Fixed Point Theorems for (ψ, ϕ)-Contractive maps in Weak non-Archimedean Fuzzy Metric Spaces and Application

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ABSTRACT
The present study introduce the notion of (ψ, ϕ)-Contractive maps in weak non-Archimedean fuzzy metric spaces to derive a common fixed point theorem which complements and extends the main theorems of [C.Vetro, Fixed points in weak non-Archimedean fuzzy metric spaces, Fuzzy Sets and System, 162(2011), 84-90] and [D.Mihet, Fuzzy ψ-contractive mappings in non-Archimedean fuzzy metric spaces, Fuzzy Sets and System, 159(2008) 739-744]. We support our result by establishing an application to product spaces.

Keywords

Common fixed points; Non-Archimedean fuzzy metric space; (ψ, ϕ)-contractive maps.

MSC: 47H10, 54H25

1. INTRODUCTION
The concept of fuzzy metric space was introduced by different authors (see, for example, [12]) in different ways and further, using these different concepts, various authors ([9, 10, 12]) proved theorems which assure the existence of a fixed point. Here, we use the notion of fuzzy metric space established by George and Veeramani. Recently, Mihet [13] and Vetro [19] introduced the concept of ψ-contractive maps in non-Archimedean and weak non-Archimedean fuzzy metric spaces respectively and proved a fuzzy version of the Banach contraction principlein these settings.

The main reason of our interest in fuzzy metric spaces is their application in engineering problems, in information systems and in quantum particle physics, particularly in concern with both string and E-infinity theory which were given and studied by El-Naschie [3-7]. Recently, fuzzy metrics have been applied to color image filtering, improving some filters when replacing some classical metrics [14-16].

Also, fixed point theorems play a central role also in the proof of existence of general equilibrium in market economics as developed in the 1950’s by noble prize winners in economics Gerard Debrew and Kenneth Arrow. In fact, an equilibrium price is a fixed point in a stable market.

In the last two decades, many researchers explored the existence of weaker contractive conditions or extended previous results under relatively weak hypotheses on metric spaces. The starting point of our paper is to follow this trend by introducing the definition of (ψ, ϕ)-contractive maps in weak non-Archimedean fuzzy metric spaces. Then, we utilize this new concept to investigate the existence of a common fixed point for a pair of maps satisfying generalized contractive conditions. Our result extends and complements the main results of Mihet [13] and Vetro [19]. Our result also fuzzifies some results in the literature (see [1, 2, 17, 20] and the references therein). We will show an application of our theorem in product spaces.

2. PRELIMINARIES
In what follows, we collect some relevant definitions, results, examples for our further use.

Definition 1.1 A fuzzy set A in X is a function with domain X and values in [0, 1].

Definition 1.2 A continuous t-norm ([18]) is a binary operation T on [0,1] satisfying the following conditions:

(i) T is commutative and associative;

(ii) $T(a, 1) = a$ for all $a \in [0,1]$;

(iii) $T(a, b) \leq T(c, d)$ whenever $a \leq c$ and $b \leq d$ ($a, b, c, d \in [0,1]$);

(iv) T is continuous.

Remark 1.1 The following are classical examples of continuous t-norm

(i) $T_M(a, b) = \min\{a, b\}$,

Minimum t-norm;

(ii) $T_H(a,b) = \begin{cases} 0 & \text{if } a = b = 0, \\ \frac{ab}{a+b+ab} & \text{otherwise,} \end{cases}$

Hamacher product;

(iii) $T_P(a, b) = ab$,

Product t-norm;

(iv) $T_N(a, b) = \begin{cases} \min\{a, b\} & \text{if } a + b > 1, \\ 0 & \text{otherwise,} \end{cases}$

Nilpotent minimum;

(v) $T_L(a, b) = \max\{a + b - 1, 0\}$,

Lukasiewicz t-norm;

(vi) $T_D(a, b) = \begin{cases} b & \text{if } a = 1, \\ a & \text{if } b = 1, \\ 0 & \text{otherwise,} \end{cases}$

Drastic t-norm.

The minimum t-norm is the pointwise largest t-norm and the drastic t-norm is the pointwise smallest t-norm, that is, $T_M(a, b) \geq T(a, b) \geq T_D(a, b)$ for any t-norm t with $a, b \in [0,1]$.

Definition 1.3 A fuzzy metric space ([12]) is a triple (X, M, $\ast$), where X is a nonempty set, $\ast$ is a continuous t-norm and M is a fuzzy set on $X^2\times$ [0, +∞) such that the following properties hold:

(KM-1) M(x, y, 0) = 0, for all x, y∈ X;

(KM-2) M(x, y, t) = 1,for all t > 0 iff x = y;

(KM-3) M(x, y, t) = M(y, x, t), for all x, y∈ X, t > 0;

(KM-4) M(x, y, ·) : [0, +∞) → [0, 1]is left continuous, for all x, y∈ X;

(KM-5) M(x, z, t + s) ≥ M(x, y, t) $\ast$ M(y, z, s),for all x, y, z∈ X and s, t > 0.

We will refer to these spaces as KM-fuzzy metric spaces.

In the above definition, if the triangular inequality (KM-5) is replaced by the following:

(NA) M(x, z, max{t, s}) ≥ M(x, y, t) $\ast$ M(y, z, s), for all x, y, z∈ X and s, t > 0,

then the triple (X, M, $\ast$) is called a non-Archimedean fuzzy metric space. It is easy to check that the triangle inequality (NA) implies (KM-5), that is, every non-Archimedean fuzzy metric space is itself a fuzzy metric space.

If, in the definition above, M is a fuzzy set on $X^2\times$ (0,+∞) and (KM-1), (KM-2),(KM-4) are replaced, respectively with (GV-1), (GV-2), (GV-4) below, then (X, M, $\ast$) is called a fuzzy metric space in the sense of George and Veeramani[8].

(GV-1) M(x, y, t) > 0, for all x, y∈ X;

(GV-2) M(x, x, t) = 1 for all t > 0 and if M(x, y, t) = 1 for some t > 0, then x = y;

(GV-4) M(x, y, ·): (0, +∞) → (0, 1]is continuous, for all x, y∈ X;

Example 1.1([19]) Let X = [0, +∞), ∗ be such that a∗ b ≤ ab for every a, b∈ [0, 1] and d be the usual metric. Also define

$M(x, y, t) = e^{-\frac{d(x,y)}{t}}$, then(X, M, $\ast$) is a non-Archimedean fuzzy metric space. Clearly, (X, M, $\ast$) is also a fuzzy metric space.

In Definition 1.3, if the triangular inequality

(KM-5) is replaced by the following:

(WNA) M(x, z, t) ≥ max{M(x, y, t) ∗ M(y, z, t/2), M(x, y, t/2) ∗ M(y, z, t)}

for all x, y, z ∈ X and t > 0, then the triple is called a weak non-Archimedean fuzzy metric space. Obviously every non-Archimedean fuzzy metric space is itself a weak non-Archimedean fuzzy metric space.

Remark 1.1([19]) Condition (WNA) does not imply that M(x, y, ·) is non-decreasing and thus a weak non-Archimedean fuzzy metric space is not necessarily a fuzzy metric space. If M(x, y, ·) is non-decreasing, then a weak non-Archimedean fuzzy metric space is a fuzzy metric space.

Example 1.2 ([19]) Let X = [0, +∞), ∗be such that a$\ast$b = ab for every a, b∈ [0, 1]. Also define M(x, y, t) by: M(x, y, 0)=0,M(x, x, t) = 1 for all t > 0, M(x, y, t) = t for x ≠ y and 0 < t ≤ 1, M(x, y, t)=t/2 for x ≠ y and 1 < t ≤ 2, M(x, y, t) = 1 for x ≠ y and t > 2.Then (X, M, $\ast$) is a weak non-Archimedean fuzzy metric space, but it is not a fuzzy metric space.

In [19], Vetro introduced the topology induced by a weak non-Archimedean fuzzy metric space and proved the following propositions.

Proposition 1.1 Let (X, M, $\ast$) be a weak non-Archimedean fuzzy metric space, then every open ball is an open set.

Proposition 1.2 Every weak non-Archi-medean fuzzy metric space (X, M, $\ast$) is Hausdorff.

Proposition 1.3 Let (X, M, $\ast$) be a weak non-Archimedean fuzzy metric space. A sequence $\{x_n\}$ in a weak non-Archimedean fuzzy metric space (X, M, $\ast$) is convergent to x∈ X if and only if $\lim_{n\to+\infty} M(x_n, x, t) = 1$, for all t > 0.

Definition 1.4 ([19]) Let (X, M,$\ast$) be a weak non-Archimedean fuzzy metric space. A sequence $\{x_n\}$ in X is called a Cauchy sequence, if for each ε ∈ (0, 1) and t > 0 there exists n(ε) ∈ ℕ such that $M(x_n, x_m, t) > 1- \varepsilon$ for all m, n ≥ n(ε).

In [9] Grabiec called the sequence G-Cauchy if $\lim_{n\to+\infty} M(x_n, x_{n+m}, t) = 1$ for each m ∈ ℕ and t > 0. A weak non-Archimedean fuzzy metric space (X, M, $\ast$) is called complete (G-complete) if every Cauchy (G-Cauchy) sequence is convergent.

In [10], Gregori and Sapena gave the following definition.

Definition 1.5 Let (X, M, $\ast$) be a fuzzy metric space in the sense of George and Veeramani. A map f: X →X is called fuzzy contractive if

$$\frac{1}{M(fx,fy,t)} - 1 \le k\left(\frac{1}{M(x,y,t)} - 1\right)$$

for each x, y∈ X and t > 0, where k is fixed in (0, 1). In this case, k is called the contractive constant of f.

3. MAIN RESULT

In this section, we prove a fixed point theorem for (ψ, ϕ)-contractive maps in weak non-Archimedean fuzzy metric spaces.

Let ψ: [0, 1] →[0, 1] and ϕ: [0, 1] → (0, 1] be such that

(1) ψ is continuous monotone non-decreasing with ψ(t) > t for all t ∈ (0, 1) and ψ(1) = 1.

(2) ϕ is lower semi-continuous with ϕ(t) = 1 if and only if t = 1.

Definition 2.1 Let X be a nonempty set, M be a fuzzy set on X^2 × [0, +∞) and f, g: X → X. We say that (f, g) is a pair of (ψ, ϕ)-contractive maps if there exist two functions ψ and ϕ, defined as above, such that for every t > 0, x, y ∈ X, with x ≠ y and M(x, y, t) > 0, the following condition holds:

(i) ψ(M(fx, gy, t)) ≥

ψ(m(x, y, t)) + ϕ(m(x, y, t)),

where

(ii) m(x, y, t) = min{M(x, y, t),

M(fx, x, t), M(gy, y, t)}.

Fix $x_0 \in X$ and define the sequence $\{x_n\}$ by

$x_1 = fx_0, x_2 = gx_1, \ldots, x_{2n+1} = fx_{2n}, x_{2n+2} = gx_{2n+1}, \ldots$.

We call $\{x_n\}$, a (f, g)-sequence of initial point x_0.

Theorem 2.1 Let $(X, M, *)$ be a complete weak non-Archimedean fuzzy metric space and let f, g: $X \to X$. Assume that (f, g) is a pair of (ψ, ϕ)-contractive maps and that for all x, $y \in X$, with $x \neq y$, there exists $t > 0$ suchthat $0 < M(x, y, t) < 1$. If there exists $x_0 \in X$ such that $M(x_0, fx_0, t) > 0$ for all $t > 0$, then f and g have a unique common fixed point.

Proof: We prove the theorem in several steps.

Step-1 Let $x_0 \in X$ such that $M(x_0, fx_0, t) > 0$ for all $t > 0$.We showthat $M(x_n, x_{n-1}, t) \to 1$ as $n \to +\infty$.

Suppose that n is an even number. Substituting $x = x_n$ and $y = x_{n-1}$ in (1) and using the properties of the functions ψ and ϕ, we obtain

$$\psi(M(x_{n+1}, x_n, t)) = \psi(M(fx_n, gx_{n-1}, t))$$
$$\geq \psi(m(x_n, x_{n-1}, t)) + \phi(m(x_n, x_{n-1}, t)$$
$$\geq \psi(m(x_n, x_{n-1}, t)).$$

Using the monotone property of the function ψ, we get

(3) $$M(x_{n+1}, x_n, t) \geq m(x_n, x_{n-1}, t).$$

Now, from (2) we have

$$m(x_n, x_{n-1}, t) = \min\{M(x_n, x_{n-1}, t), M(fx_n, x_n, t), M(gx_{n-1}, x_{n-1}, t)\}$$
$$= \min\{M(x_n, x_{n-1}, t), M(x_{n+1}, x_n, t), M(x_n, x_{n-1}, t)\}.$$

If $M(x_{n+1}, x_n, t) < M(x_n, x_{n-1}, t)$, then $m(x_n, x_{n-1}, t) = M(x_{n+1}, x_n, t)$, it furthermore implies that

$$\psi(M(x_{n+1}, x_n, t)) \geq \psi(M(x_{n+1}, x_n, t)) + \phi(M(x_{n+1}, x_n, t)).$$

which is a contradiction.

So, we have

(4) $$M(x_{n+1}, x_n, t) \geq m(x_n, x_{n-1}, t)$$
$$= M(x_n, x_{n-1}, t) \geq M(x_0, fx_0, t) > 0.$$

Similarly, we can obtain the inequality (4) also in the case that n is an odd number. Therefore, the sequence $\{M(x_{n+1}, x_n, t)\}$is monotone non-decreasing and bounded and so

$$\lim_{n\to+\infty} M(x_{n+1}, x_n, t) = \lim_{n\to+\infty} m(x_n, x_{n-1}, t)$$
$$= r, \text{ where } 0 < r \leq 1.$$

We claim that $\lim_{n\to+\infty} M(x_{n+1}, x_n, t)) = r = 1$. Infact, if $0 < r < 1$ then, as ϕ is lower semi-continuous, from

$$\psi(M(x_{n+1}, x_n, t)) \geq \psi(m(x_n, x_{n-1}, t)) + \phi(m(x_n, x_{n-1}, t)),$$

for $n \to +\infty$, we get

$$\psi(r) \geq \psi(r) + \phi(r),$$

which is a contradiction since $\phi(r) > 0$. Hence,

(5) $$\lim_{n\to+\infty} M(x_{n+1}, x_n, t)) = 1.$$

Step-2 Next we prove that the (f, g)-sequence $\{x_n\}$ of initial point x_0 is a Cauchy sequence. For this it is sufficient to show that the subsequence $\{x_{2n}\}$ is a Cauchy sequence. Suppose that $\{x_{2n}\}$ is not a Cauchy sequence. Then there exists $\varepsilon > 0$ for which we can find subsequences $\{x_{2m(k)}\}$ and $\{x_{2n(k)}\}$ such that n(k) is the smallest index for which $n(k) > m(k) > k$,

$$M(x_{2m(k)}, x_{2n(k)}, t) \leq 1 - \varepsilon.$$

This implies that

$$M(x_{2m(k)}, x_{2n(k)-2}, t) > 1 - \varepsilon.$$

From (2) and (WNA) we have

$$1 - \varepsilon \geq M(x_{2m(k)}, x_{2n(k)}, t)$$
$$\geq M(x_{2m(k)}, x_{2n(k)-2}, t) * M(x_{2n(k)-2}, x_{2n(k)-1}, \tfrac{t}{2}) * M(x_{2n(k)-1}, x_{2n(k)}, \tfrac{t}{4})$$
$$> (1-\varepsilon) * M(x_{2n(k)-2}, x_{2n(k)-1}, \tfrac{t}{2}) * M(x_{2n(k)-1}, x_{2n(k)}, \tfrac{t}{4}).$$

Letting $k \to +\infty$and using (5) we conclude that

(6) $$\lim_{k\to+\infty} M(x_{2m(k)}, x_{2n(k)}, t) = 1 - \varepsilon.$$

Moreover, from

$$M(x_{2m(k)}, x_{2n(k)+1}, t) \geq M(x_{2m(k)}, x_{2n(k)}, t) * M(x_{2n(k)}, x_{2n(k)+1}, \tfrac{t}{2})$$

and

$$M(x_{2m(k)}, x_{2n(k)}, t) \geq M(x_{2m(k)}, x_{2n(k)+1}, t) * M(x_{2n(k)+1}, x_{2n(k)}, \tfrac{t}{2}),$$

letting $k \to +\infty$, we obtain

$$\liminf_{k\to+\infty} M(x_{2m(k)}, x_{2n(k)+1}, t) \geq 1 - \varepsilon$$

and

$$1 - \varepsilon \geq \limsup_{k\to+\infty} M(x_{2m(k)}, x_{2n(k)+1}, t).$$

It implies that

(7) $$\lim_{k\to+\infty} M(x_{2m(k)}, x_{2n(k)+1}, t) = 1 - \varepsilon.$$

Analogously, one can show that

(8) $$\lim_{k\to+\infty} M(x_{2m(k)-1}, x_{2n(k)+1}, t)$$
$$= \lim_{k\to\infty} M(x_{2n(k)}, x_{2m(k)-1}, t) = 1 - \varepsilon.$$

Now, putting $x = x_{2m(k)-1}$, $y = x_{2n(k)}$ in (1) we have

$$\psi(M(x_{2m(k)}, x_{2n(k)+1}, t))$$
$$= \psi(M(fx_{2mk-1}, gx_{2n(k)}, t))$$
$$\geq \psi(m(x_{2mk-1}, x_{2n(k)}, t))$$
$$+ \phi(m(x_{2mk-1}, x_{2n(k)}, t)).$$

Finally, as ϕ is lower semi-continuous, letting $k \to +\infty$ we get

$$\psi(1-\varepsilon) \geq \psi(1-\varepsilon) + \phi(1-\varepsilon),$$

which is a contradiction. Thus the (f, g)-sequence $\{x_{2n}\}$ is a Cauchy sequence and hence also the (f, g)-sequence $\{x_n\}$ is Cauchy. Since the weak non-Archimedean fuzzy metric space

X is complete, therefore there exists x such that $x_n \to x$ as $n \to +\infty$.

Step-3 Let us now prove that x is a common fixed point of f and g, i.e., x = fx = gx. If fx ≠ x, then there exists t > 0 suchthat 0 < M(x, fx, t) < 1. From (2), we have

$$m(x, x_{2n-1}, t) = \min\{M(x, x_{2n-1}, t), M(fx, x, t), M(gx_{2n-1}, x_{2n-1}, t)\}$$
$$= \min\{M(x, x_{2n-1}, t), M(fx, x, t), M(x_{2n}, x_{2n-1}, t)\}.$$

Letting the limit as $n \to +\infty$, we obtain

$$\lim_{n\to+\infty} m(x, x_{2n-1}, t) = M(fx, x, t).$$

Now, from

$$\psi(M(fx, x_{2n}, t)) = \psi(M(fx, gx_{2n-1}, t)) \geq \psi(m(x, x_{2n-1}, t)) + \phi(m(x, x_{2n-1}, t))$$

Taking the limit as $n \to +\infty$, we obtain

$$\psi(M(fx, x, t)) \geq \psi(M(fx, x, t)) + \phi(M(fx, x, t)).$$

which is a contradiction and therefore fx = x.

Analogously, we obtain that gx = x and thus x is a common fixed point of f and g.

Step-4 We prove the uniqueness of the common fixed point of f and g.

Assume that x, y ∈ X are two distinct common fixed points of f and g, then

$$\psi(M(x, y, t)) = \psi(M(fx, gy, t)) \geq \psi(m(x, y, t)) + \phi(m(x, y, t))$$

which is a contradiction as m(x, y, t) = M(x, y, t) and therefore x = y.

As consequence of Theorem 2.1, we state the following result.

Corollary 2.1 Let (X, M, ⋆) be a complete weak non-Archimedean fuzzy metric space and let f: X → X. Assume that (f, f) is a pair of (ψ, ϕ)-contractive maps and that for all x, y ∈ X, with x ≠ y, there exists t > 0 such that 0 < M(x, y, t) < 1. If there exists $x_0 \in X$ suchthat $M(x_0, fx_0, t) > 0$ for all t > 0, then f has a unique fixed point.

4. APPLICATION

In this section, we give an application of our main result to the product space X × X.

Theorem 3.1 Let (X, M, ⋆) be a complete weak non-Archimedean fuzzy metric spaces and let F, G: X × X→ X. Assume that (F(·,y), G(·,y)) is a pair of (ψ, ϕ)-contractive maps for each y ∈ X and (F(z(y),y), G(z(y),y)) is a pair of (ψ, ϕ)-contractive maps for each z: X → X. Suppose also that for each x, y, u, v ∈X suchthatF(x, y) ≠ G(u, v) and t > 0, the following condition holds:

(9) $\psi(M(F(x, y), G(u, v), t)) \geq \psi(m(x, u, t)) + \phi(m(x, u, t))$,

where

(10) $m(x, u, t) = \min\{M(x, u, t), M(F(x, y), x, t), M(G(u, v), u, t)\}$.

Then there exists exactly one point w in X, such that F(w, w) = w = G(w, w).

Proof: Fix y = v ∈ X. Let f, g : X → X be such that fx=F(x, y) and gu=G(u, y), for all x, u ∈ X. Then, condition (9) reduces to condition (1) and so, by Theorem 2.1, the pair (f, g) has a unique common fixedpoint z(y), thatis f(z(y)) = z(y) = g(z(y)). Now, we can applyTheorem 3.1 to the self-mappings F(z(y),y) and G(z(y),y) on X and so we deduce that there exists a unique point w such that F(z(w),w) = G(z(w),w) = z(w) = w. Thiscompletes the proof.

5. CONCLUSIONS

In this work we have dealt with a new class of contractive maps in fuzzy metric spaces. We considered these maps in a more general setting called weak non-Archimedean fuzzy metric space. Moreover, we proved that if the space is complete, everypair of (ψ, ϕ)-contractive mapshas a uniquecommon fixedpoint. The future scope is to weaken the contractive condition and the hypotheses on the space.

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