{u}_{j-1}) - \varepsilon_j] \\ &+ \tilde{W}_j^T[z_{j,1}S_j(X,\bar{u}_{j-1}) - \sigma_j\hat{W}_j], \quad (38)\end{aligned}$$

where $c_{j,m_j} = \frac{1}{2}c_{j,1} + (\bar{g}^2_{\lambda_{j,m_j-1}}/c_{j,1}) + (1 + \frac{2c^2_s}{\sigma_j})\bar{g}^2_{\lambda_{j,m_j-1}}$.

Using the inequalities that

$$\begin{cases} g_{\lambda_{j,m_j}}z_{j,m_j}\tilde{W}_j^T S_j(X,\bar{u}_{j-1}) \leq \frac{2c^2_s}{\sigma_j}\bar{g}^2_{\lambda_{j,m_j}}z^2_{j,m_j} + \frac{\sigma_j}{8}\tilde{W}_j^T\tilde{W}_j, \\ z_{j,1}\tilde{W}_j^T S_j(X,\bar{u}_{j-1}) \leq \frac{2c^2_s}{\sigma_j}z^2_{j,1} + \frac{\sigma_j}{8}\tilde{W}_j^T\tilde{W}_j, \\ g_{\lambda_{j,m_j}}z_{j,m_j}\varepsilon_j \leq \bar{g}^2_{\lambda_{j,m_j}}z^2_{j,m_j} + \frac{1}{4}\varepsilon^{*2}_j, \\ -\tilde{W}_j^T\sigma_j\hat{W}_j \leq -\frac{\sigma_j}{2}\tilde{W}_j^T\tilde{W}_j + \frac{\sigma_j}{2}W_j^T W_j, \end{cases} \quad (39)$$

we have

$$\begin{aligned}\dot{V}_{j,m_j} &\leq -\frac{1}{2}c_{j,1}\sum_{k=1}^{m_j-1} z^2_{j,k} - [\frac{1}{2}c_{j,1} - \frac{2c^2_s}{\sigma_j}]z^2_{j,1} - \frac{\sigma_j}{4}\tilde{W}_j^T\tilde{W}_j \\ &+ \frac{\sigma_j}{2}W_j^T W_j + \frac{1}{4}\varepsilon^{*2}_j. \quad (40)\end{aligned}$$

Let $\varrho_j = \frac{\sigma_j}{2}W_j^T W_j + \frac{1}{4}\varepsilon^{*2}_j$, and choose

$$\begin{aligned} c_{j,1} &\geq \frac{4c^2_s}{\sigma_j} \quad (41) \\ \sigma_j &\geq 2c_s\sqrt{2\lambda_{max}(\Gamma^{-1})}. \quad (42)\end{aligned}$$

we have

$$\dot{V}_{j,m_j} \leq -c_{j,1}V_{j,m_j} + \varrho_j. \quad (43)$$

Thereby, we obtain the result that

$$0 \leq V_{j,m_j} \leq \rho_j + [V_{j,m_j}(0) - \rho_j]e^{-c_{j,1}t}, \forall t \geq 0. \tag{44}$$

where $\rho_j = \varrho_j / c_{j,1} > 0$. The inequality (44) means that V_{j,m_j} eventually is bounded by ρ_j. Using (34), we conclude that control u_j is also bounded. Thus, all signals of the closed-loop system remain bounded.

2) From (44), we have

$$\begin{aligned} \sum_{k=1}^{m_j} z_{j,k} &< \rho_j + [V_{j,m_j}(0) - \rho_j]e^{-c_{j,1}t} \\ &< \rho_j + V_{j,m_j}(0)e^{-c_{j,1}t} \end{aligned} \tag{45}$$

which implies that given $\epsilon_j > \sqrt{\rho_j}$, there exists T such that for all $t \geq T$, the tracking error satisfies

$$|z_{j,1}| = |y_{j,1} - yd_j| < \epsilon_j \tag{46}$$

where ϵ_j is the size of a small residual set which depends on the NN approximation error ε_j and the controller parameters $c_{j,1}, \sigma_j$ and Γ_j. By increasing the values of $c_{j,1}$ and reducing the value of $\lambda_{max}(\Gamma^{-1})$, the quantity ρ_j can be made arbitrary small. Thus, the tracking error $z_{j,1}$ can be made arbitrary small. This concludes the proof. $\square$

5. CONCLUSION

The main contribution of this paper has presented robust adaptive control for a general class of MIMO pure-feedback nonlinear system by employing a single NN to approximate the lumped uncertain nonlinearities. The proposed control can eliminate the circularity problem completely, and guarantees SGUUB of all the signals in the closed-loop and convergence of the tracking error to an arbitrarily small residual set. In the future, investigation on a general class of nonaffine nonlinear systems will be interesting research topics in this field.

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