

SER Analysis of OFDM System over Rayleigh Fading Channel

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ABSTRACT

In this paper, we present an efficient technique for the SER of OFDM system over Rayleigh fading channels, using well known characteristics function (CHF) based analysis approach. The exact probability density function of a sum of Rayleigh random vectors are used to derive the expression for the error rates of OFDM system. The average SER expressed in terms of the exponential and Bessel functions. Further to improve the system performance error-rate analysis is extended to a system using multichannel reception with maximal ratio combining.

Keywords

OFDM, Digital Communication, Rayleigh Fading channel, MRC diversity.

1. INTRODUCTION

ORTHOGONAL frequency-division multiplex (OFDM) is an effective technique for mitigating the effects of delay spread in frequency selective fading channels [1]. The basic principle of OFDM is to split the high data stream into number of lower rate data streams, which are transmitted simultaneously over a number of sub-carriers. Using the overlapping multi-carrier modulation technique almost 50% bandwidth can be saved [2]. In an OFDM system, IFFT and FFT processing are used in the modulation and demodulation of the signals. One important attribute of this scheme is the spreading of a fade over many bits. Rather than having a few adjacent bits completely destroyed, we now have all the bits only slightly affected by a fade [3].

Many error-rate performance studies of the OFDM systems can be found in the literature [1]–[7]. The complexity of the error-rate analysis depends on the employed digital-modulation method and the used channel-fading model. By far, the most common data-modulation method employed for the OFDM is either M-phase shift keying (PSK) or M-quadratic amplitude modulation [8]. A great number of distributions exist that well describe the statistics of the mobile radio signal [9]–[10]. Among them, the Rayleigh distribution is the basic model to characterize the fading channels.

The Bit Error Rate (BER) performance of OFDM systems communicating over Rayleigh fading channels has been extensively studied. In [11], used the pilot symbol along with the previously known channel coefficients for fast Rayleigh faded channels. Estimation of OFDM system in Rayleigh faded channel is provided by many techniques, in [12], Timing phase estimator for OFDM system in Rayleigh faded environment is proposed with low complexity. In [13] an accurate analytical solution for the bit error rate (BER) of a DS-CDMA system

operating over Rayleigh fading channels has been derived. In [14] diversity at the receiver is a well-known promising avenue for improving mean signal strength and reducing signal level fluctuations in fading channels, where the multiple received copies can be combined intelligently to provide a higher average received signal-to-noise ratio (SNR). In [1] The combination of OFDM and diversity has been becoming popular in wireless communications. In [9] and [15], Proakis derives the average symbol-error probability (ASER) of MPSK with maximal-ratio combining (MRC) in Rayleigh fading. In [16] presents finite-range integral expressions for MPSK and MDPSK in Rician fading.

In this paper we present the SER analysis of QPSK-OFDM systems over Rayleigh fading channel. Our Technical contributions are summarized below. We first calculate the pdf of sum of Rayleigh random phase vectors in an integral form by using CHF method. Then we use this pdf to evaluate the error rates performance for QPSK modulated OFDM system in frequency selective Rayleigh fading channels for one tap and two-tap. We express the average SER in terms of exponential function and Bessel functions. Further to improve the system performance we use MRC diversity technique.

The rest of the paper organized as follows. The section 2 concerns with the system model. In section 3 we derive integral expression for the pdf of a sum of the random phase vectors. In section 4 SER expressions for QPSK-OFDM system over frequency selective fading channels are derived. Error-rate analysis is extended to a system using multichannel reception with maximal ratio combining in 5. The section 6 discusses about the numerical results and finally section 7 concludes the work.

2. SYSTEM MODEL

Consider an OFDM system with N sub-carriers. Let $X(k)$ is the k^{th} OFDM data block to be transmitted with N subcarriers. These data are used to modulate N orthogonal sub carriers. Then IDFT is used to modulate the input signal. After modulation signal can be represented as [17]:

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) \exp(j2\pi kn/N) \quad n=0, 1, \dots, N-1 \quad (1)$$

According to the nature of the interaction between the wave and obstacle the signal can be reflected diffracted or diffused. This phenomenon referred to as multipath propagation. The channel impulse response of a multipath fading channel is modeled as $|h(n)|e^{-j\theta(n)}$. In our analysis we assume that frequency synchronization are achieved at the receiver side $r(n)$ can be represented in frequency domain as:

$$R(k) = X(k) * H(k) + N \quad (2)$$

Where N is an independent identically distributed (i.i.d) complex Gaussian noise component with zero mean and unit variance. The amplitude of $H(k)$ is modeled as a Rayleigh RV with PDF.

3. JOINT CHF & PDF FOR RAYLEIGH RVS

In this section, we derive an integral expression for the pdf of the amplitude of Rayleigh random phase vectors.

Consider a circularly symmetric random variable is of the form $Z = X_n + jY_n$, where real and imaginary parts are zero mean independent and identically distributed (iid). The magnitude is $|z|$ and the phase ϕ is uniformly distributed over $[0, 2\pi]$.

The joint CHF of the RVs X_n and Y_n is defined as [19]:

$$\Psi_{X_n Y_n}(\eta_1, \eta_2) = E_{X_n Y_n} [\exp(j\eta_1 X + j\eta_2 Y)] \quad (3)$$

With the polar-coordinate substitution and after some simplification eq. (3) can be written as [22, eq. (27)]:

$$\Psi_{X_n Y_n}(\eta_1, \eta_2) = \int_0^\infty f(r) \cdot \left[\frac{1}{2\pi} \int_0^{2\pi} \exp(j\eta r \cos(\phi)) d\phi \right] dr \quad (4)$$

Where $\frac{1}{2\pi}$, the pdf of ϕ . The uniform distribution of ϕ is $[0, 2\pi]$. Using the integral form of the zeroth-order Bessel function [21] i.e. $J_0(z) = \frac{1}{2\pi} \int_0^{2\pi} \exp(jz \cos(\phi)) d\phi$. so finally equation (4) reduces to the following form:

$$\Psi_{X_n Y_n}(\eta) = \int_0^\infty f(r) J_0(\eta r) dr \quad (5)$$

This equation is used to derive the expressions for the joint CHF for Rayleigh distributions.

The Rayleigh joint distribution is given by

$$f(r, \theta) = \frac{r}{\pi\Omega} \exp\left(-\frac{r^2}{\Omega}\right), r \geq 0 \quad (6)$$

The r and ϕ independent random variables, the individual probability density functions are given by:

$$f(r) = \frac{2}{\Omega} r \exp\left(-\frac{r^2}{\Omega}\right) \quad r \geq 0 \quad (7)$$

$$f(\phi) = \frac{1}{2\pi} \quad -\pi \leq \phi \leq \pi \quad (8)$$

To calculate the joint CHF of X and Y we have from eq. (5):

$$\Psi_{X_n Y_n}(\eta) = \int_0^\infty f(r) J_0(\eta r) dr \quad (9)$$

$$\Psi_{X_n Y_n}(\eta) = \int_0^\infty \frac{2}{\Omega} r \exp\left(-\frac{r^2}{\Omega}\right) J_0(\eta r) dr \quad (10)$$

Now using integral identity [20, eq. (6.631.4)] and further some simplification we have:

$$\Psi_{X_n Y_n}(\eta) = \exp\left(-\frac{\Omega\eta^2}{4}\right) \triangleq \Psi_{XY}(\eta) \quad (11)$$

Where $J_0(\cdot)$ is zeroth order Bessel function of first kind [21]. Because the channel tap coefficients are independent, the joint CHF becomes [7]:

$$\Psi_{X_n Y_n}(\eta) = \prod_0^{M-1} \exp\left(-\frac{\Omega\eta^2}{4}\right) \triangleq \Psi_{XY}(\eta) \quad (12)$$

The pdf of the amplitude is given by [7, eq. 11] as:

$$f_{XY}(r) = \int_0^\infty \Psi_{XY}(\eta) J_0(\eta r) \eta d\eta \quad (13)$$

$$f_{XY}(r) = \int_0^\infty \prod_0^{M-1} \exp\left(-\frac{\Omega\eta^2}{4}\right) J_0(\eta r) \eta d\eta \quad (14)$$

the integral representation of the pdf in (14) is exact, which is further used to evaluate the error rate.

4. SER PERFORMANCE ANALYSIS USING SINGLE-CHANNEL RECEPTION

The conditional SER of a particular modulation is given by $Q(Sr)$, where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(-\frac{t^2}{2}\right) dt$. Using the alternative representation of above equation as given in [25].

$$Q(Sr) = \frac{1}{\pi} \int_0^{\pi/2} \exp\left(-\frac{S^2 x^2}{2\sin^2(\phi)}\right) d\phi \quad (15)$$

The symbol error rate for QPSK is given by [23]:

$$= \text{erfc}(\sqrt{\gamma}) - .25 \text{erfc}^2(\sqrt{\gamma}) \quad (16)$$

Using the relationship between Q -function and erfc function [24 pp.649, eq. (F.12)], we have

$$= 2Q(Sr) - Q^2(Sr) \quad (17)$$

If the SNR is high (as is necessary for practical QPSK systems) the probability of symbol error may be approximated as:

$$\approx 2Q(Sr) \quad (18)$$

Where S is a factor related to symbol to noise ratio (SNR). So for our system model the average error rate, denoted by $P(S)$, is given by:

$$P(S) = \int_0^\infty 2Q(Sr) f_{XY}(r) dr \quad (19)$$

$$P(S) = \int_0^\infty \left[\frac{2}{\pi} \int_0^{\pi/2} \exp\left(-\frac{S^2 r^2}{2\sin^2(\phi)}\right) d\phi \right] f_{XY}(r) dr \quad (20)$$

$$P(S) = \int_0^\infty \left[\frac{2}{\pi} \int_0^{\pi/2} \exp\left(-\frac{S^2 r^2}{2\sin^2(\phi)}\right) d\phi \right] \times \left[r \int_0^\infty \Psi_{XY}(\eta) J_0(\eta r) \eta d\eta \right] dr \quad (21)$$

To simplify the equation exchange the order of integration, so equation (21) can be rewritten as:

$$P(S) = \frac{1}{\pi} \int_0^\infty \Psi_{XY}(\eta) \eta d\eta \times \int_0^{\pi/2} \left(\int_0^\infty r J_0(\eta r) \exp\left(-\frac{S^2 r^2}{2\sin^2(\phi)}\right) dr \right) d\phi \quad (22)$$

In eq. (22) solve inner integral by using an integral identity [20, eq. (6.631.4)], so eq. (19) can be expressed as:

$$\int_0^{\pi/2} \left(\int_0^\infty r \exp\left(-\frac{S^2 r^2}{2\sin^2(\phi)}\right) J_0(\eta r) dr \right) d\phi = \left(\int_0^{\pi/2} \frac{\sin^2(\phi)}{S^2} \exp\left(-\frac{.5\eta^2}{S^2} \sin^2(\phi)\right) d\phi \right) \quad (23)$$

$$= \frac{\pi}{4S^2} \exp\left(-\frac{.25\eta^2}{S^2}\right) (I_0\left(\frac{.25\eta^2}{S^2}\right) - I_1\left(\frac{.25\eta^2}{S^2}\right)) \quad (24)$$

Where $I_0(\cdot)$ and $I_1(\cdot)$ is modified bessels function of zeroth order and first order respectively [21]. Substituting eq. (24) into eq. (22), we can get final SER expression for the Rayleigh fading distribution.

$$P(S) = \int_0^\infty \frac{1}{2S^2} \prod_0^{M-1} \exp\left(-\frac{\Omega\eta^2}{4}\right) \exp\left(-\frac{.25\eta^2}{S^2}\right) (I_0\left(\frac{.25\eta^2}{S^2}\right) - I_1\left(\frac{.25\eta^2}{S^2}\right)) \eta d\eta \quad (25)$$

Above eq. (25) can be useful for numerical computation of SER for OFDM signals transmitted over Rayleigh channels.

5. SER ANALYSIS OF OFDM SYSTEM WITH MRC DIVERSITY

The conditional error rate expression for maximal ratio combining (MRC) is $Q(S\sqrt{\sum_{j=0}^{M-1} r_j})$ and given by [24]

$$Q(S\sqrt{\sum_{j=0}^{M-1} r_j}) = \frac{1}{\pi} \int_0^{\pi/2} \exp\left(-\frac{S^2 \sum_{j=0}^{M-1} r_j^2}{2\sin^2(\phi)}\right) d\phi \quad (26)$$

Where r_j is the fading amplitude in the j th branch and S is a factor related to signal to noise ratio (SNR)

Averaging (26) with respect to the joint PDF of the fading amplitudes, we obtain the average error rate, denoted by $P_{MRC}(S)$, as derived in [7] for the BPSK modulated signal transmitted over Nakagami- m fading channel. Here we utilize the same expression for the QPSK signal transmitted over Rayleigh fading channel.

Using [7, eq. 39] we have error rate for QPSK-OFDM system as:

$$P_{MRC}(S) = \frac{1}{\pi} \int_0^{\pi/2} d\phi \left[\prod_{j=0}^{M-1} \int_0^\infty \exp\left(-\frac{S^2 r_j^2}{2\sin^2(\phi)}\right) r_j \Psi_{XY}^j(\eta) J_0(\eta r_j) \eta d\eta dr_j \right] \times \quad (27)$$

$$P_{MRC}(S) = \frac{1}{\pi} \int_0^{\pi/2} \prod_{j=0}^{M-1} M_j(\phi) d\phi$$

Where

$$M_j(\phi) = \int_0^\infty \frac{(\sin^2(\phi))}{S^2} \exp\left(-\frac{\eta \sin^2(\phi)}{2S^2}\right) \Psi_{XY}^j(\eta) d\eta$$

is the moment generating function of r_j .

6. NUMERICAL RESULTS

In this section the error rate performance of an OFDM system over Rayleigh fading channel is analytically evaluated. Fig.1 illustrates the average BER performance versus the SNR in a QPSK modulated OFDM system in the presence of frequency – selective Rayleigh fading channel with single tap system.

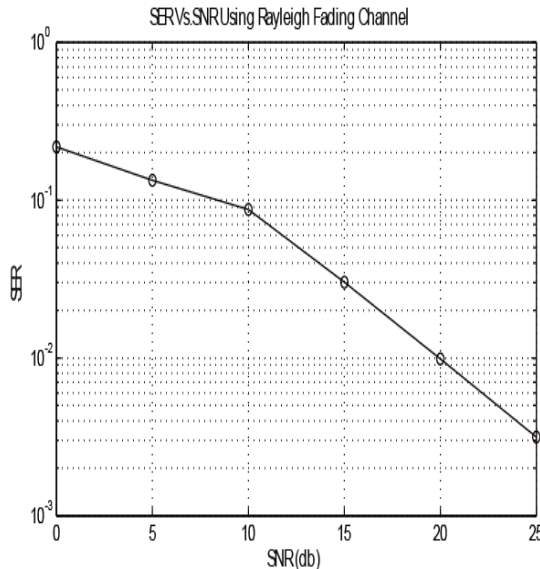


Fig.1 SER Vs SNR for OFDM system with single tap

Fig. 2 shows the error rate curves for QPSK-OFDM system over a two-tap Rayleigh fading channel with and without MRC diversity. We observe in Fig. 2 that with dual branch MRC diversity, the slopes of the error rate curves become larger and significant gains are obtained. For example, in the two-tap frequency selective Rayleigh fading channel at SNR=10 the error rate is 15.4×10^{-3} , and with dual branch MRC reception for the same SNR we have error rate 1.44×10^{-4} .

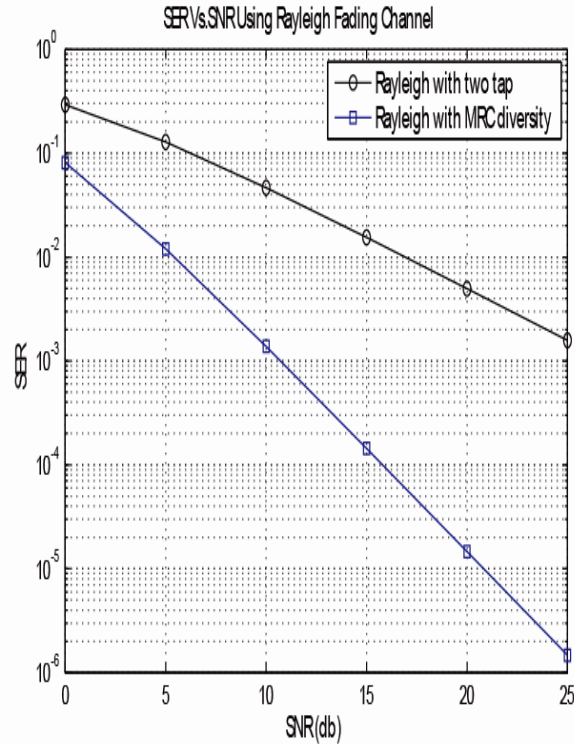


Fig.2 SER Vs SNR for OFDM system with two tap

7. CONCLUSION

In this paper we have investigated the error probability of OFDM systems using QPSK demodulation over frequency selective Rayleigh fading channels. Based on the joint distribution of two RVs, an explicit analytical expression for both the pdf and the SER has been derived. Basically in this paper we have outlined a simple CHF approach for deriving closed form error rates for signals in Rayleigh fading channels. Finally it has been concluded that, by using diversity technique we can improve the system performance.

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