

Extraction of Depth Elevation Model (DEM) from High Resolution Satellite Imagery using Shape from Shading Approach

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ABSTRACT

Shading information in satellite imagery will give user an immediate appreciation for the surface topography. In this paper, an attempt has been made to reconstruct 3D shape or Depth elevation model (DEM) of satellite imagery using Shape from Shading approach. Three widely used shape from shading algorithms – Pentland's Linear approach, Lee and Rosenfeld's approach and Horn's approach were used to recover shape of the satellite imagery and the result were compared. Each of this approaches were modified to get the better result.

Keywords

Depth Elevation Mode (DEM), Shape from Shading (SFS), Geographical Information System(GIS), Pixel distance.

1. INTRODUCTION

Depth Elevation Model (DEM), an important source of information for geomorphology, is usually used to express a topographic surface in three dimensional and to imitate essential natural geography. Jaurequi (1984) defines a digital elevation model (DEM) as a statistical representation of continuous surface of the ground by a large number of selected points with known x, y, z co-ordinate fields [1]. Aronoff (1990) defines DEM as a set of elevation measurements for locations distributed over the land surface and it carries different names: digital elevation model (DEM), digital terrain model (DTM), digital terrain data (DTD) [1]. Meijerink et al. [2] differentiates a DEM from DTM, DTM is defined as spatial distribution of terrain attributes, a topographic map in digital format, that not only consist of the DEM but also slope, aspect, the types of land use, settlements, types of drainage patterns, and so on. In general, digital elevation model (DEM) is a digital file consisting of terrain elevation and a DTM consists of additional terrain information. DEM is a very effective tool for terrain analysis: many terrain attributes can be derived from DEM (such as slope, aspect, slope type, watershed and standard flow path) and these attribute can be displayed with both image and attribute database, with the help of Geographical Information System (GIS).

Deriving 3D terrain information from digital data is serving for multi-disciplinary sciences today. Other than time consuming and tedious constant field survey, digital photogrammetric tools allows for the accurate data collection from imagery/ Researchers have investigated

various methods of generating Depth Elevation Model (DEM) using digital remote sensing data. Traditional approach for generating surface information makes use of contour lines extracted from topographical sheets, which provide accurate terrain elevations but require careful scrutiny. Traditional approach to generate surface manually is costly, slow, tedious, and imprecise operation as it requires skilled artists with good insight into cartography. Automatic generated shaded maps become most important when the interpreter's time is limited, as in aviation, for users that are not trained cartographers, and for small scale maps, where contours degenerate into messy tangles of lines. One method similar to photogrammetry is to use two images at a time for the reconstruction of 3D stereo model in which height information can be extracted.

Shape of a 2D-imagery can be recovered using various techniques: shape from shading, shape from stereo, shape from motion, shape from texture etc. In this paper we have explained only shape from shading techniques. The shape from shading (SFS) problem is to compute three dimensional shape from the brightness value of any image of that surface. The recovered shape can be expressed [3] in form of

- Depth $Z(x,y)$: It is defined as relative distance of camera to surface points or relative height from xy-plane.
- Surface normal (n_x, n_y, n_z) : It is defined as the orientation of a vector perpendicular to the tangent plane on the object surface.
- Surface gradient (p,q) : it is defined as rate of change of in x-direction and y-direction.

$$p = \frac{\partial z}{\partial x} \quad (1)$$

$$q = \frac{\partial z}{\partial y} \quad (2)$$

- Surface slant(ϕ) and tilt(θ): It is related to surface normal as

$$n_x = l \sin \phi \cos \theta \quad (3)$$

$$n_y = l \sin \phi \sin \theta \quad (4)$$

$$n_z = l \cos \phi \quad (5)$$

Many works have been done to process the given the 2D image using many techniques. The Shape from Shading (SFS) Technique was first developed by B K P Horn in early 1970s, and since then many approaches have been emerged. SFS techniques can be divided into

four groups: minimization approach, propagation approach, local approach and linear approach. Minimization Approaches find the solution by minimizing an energy function. One of the earlier minimization approaches, which recovered the surface gradients, was by Ikeuchi and Horn [4]. Brooks and Horn [5] minimized the same energy function, in terms of the surface normal. Frankot and Chellappa [6] enforced integrability in Brooks and Horn's algorithm in order to recover integrable surfaces (surfaces for which $zxy = zyx$). Propagation Approaches propagate the shape information from a set of surface points (e.g. singular points) to the whole image. Horn's characteristic strip method [7], [8] is essentially a propagation method. A characteristic strip is a line in the image along which the surface depth and orientation can be computed if these quantities are known at the starting point of the line. Bichsel and Pentland [9] proposed a minimum downhill approach for SFS which converged in less than ten iterations. Local Approaches derive shape based on the assumption of the surface type. Pentland's local approach [10] recovered shape information from the intensity, and its first and second derivatives. He used the assumption that the surface is locally spherical at each point. Under the same spherical assumption, Lee and Rosenfeld [11] computed the slant and tilt of the surface in the light source coordinate system using the first derivative of the intensity. Linear Approaches compute the solution based on the linearization of the reflectance map. Pentland [12] used the linear approximation of the reflectance function in terms of the surface gradient, and applied a Fourier transform to the linear function to get a closed form solution for the depth at each point. Tsai and Shah [13] applied the discrete approximation of the gradient first, then employed the linear approximation of the reflectance function in terms of the depth directly. Their algorithm recovered the depth at each point using a Jacobi iterative scheme. Hind Taud et al. [14] developed a method to generate digital elevation models by means of the dilation of contour lines stored in a raster grid. Prima et al. [15] proposed a method to generate a digital elevation model (DEM) from contour lines. Luuk Spreeuwiers et al. [16] suggested a method for automatic construction of DEM using aerial video image sequences. In this paper, the approaches by three different contributors are reviewed - Horn, Alex P. Pentland and Lee & Rosenfeld. Horn used minimization approach by minimizing the integrability constraints. Pentland used linear approach which reduces the non-linear problem into a linear one through the linearization of the reflectance map. The idea is based on the assumption that the lower order components in the reflectance map dominate. Lee & Rosenfeld approximated the local surface regions by spherical patches. An attempt has been made to improvise each of the three approaches, and to model hybrid approach that reconstruct DEM efficiently.

2. PENTLAND'S LINEAR APPROACH

Linear approaches reduce the non-linear problem into a linear one through the linearization of the reflectance map. The idea is based on the assumption that the lower order components in the reflectance map dominate. It uses the assumptions about the reflectance function, rather than using assumptions about the surface shape. Pentland algorithm used the linear approximation of the reflectance map in p and q (p and q are the surface gradients: rate of change of depth in x and y directions). The reflectance function can be expressed as follows:

$$I(x, y) = R(p, q) = \frac{\cos \sigma_s + p \cos \tau_s \sin \sigma_s + q \sin \tau_s \sin \sigma_s}{\sqrt{1 + p^2 + q^2}} \quad (6)$$

where σ_s and τ_s are respectively the slant and tilt of the light source. By taking the Taylor series expansion of the reflectance function about $p = p_0$ and $q = q_0$, ignoring the higher order terms and considering the resultant for the Lambertian reflectance ($p_0 = q_0 = 0$), the above equation reduces to

$$I(x, y) = R(p, q) = \cos \sigma_s + p \cos \tau_s \sin \sigma_s + q \sin \tau_s \sin \sigma_s \quad (7)$$

When $\sigma > 45^\circ$ (i.e., the illuminant is more than 45 degrees from the viewer) this approximation to the Lambertian reflectance function is accurate to within 10% for $-0.2 < p, q < 0.2$, a range of typical mountainous terrain in aerial imagery. It is accurate over the range $-1 < p, q < 1$ when $\sigma > 75^\circ$. Results obtained using this method have shown that this approach is ideally suited for the recovery of detailed surface shape with relatively small region. Because there is no smoothness constraint it can be directly applied to the complex surface such as mountainous or hilly region. This algorithm gives a non-iterative, closed form solution using Fourier Transform. For mathematical simplicity, Pentland has assumed the orthographic projection onto the x - y plane, that the surface is not self-shadowing.

3. LEE & ROSENFELD'S LOCAL APPROACH

Local approaches derive the shape by assuming local surface type. They use the intensity derivative information and assume spherical surface. Lee and Rosenfeld approximated the local surface regions by spherical patches. The slant and tilt of the surface were first computed in the light source coordinate, then transformed back to the viewer coordinate. They proved that the tilt of the surface normal could be obtained from:

$$\tau = \arctan \frac{I_y \cos \tau_s - I_x \sin \tau_s}{I_x \cos \tau_s \cos \sigma_s + I_y \cos \sigma_s \sin \tau_s} \quad (8)$$

where, I_x and I_y are intensity derivatives along the x and y directions, σ_s is the slant of the light source, and τ_s is the tilt of the light source. If the surface has uniform reflectance, and if the reflectance map is given by $I = \vec{N} \cdot \vec{S}$ (Lambertian), then the brightest point has its surface normal pointing toward the light source, and the cosine value of surface slant can be obtained by the ratio of its intensity and ρ . This approach is an improvement of Pentland's first approach, since it involves only the first derivatives of the intensity rather than the second derivatives. This makes it less sensitive to noise. However, the local spherical assumption of the surface limits its application.

The major part in the implementation of Lee and Rosenfeld's algorithm is the rotation of the image from the viewer coordinates to the light source coordinates, and the computation of the intensity gradient in the light source coordinates. For this method, there are no parameters which needed to be determined.

4. HORN'S MINIMIZATION APPROACH

Horn combined the brightness constraint, the smoothness constraint and the unit normal constraint and minimized the following energy function:

$$\iint \left((I - R)^2 + \lambda (\|\vec{N}_x\|^2 + \|\vec{N}_y\|^2) + \mu (\|\vec{N}_x\|^2 - 1) \right) dx dy \quad (9)$$

where \vec{N} is the surface normal, λ is a scalar that weighs the relative importance of the smoothness term and μ is a Lagrangian multiplier.

The minimization of the above function was done through variational calculus. The function given in above equation has the Euler equation

$$(I - \bar{N} \cdot \vec{S}) + \lambda \nabla^2 \bar{N} - \mu \bar{N} = 0 \quad (10)$$

A discrete approximation to the Laplacian operator was used in order to change the Euler equation into the following discrete form

$$(I_{i,j} - \bar{N}_{i,j} \cdot \vec{S}) + \frac{4\lambda}{\epsilon^2} (\bar{N}_{i,j} - \bar{N}_{i,j}) - \mu_{i,j} \bar{N}_{i,j} = 0 \quad (11)$$

Here $\bar{N}_{i,j}$ is the average of the normals in the neighborhood around the point (i, j). Assuming known light source direction, \vec{S} , the iterative scheme for \bar{N} was developed by rearranging the above equation and taking only the direction of the vector by dropping the constant term. The iterative scheme for \vec{S} was derived by assuming \bar{N} is known, then setting the partial derivative of the energy function with respect to \vec{S} to zero and solving it. Here, ϵ is the distance between adjacent pixels in the image.

Horn's algorithms require that the shape at occluding boundaries (either the surface normal or the surface gradient) be known. However, information at occluding boundaries is difficult to obtain, especially for the surface gradient, since the surface gradient at the occluding boundary has at least one infinite component. Another disadvantage is the slow convergence of the algorithms.

5. WEIGHTED ENHANCEMENT AND PENTLAND'S APPROACH

The input image considered throughout the paper is a mountainous region which has gradual ups and downs with some peak points. But the shape reconstructed using Pentland's approach shows that it has cliffs (steep slope). This inaccuracy emerged out because of the assumption that Pentland has made. The assumption was that the surface should not be self-shadowing. It means the surfaces should be orthographic; the illuminant should be on the top of the surface. Closer look at the reconstructed DEM shows that there is steep fall in the height on the pixels where there is shadow of the nearby high cliffs, however which is not true. An attempt was made to enhance image globally which basically works on the enhancing the pixel values of the shaded region. Here, globally means taking the average intensity of the whole image and then enhancing the image with respect to that. The image enhancement algorithm used basically works on the intensity values of the image. It checks the range of the intensity values of the image, i.e., the maximum intensity pixel value and the minimum one. Calculate the average intensity, finds the shadowed region, enhance the intensity of the shadowed region globally (all over the image) and then applies the Pentland's Algorithm on this enhanced image. The image enhancement algorithm is summarized as follows:

- Step 1: Find the maximum, *max* and minimum, *min* intensity in the image.
- Step 2: Calculate the average, *avg* intensity considering all pixels in the image.
- Step 3: Calculate $\beta = \left(\frac{avg + min}{2} \right)$

The sum of β and constant δ gives the threshold of the pixel value for which the enhancement has to be done.

- Step 4: Calculate $\alpha = \{(k_1 * avg) + (k_2 * max)\}$

where k_1 is the weight for the avg and k_2 is the weight for the max and $0 \leq k_1, k_2 < 1$.

- Step 5: The threshold ($\beta + \delta_1$) is applied to the image so that the shaded pixels (or low intensity pixels) gets filtered out.
- Step 6: Let the filtered pixel be $I'(x,y)$, where $I'(x,y)$ is the intensity value of the pixel at (x,y) position in the image.

$$I'(x,y) = (I(x,y) * k_3) + \alpha + \delta_2 \quad (12)$$

k_3 is the intensity booster for the $I(x,y)$ and $k_1 + k_2 + k_3 = 1$.

6. MASKED ENHANCEMENT TECHNIQUE

The masked image enhancement is done with the help of mask run throughout the image plane. This mask is applied to the image locally at first and then it's applied to the image globally. The mask works on the principle of finding the local maxima of the image pixel. The mask finds out the local maxima around each pixel consideration. Now, this local maxima is used to enhance the pixel (i, j). The procedure for enhancing is as follows.

- Step 1: Determine the local maxima for a mask centered at (i, j) (exclude center pixel on which the mask is applied).
- Step 2: If the local maximum is found to be less than the intensity value at (i, j), then it is concluded that the pixel (i, j) is not in shaded region and hence does not need enhancement else enhance the pixel at (i, j).

To enhance the pixel, we add an enhancement factor Ω to the pixel (i, j). The procedure to compute Ω is as follows:

The average of intensity values of all the pixels in the mask, excluding the pixel (i, j) and the local maxima value is calculated. Let this average value be denoted by β . Now, add β with the local maxima value and find their average. Denote this value by μ . Store the value of the intensity value of the pixel (i, j) in ρ . If ρ is greater than or equal to 255, then divide ρ by a dividing factor ψ . The factor ψ corresponds to the mask size. If mask size is 'n', then ψ becomes (n-2). The value 2 is subtracted from 'n', owing to the fact that the pixel (i, j) and the local maxima values were not added earlier in the computation of β . If the value of ρ is less than 255, then the value μ is added to the pixel (i, j).

The shaded region in input image is enhanced according to their actual height. This method is particularly used to enhance those regions where we have a cliff and its immediate corresponding region is shaded by the shadow of the cliff.

7. ALGORITHM FOR FINDING MAXIMAL AND MINIMAL EDGE

In a satellite imagery there are many regions which are under shadow. So shape from shading algorithm gives inappropriate results. There is a sharp decrease in pixel intensity near edges. To draw a 3D terrain a depth map of the edges is needed. Depth map is an array containing the height of each pixels lying in the edge. It is assumed that there is no sudden change in height near edges. All the pixels just next to edge-pixels are set to same height as of the corresponding edge-pixel. In order to determine whether an edge is maximal or minimal, the direction of shadow is needed. The direction of shadow can be determined if there is sudden change in pixel intensity.

Let the edge pixel under consideration be $P'(i, j)$. The set of pixels which occurs before the P' along the direction of sunlight is called

early-pixels and those which occurs after, is called late-pixels. Early-pixels of P' are brighter than its late pixel if P' is in maximal edge. Late pixels of the P' are brighter than its early pixels if P' is in minimal edge. We can find the early and late pixels by method described below.



Figure 1. The arrow shows the direction of light. All the pixels intersecting sun ray and edges are levelled as Maxima or Minima according to the direction of shadow.

Equation of sunlight for an image of size m x n can be defined as

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1} \quad (13)$$

where (x_2, y_2) is given point and (x_1, y_1) is $(m/2, n/2)$ and the slope of sunlight can be defined as $\frac{y_2-y_1}{x_2-x_1}$. If slope of the sunlight is more than 1 and less than -1, x-intercept is more than y-intercept. First early pixel can be determined using equation below

$$((x_i - 1) - x_1) * \frac{y_2 - y_1}{x_2 - x_1} + y_1 \quad (14)$$

and to find K^{th} early pixel

$$((x_i - K) - x_1) * \frac{y_2 - y_1}{x_2 - x_1} + y_1 \quad (15)$$

Similarly, to find first late point

$$((x_i + 1) - x_1) * \frac{y_2 - y_1}{x_2 - x_1} + y_1 \quad (16)$$

And to find K^{th} late point we use the formula

$$((x_i + K) - x_1) * \frac{y_2 - y_1}{x_2 - x_1} + y_1 \quad (17)$$

If slope of sunlight is between -1 to 1, Y-intercept is more than X-intercept. To find first early pixel

$$((y_i - 1) - y_1) * \frac{x_2 - x_1}{y_2 - y_1} + x_1 \quad (18)$$

To find K^{th} early pixel

$$((y_i - K) - y_1) * \frac{x_2 - x_1}{y_2 - y_1} + x_1 \quad (19)$$

Similarly, to find first early pixel

$$((y_i + 1) - y_1) * \frac{x_2 - x_1}{y_2 - y_1} + x_1 \quad (20)$$

And, to find K^{th} early pixel

$$((y_i + K) - y_1) * \frac{x_2 - x_1}{y_2 - y_1} + x_1 \quad (21)$$

The maximal and minimal edges for the input images is shown in figure 2 and 3.

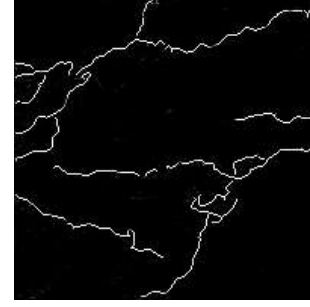


Figure 2. Maximal edges.

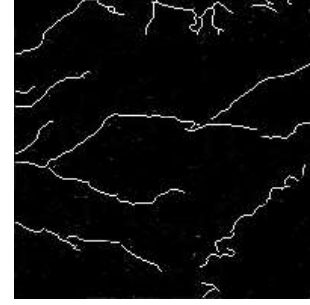


Figure 3. Minimal edges.

8. PIXEL DISTANCE ALGORITHM FOR FINDING 3D TERRAIN BASED ON PIXEL INTENSITY

This algorithm does not use pixel intensity to find the height instead it uses depth map and edge map only. The maximal edge and minimal edge is determined by using the algorithm written earlier. Height of all the pixels in maximal and minimal edge is determined using depth map. For each pixel determine one maximal and one minimal edge nearest to it in opposite direction. Let the point of consideration be P_i , P_{max} be the maximal edge pixel nearest to P_i and P_{min} be the minimal edge pixel nearest to P_i . Let D_1 and D_2 be the distance of P_{max} and P_{min} from P_i . Height of P_{max} and P_{min} is already known from the depth map. Now we calculate the height of P_i by the formula

$$\frac{(D_1 H_{max} + D_2 H_{min})}{(D_1 + D_2)} \quad (22)$$

9. HYBRID ALGORITHM

The basic concept of hybrid algorithm is to combine all the three algorithms. For this purpose, we can choose a local function $f(x)$ and then linearize $f(x)$ over the entire image so that the entire image can be processed and then apply a smoothing function over the image to smoothen out the entire image. Thus, this way we enhance the image locally and then spread out the local enhancement function over the entire image by using the linearizing function. Thus, to get better results, smoothing function is used over the entire image.

10. RESULT AND DISCUSSION

The study area taken into consideration is portion of LISS III Satellite Imagery Data of East Sikkim with resolution of 23m approx shown in Figure 4.

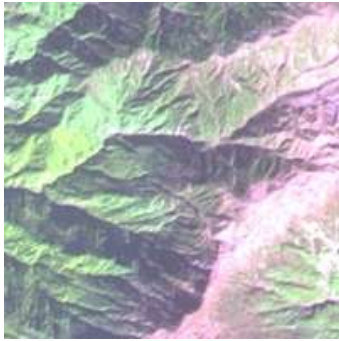


Figure 4: LISS III Satellite Imagery Data taken as input for various SFS approach

Figure 5- a) is the DEM reconstructed using Pentland's linear approach. The shape recovered using this algorithm as can be seen, is not accurate. The input image shown in Figure 4 is a mountainous region which has gradual ups and downs with some peak points. But the result is showing that it has cliffs (steep slope). This inaccuracy emerged out because of the assumption that Pentland has made. The assumption was that the surface should not be self-shadowing. It means the surfaces should be orthographic; the illuminant should be on the top of the surface. Figure 5-b) is the DEM reconstructed using enhanced Pentland's approach described in section V. The value used for scaling constant $k_1 = 0.35$; $k_2 = 0.25$; $k_3 = 0.4$; $\delta_1 = 30$ and $\delta_2 = 50$. The results shows satisfactory for self shadowing result as the shaded region is enhanced prior to applying Pentland's algorithm. Figure 5- c) is the DEM reconstructed using Lee and Rosenfeld's approach. Their method estimates the depth using local spherical assumption and intensity derivatives. This makes the algorithm unsuitable for non-spherical surfaces, and very sensitive to noise. However, the results can be improved by pre-filtering the input images in order to reduce the noise. Figure 5- d) is the DEM reconstructed using enhanced Lee and Rosenfeld' approach where shaded region is enhanced locally using booster described in section VI. Figure 5-e) is the DEM reconstructed using Horn's approach. Similar to Pentland's and Lee & Rosenfeld's this approach true does not work efficiently for the shaded region. Figure 5-f) shows the reconstructed DEM using Pixel Distance Approach. This method does not use pixel intensity to find the height instead it uses depth map and edge map only. The maximal edge and minimal edge is determined using the algorithm written earlier. Height of all the pixels in maximal and minimal edge is determined using depth map. The result obtained is more accurate compared to other method.

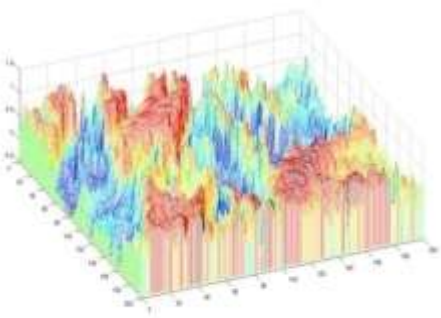
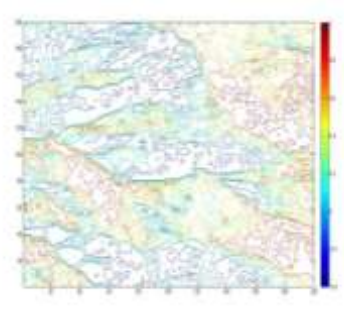
Figure 5-g) shows the DEM reconstructed using Hybrid approach. This approach combines features of the entire three algorithms.

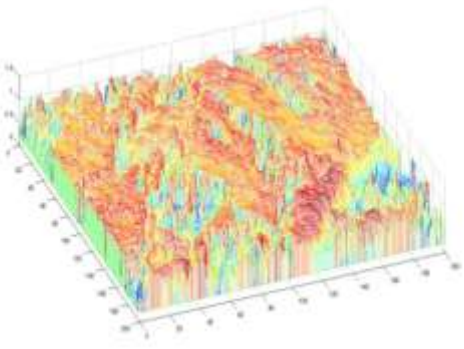
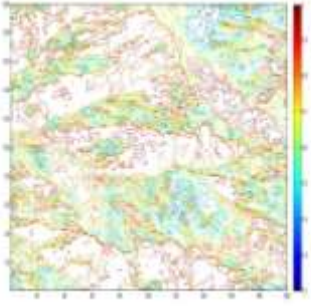
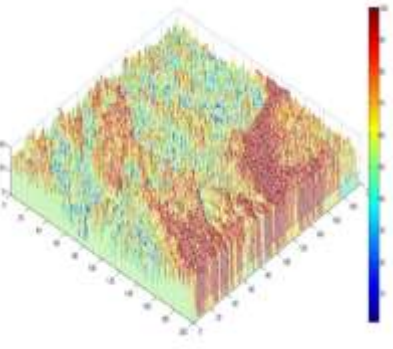
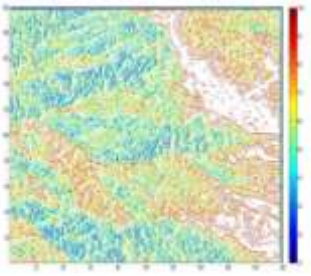
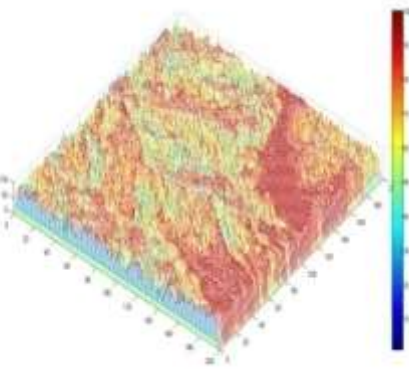
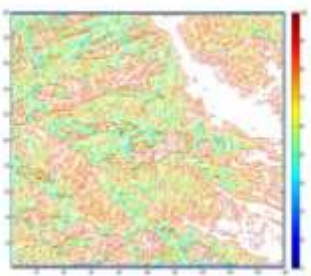
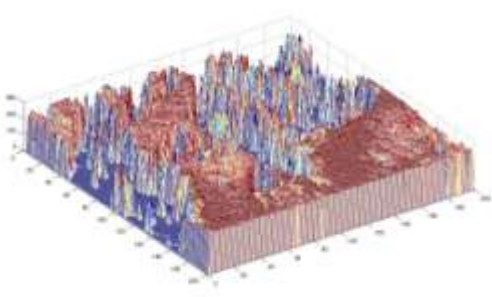
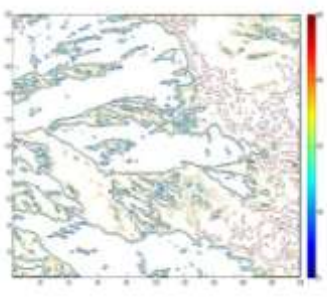
11. SUMMARY AND CONCLUSION

Three well known SFS algorithm were studied and implemented for generating DEM of the satellite imagery. The main difficulty encountered in creating DEM of the mountainous region is the shaded art of the images. The problem of Pentland's algorithm was that it provided incorrect result when a high surface has shadow on it. This was solved by enhancing the image by scaling pixels with shadow on it. The problem of Lee-Rosenfeld was that it provides incorrect result when a high surface has shade on it (i.e. low intensity pixel value) and the local spherical assumption of the surface limits its application. These problems were solved by enhancing the image by scaling with respect to the local maxima. It enhances those pixel values that have a local maxima value quite variant from itself, thereby enhancing the image's shaded regions. The problem with Horn's algorithm is that it does not work on shaded areas. Also, it needs to essential components to function. These components are external depth map and an edge map. This can be done by linear SFS method. The maxima and minima are found out and are used to build slopes between themselves. By this way, the shaded region problem can be solved as it is assumed that the maxima lies on elevated point and the maxima lies on a point that is lower in height, since it is having shade on it. To gain a better result, all pixel values could be found out using 180 degree iteration. Another method employed for the achievement of enhanced image is the nearest min-max method. In this method, the nearest maxima and minima from the pixel under consideration are found and these two values are used to calculate the height at that point. The hybrid algorithm is designed considering the advantage of all the three algorithm- Pentland, Lee-Rosenfeld and Horn algorithms. This algorithm solves most of the problems of all the three previous algorithms. Thus, the hybrid algorithm gives better and accurate result in most of the aspects than that of its predecessors.

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SL. No.	SFS Algorithm Used	3D Surface Reconstructed	Contour Map Generated
a)	Pentland's Linear Approach		

b)	Enhanced Pentland's Linear Approach		
c)	Lee-Rosenfeld's Approach		
d)	Enhanced Lee-Rosenfeld's Approach		
e)	B. K. P. Horn's Approach		

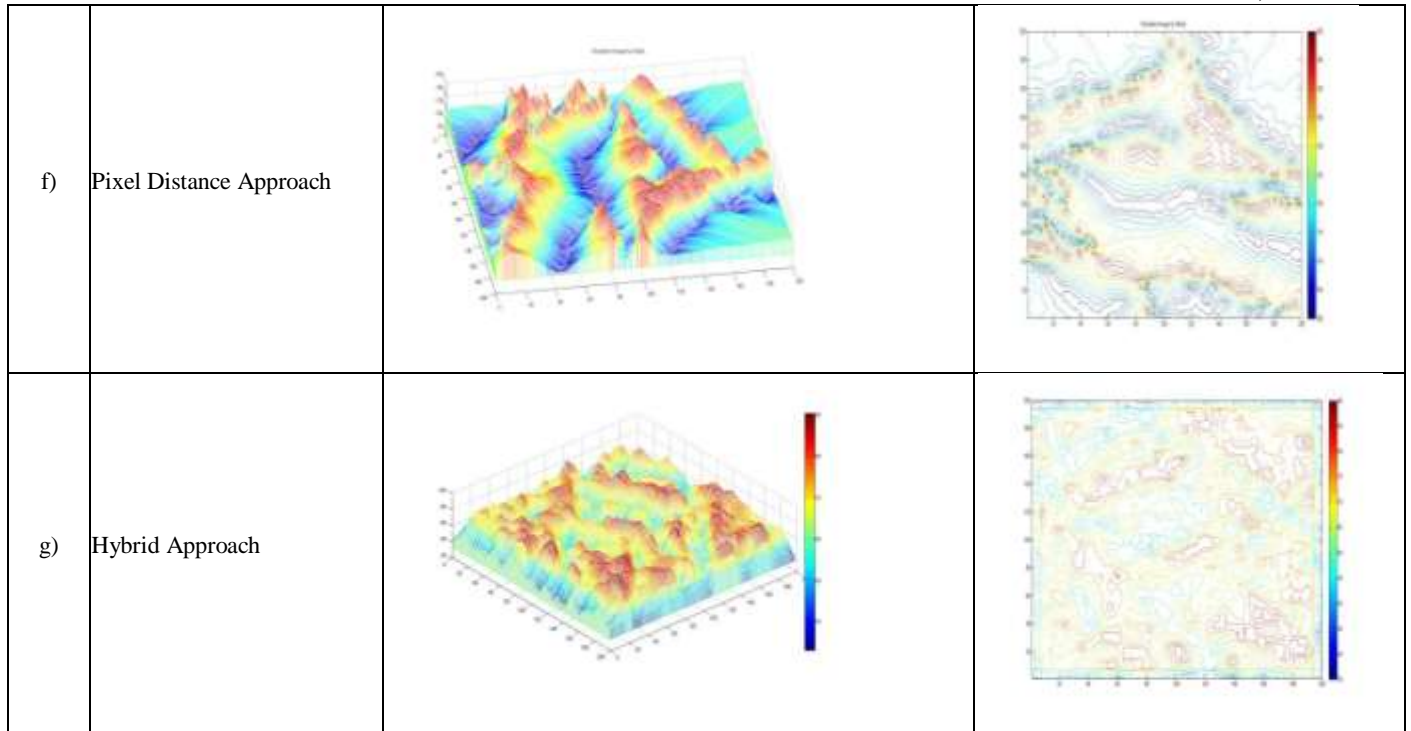


Figure 5: Depth elevation model (DEM) reconstructed from satellite imagery using various approach.

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