Directable Fuzzy Automata

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ABSTRACT

The aim of this paper is testing the directability of a given fuzzy automaton. A fuzzy automaton is directable if there exists a word, a directing word, which takes each state of a fuzzy automaton to a single state with some membership value. In this paper, we proposed a method for testing the directability of a fuzzy automaton using the mergeability relation.

Keywords

Directable, Mergeable, Congruence and Directing Congruence.

1. INTRODUCTION

Fuzzy set is a generalization of a classical set was introduced by Zadeh in 1965 [11]. This concept is applied in different discipline including medical sciences, artificial intelligence, pattern recognition and automata theory. Fuzzy ideas applied in automata was first proposed by Wee in 1967 [10]. Santos proposed fuzzy automata as a model of pattern recognition [9]. J. N. Mordeson and D. S. Malik gave a detailed account of fuzzy automata and applications in their book 2002 [8].

A fuzzy automaton is directable if there exists a word, a directing word, which takes each state of a fuzzy automaton to a single state with some membership value.

Using the concept of fuzzy recognizer, word recognized by fuzzy recognizer and the language recognized by fuzzy recognizer we prove that for any directable fuzzy automaton, the set of all directing words of a fuzzy automaton belongs to the language recognized by fuzzy recognizer.

We provide a necessary and sufficient condition for a fuzzy automaton to be directable, congruence of a fuzzy automaton to be directing. Further, we proposed a method for testing the directability of a fuzzy automaton using the mergeability relation.

Finally, We proposed an algorithm to find whether a fuzzy automaton is directable or not, using the mergeability relation of the states with example.

2. PRELIMINARIES

Let X denote a universal set. Then a fuzzy set A in X is set of ordered pairs: $A = \{(x,\ \mu_A(x)|x \in X\}$, $\mu_A(x)$ is called the membership function or grade of membership of x in A which maps X to the membership space $[0, 1]$ [12].

A finite fuzzy automaton is a system of 5 tuples, $M = (Q,\ \Sigma,\ f_M,\ q_0,\ F)$,
where, Q is set of states, Σ is input symbols, f_M is transition function from $Q \times \Sigma \times Q \rightarrow [0, 1]$, q_0 is an initial state and $q_0 \in Q$, and $F \subseteq Q$ set of final states. The transition in a fuzzy automaton is as follows:
$f_M(q_i, a, q_j) = \mu$, $0 \leq \mu \leq 1$, means that when M is in state q_i and reads the input a will move to the state q_j with weight function μ.
f_M can be extended to $Q \times \Sigma^* \times Q \rightarrow [0, 1]$ by,

$$f_M(q_i,\ \epsilon,\ q_j) = \begin{cases} 1 & \text{if } q_i = q_j \\ 0 & \text{if } q_i \neq q_j \end{cases}$$

$$f_M(q_i, w, q_m) = Max\{Min\{f_M(q_i, a_1, q_1), f_M(q_1, a_2, q_2), .., f_M(q_{m-1}, a_m, q_m)\}\}$$

for $w = a_1\ a_2\ a_3\ ...\ a_m \in \Sigma^*$, where Max is taken over all the paths from q_i to q_m.
We can represent transitions more conveniently by the matrix notation as follows:
For each $a \in \Sigma$, we can form a $n \times n$ matrix $F(a)$ whose $(i, j)^{th}$ element is $f_M(q_i,\ a,\ q_j)$. Let $w \in \Sigma^*$ and $w = a_1\ a_2\ a_3\ ....\ a_m$ then $F(w) = F(a_1) \circ F(a_2) \circ ..... \circ F(a_m)$.
In other words, $F(w)$ is the fuzzy sum of fuzzy products of weights taken over the paths in the automaton [4].
Throughout this paper, we consider a fuzzy automaton without initial state and final state and M denotes $M = (Q, \Sigma, f_M)$, f_M is transition function from $Q \times \Sigma \times Q \rightarrow [0, 1]$.
A fuzzy automaton M is called deterministic if for each $a \in \Sigma$ and $q_i \in Q$, there exists a unique state q_a such that $f_M(q_i,\ a,\ q_a) > 0$ otherwise it is called nondeterministic [3].
Let $M' = (Q',\ \Sigma,\ f_{M'})$, $Q' \subseteq Q$ and $f_{M'}$ is the restriction of f_M. The fuzzy automaton M' is called a subautomaton of M if
(i) $f_{M'} : Q' \times \Sigma \times Q' \rightarrow [0, 1]$ and
(ii) For any $q_i \in Q'$ and $f_{M'}(q_i, u, q_j) > 0$ for some $u \in \Sigma^*$, then $q_j \in Q'$. M is said to be strongly connected if for every $q_i,\ q_j \in Q$, there exists $u \in \Sigma^*$ such that $f_M(q_i,\ u,\ q_j) > 0$. Equivalently, M is strongly connected if it has no proper subautomaton [8].
An equivalence relation R on Q in M is called a congruence relation if for all $q_i, q_j \in Q$ and $a \in \Sigma$, $q_i\ R\ q_j$ implies that, then there

exists $q_l, q_k \in Q$ such that $f_M(q_i, a, q_l) > 0$, $f_M(q_j, a, q_k) > 0$ and $q_l \; R \; q_k$ [1, 2].
Let M be a fuzzy automaton. The quotient fuzzy automaton determined by the congruence $\cong$ is a fuzzy automaton
$M/\cong \; = (Q/\cong, \Sigma, f_{M/\cong})$, where $Q/\cong = \{Q_i = [q_i]\}$ and
$f_{M/\cong}(Q_1, a, Q_2) = Min\,\{f_M(q_1, a, q_2) > 0 \,/\, q_1 \in Q_1,\ q_2 \in Q_2$ and $a \in \Sigma\}$ [7].

3. DIRECTABLE FUZZY AUTOMATA AND DIRECTING CONGRUENCES

Let M be a fuzzy automaton. For every $q_i \in Q$, if there exists a state $q_j \in Q$ and $u \in \Sigma^*$ such that $f_M(q_i,\ u,\ q_j) > 0$. In that case, the word u is said to be a directing word of M. If M has a directing word, then we say that M is a directable fuzzy automaton. The set of all directing words of M is denoted by $DW(M)$ [6].
We say that two states $q_i, q_j \in Q$ are said to be mergeable or reducible if there exists a word $u \in \Sigma^*$ and $q_j \in Q$ such that $f_M(q_i, u, q_k) > 0 \Leftrightarrow f_M(q_j, u, q_k) > 0$ [5].
Let M be a fuzzy automaton. The set of all equivalence relations on a set Q is denoted by $Eq(Q)$. Let $\delta_M \in Eq(Q)$. If any two states $q_i, q_j \in Q$ is called δ_M-Mergeable, then there exists $(q_k, q_l) \in \delta_M$ such that $f_M(q_i, w, q_k) > 0$ and $f_M(q_j, w, q_l) > 0$, for some $w \in \Sigma^*$.
Let M be a fuzzy automaton. Let ρ be the congruence relation on the states set Q in M. If ρ is called directing congruence, then the quotient fuzzy automaton M/ρ is a directable fuzzy automaton.

3.1 Fuzzy Recognizer [8]

Let Q and Σ be finite subsets. A fuzzy recognizer is a five tuple $M_1 = (Q,\ \Sigma,\ f_{M_1},\ \psi,\ \xi)$, where,
(i) Q is a finite nonempty set of states,
(ii)Σ is a finite nonempty set of input symbols,
(iii) $f_{M_1} : Q \times \Sigma \times Q \rightarrow [0,1]$ is a function, called the fuzzy transition function,
(iv)ψ is a initial fuzzy subset of Q, i.e., $\psi : Q \rightarrow [0,1]$, called the initial fuzzy state, and
(v)ξ is a final fuzzy subset of Q, i.e., $\xi : Q \rightarrow [0,1]$, called the fuzzy subset of final states.
Note
Clearly, if $M_1 = (Q,\ \Sigma, f_{M_1}, \psi, \xi)$ is a fuzzy recognizer, then $M = (Q,\ \Sigma,\ f_M)$ is a fuzzy automaton. We call M the fuzzy automaton associated with the fuzzy recognizer M_1.

3.2 Word Recognized by Fuzzy Recognizer[8]

Let $M_1 = (Q,\ \Sigma, f_{M_1},\ \psi,\ \xi)$ be a fuzzy recognizer. Let $w \in \Sigma^*$. Then w is said to be recognized by M_1 if
$\vee_{q_i \in Q}(\psi(q_i) \wedge (\vee_{q_j \in Q}\{f_{M_1}(q_i,\ w,\ q_j) \wedge \xi(q_j)\})) > 0$.

3.3 Language Recognized by Fuzzy Recognizer[8]

Let $M_1 = (Q,\ \Sigma, f_{M_1},\ \psi,\ \xi)$ be a fuzzy recognizer.
Let $L(M_1) = \{w \in \Sigma^* \,/\, w \ is \ recognized \ by \ M_1\}$. $L(M_1)$ is called the language recognized by the fuzzy recognizer M_1, and the set of all fuzzy recognizable language by M_1 associated with $M = (Q,\ \Sigma,\ f_M)$ is denoted by $FRec(M)$.

4. PROPERTIES OF DIRECTABLE FUZZY AUTOMATA AND DIRECTING CONGRUENCES

LEMMA 4.1. *For any directable fuzzy automaton* $M = (Q, \Sigma, f_M)$, $\Sigma^* \; DW(M) \; \Sigma^* = DW(M)$

Proof:
Since M is a directable fuzzy automaton, there exists a directing word $w \in \Sigma^*$. Now,$w \in DW(M) \Rightarrow \lambda w \lambda \in \Sigma^* DW(M) \Sigma^*$.
Therefore, $DW(M) \subseteq \Sigma^* DW(M) \Sigma^*$——(1)
Take $w_1 \in \Sigma^* DW(M) \Sigma^*$, $w_1 = uwv$, $w \in DW(M)$and $u, v \in \Sigma^*$. Since w is a directing word, we have $f_M(q_i, uwv, q_k) > 0$, for all $q_i \in Q$, and some $q_k \in Q$. Therefore, $w_1 = uwv$ is also a directing word. Hence, $w_1 \in DW(M)$.
That is, $\Sigma^* DW(M) \Sigma^* \subseteq DW(M)$ —–(2)
From (1) & (2), $\Sigma^* DW(M) \Sigma^* = DW(M)$.

LEMMA 4.2. *For any directable fuzzy automaton* $M = (Q, \Sigma, f_M)$, $DW(M) \in FRec(M)$

Proof:
Consider any directable fuzzy automaton, we associate a fuzzy recognizer $M_d = (2^Q,\ \Sigma,\ f_{M_d},\ \psi(Q),\ \xi(S))$, where S is set of all singleton sets. $\psi(Q) = 1$ and $\psi(Q_1) = 0, \forall Q_1 \in 2^Q$. $\xi(S) = 1$ for singleton set $S \in 2^Q$ and $\xi(Q_2) = 0, \forall Q_2 \in 2^Q$. The transition function f_{M_d} in M_d is defined by
$f_{M_d}(P, a, T) = Min\,\{f_M(q_i, a, q_j),\ q_i \in P,\ q_j \in T\}$, $P, T \in 2^Q$ for $a \in \Sigma$.
Clearly, M_d is a deterministic fuzzy automaton. Let $w \in \Sigma^*$ be a directing word of M. That is, $f_{M_d}(Q, w, S) = \vee\left\{\psi(Q) \wedge f_{M_d}(Q, w, S) \wedge \xi(S)\right\} > 0$.
w is recognized by M_d. Hence, the set of all directing words is a language recognized by the fuzzy automaton M_d. Hence, $DW(M) \in FRec(M)$.

THEOREM 4.1. *A fuzzy automaton* $M = (Q, \Sigma, f_M)$ *is directable if and only if all pairs of states of Q in M are mergeable.*

Proof:
A fuzzy automaton M is directable. Since it is directable, there exists a directing word $u \in \Sigma^*$ and $q_j \in Q$ such that $f_M(q_i, u, q_j) > 0$, for every $q_i \in Q$. Let $q_k, q_l \in Q$. Then by the hypothesis, we have
$f_M(q_k, u, q_j) > 0 \Leftrightarrow f_M(q_l, u, q_j) > 0$. Therefore, q_k and q_l are mergeable.
Conversely, Assume that M is not a directable fuzzy automaton. That is, we assume that all states are mergeable in two states q_k and q_l in Q. Then there exists $w_1 \in \Sigma^*$ such that $f_M(q_i, w_1, q_k) > 0$ and $f_M(q_j,\ w_1,\ q_l) > 0$, for $q_i's, q_j's \in Q$.
Now, take the states q_k and q_l. By the hypothesis, the states q_k and q_l are mergeable. That is, there exists a word $w_2 \in \Sigma^*$ and $q_m \in Q$ such that $f_M(q_k, w_2, q_m) > 0 \Leftrightarrow f_M(q_l, w_2, q_m) > 0$, for every $q_k, q_l \in Q$.
Now, $f_M(q_i, w_1 w_2, q_m) > 0$, for every $q_i \in Q$, which is a contradiction to our assumption. Hence, M is a directable fuzzy automaton.

THEOREM 4.2. *Let* $M = (Q, \Sigma, f_M)$ *be a fuzzy automaton. A congruence* ρ *of a fuzzy automaton* M *is directing if and only if all pairs of states on* Q *in* M *are* ρ*-mergeable.*

Proof:
Assume that congruence ρ of a fuzzy automaton M is directing. By the definition of congruence it has an equivalence class say $Q_1 = \{P_1, P_2 .... P_n\}$. Now, consider the quotient fuzzy automaton of M say $M/\rho = (Q_1,\ \Sigma,\ f_{M/\rho})$.
Since congruence ρ of M is directing, then there exists a directing word say $w \in \Sigma^*$ and $P_j \in Q_1$ such that $f_{M/\rho}(P_i, w, P_j) > 0$, for every $P_i \in Q_1$.
Let $q_i, q_j \in Q$. Then either $q_i, q_j \in P_i$ or $q_i \in P_i$ and $q_j \in P_k$.
Let $q_i, q_j \in P_i$. Then $f_M(q_i, w, q_k) > 0$ and $f_M(q_j, w, q_l) > 0$, for some $q_k, q_l \in P_j$. This implies that $(q_k, q_l) \in \rho$.
Now, let $q_i \in P_i$ and $q_j \in P_k$, for $i \neq k$ be any two states in Q. Then $f_M(q_i, w, q_m) > 0$ and $f_M(q_j, w, q_n) > 0$, for some $(q_m, q_n) \in P_j$. This implies that $(q_m, q_n) \in \rho$. All pairs of states on Q in M are ρ-mergeable.
Conversely, assume that all pairs of states on Q in M are ρ-mergeable. Since ρ is congruence we can construct the quotient fuzzy automaton say $M/\rho = (Q_1,\ \Sigma,\ f_{M/\rho})$, where, $Q_1 = (P_1, P_2 .... P_n)$.
Assume that in M/ρ all states of Q_1 are merged in two distinct states P_j and P_k. That is, there exists a word $w_1 \in \Sigma^*$ such that $f_{M/\rho}(P_i, w_1, P_j) > 0$ and $f_{M/\rho}(P_i, w_1, P_k) > 0$, for some i.
Let $q_i \in P_j$ and $q_j \in P_k$. Since all pairs of Q in M are ρ-mergeable, there exists a word $w_2 \in \Sigma^*$ such that $f_M(q_i, w_2, q_r) > 0$, $f_M(q_j, w_2, q_s) > 0$, for some $(q_r, q_s) \in P_l$.
Let $w = w_1 w_2 \in \Sigma^*$. Then $f_{M/\rho}(P_i, w, P_l) > 0$, for every $P_i \in Q_1$.
It is a contradiction to our assumption. Hence, M/ρ is a directable fuzzy automaton.

5. ALGORITHM FOR TESTING DIRECTABILITY

The algorithm for testing directability contains two auxillary data structures. A $n \times n$ fuzzy matrix $M[i,j]$ and list NewPair of pairs of states.
We assume that $Q = \{q_1, q_2, q_3 ..... q_n\}$. If the pair (q_i, q_j) is mergeable, then $0 < M[i,j] \leq 1$. Since it suffices to consider just the pairs (i,j), where $1 \leq i < j \leq n$, we actually need just upper part of M.
A pair (q_i, q_j) appears in NewPair when q_i and q_j have found to be mergeable. For finding further mergeable pairs, we use inverted transition table.
The inverted transition table $I = (I[q_k, a])$, for any $q_k \in Q, a \in \Sigma$ is defined as follows:
$I[q_k, a] = \{q_j \in Q / f_M(q_j, a, q_k) > 0\}$, for any $q_k \in Q, a \in \Sigma$.

Algorithm:

Step1:
[Initialize M and NewPair] $M[i,j] = 0$ for all $1 \leq i < j \leq n$, and NewPair = $\{\epsilon\}$ (the empty list).
Step2:
Form the inverted transition table I.
Step3:
Find all pairs $(q_k, a) \in Q \times \Sigma$ for which $|I[q_k, a]| > 1$. For every such (q_k, a) consider each pair $q_i, q_j \in I[q_k, a]$ with $i < j$. If $M[i,j] = 0$, let $M[i,j] = Min\{f_M(q_i, w, q_k), f_M(q_j, w, q_k)\}$

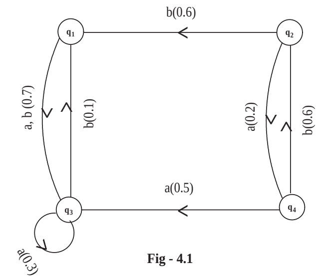

Fig - 4.1

Example

for some $w \in \Sigma^*$ and append (q_i, q_j) to NewPair.
Step4:
Until NewPair = $\{\epsilon\}$ do the following. Delete the first pair from NewPair. Suppose it (q_l, q_m). From I find all pairs $(q_i, q_j),\ i < j$, such that for some $a \in \Sigma$,
$q_i \in I[q_l, a]$ and $q_j \in I[q_m, a]$, or $q_i \in I[q_m, a]$ and $q_j \in I[q_l, a]$.
If $M[i,j] = 0$, let $M[i,j] = Min\{f_M(q_i, w, q_l), f_M(q_j, w, q_l)\}$ for some $w \in \Sigma^*$and append (q_i, q_j) to NewPair.
Step5:
If $M[i,j] > 0$ whenever $1 \leq i < j \leq n$, then M is a directable fuzzy automaton, otherwise not.
Step 1:
Initialize Fuzzy Matrix M

$$M[i,j] = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$NewPair = \{\epsilon\}$.
Step 2:
Inverted transition table I

$I[q_1, a] = \{\phi\}$ $\quad$ $I[q_1, b] = \{q_2, q_3\}$
$I[q_2, a] = \{\phi\}$ $\quad$ $I[q_2, b] = \{q_4\}$
$I[q_3, a] = \{q_1, q_3, q_4\}$ $\quad$ $I[q_3, b] = \{q_1\}$
$I[q_4, a] = \{q_2\}$ $\quad$ $I[q_4, b] = \{\phi\}$.

Step 3:
From the above inverted transition table I,find all pairs such that $|I[q_k, a]| > 1$.
$I[q_3, a] = \{q_1, q_3, q_4\}$ $\quad$ $I[q_1, b] = \{q_2, q_3\}$.
Now consider each pair (q_i, q_j) with $i < j$, we get
$\{(q_1, q_3), (q_1, q_4), (q_2, q_3), (q_3, q_4)\}$.
$M[1,3] = Min\{f_M(q_1, a, q_3), f_M(q_3, a, q_3)\}$
$= Min\{0.7, 0.3\} = 0.3$.
$M[1,4] = Min\{f_M(q_1, a, q_3), f_M(q_4, a, q_3)\}$
$= Min\{0.7, 0.5\} = 0.5$.
$M[2,3] = Min\{f_M(q_2, b, q_1), f_M(q_3, b, q_1)\}$
$= Min\{0.6, 0.1\} = 0.1$.
$M[3,4] = Min\{f_M(q_3, a, q_3), f_M(q_4, a, q_3)\}$
$= Min\{0.3, 0.5\} = 0.3$.
Therefore, the fuzzy matrix and NewPair list becomes,

$$M[i,j] = \begin{pmatrix} 0 & 0 & 0.3 & 0.5 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$NewPair = \{(q_1, q_3), (q_1, q_4), (q_2, q_3), (q_3, q_4)\}$.
Step 4:
Delete the first pair from NewPair list. That is, delete the pair

(q_1, q_3).
From I find all pairs $(q_i, q_j), i < j$. $\{q_2, q_3\} \in I[q_1, b]$and $\{q_1\} \in I[q_3, b]$.
The required pairs are
$\{(q_1, q_2)(q_1, q_3)\}$. Already $M[1, 3] > 0$. Therefore, leave the pair (q_1, q_3).
$M[1, 2] = Min\{f_M(q_1, aa, q_3), f_M(q_2, aa, q_3)\}$
$= Min\{0.3, 0.2\} = 0.2$.
Now add the pair (q_1, q_2) to NewPair.
Therefore, the fuzzy matrix and NewPair list becomes,

$$M[i,j] = \begin{pmatrix} 0 & 0.2 & 0.3 & 0.5 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$NewPair = \{(q_1, q_4), (q_2, q_3), (q_3, q_4), (q_1, q_2)\}$.
Delete the pair (q_1, q_4) in NewPair list and no such (q_i, q_j) exists for the pair (q_1, q_4) in I.
The fuzzy matrix and NewPair list becomes,

$$M[i,j] = \begin{pmatrix} 0 & 0.2 & 0.3 & 0.5 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$NewPair = \{(q_2, q_3), (q_3, q_4), (q_1, q_2)\}$.
Delete the pair (q_2, q_3). From I find all pairs $(q_i, q_j), i < j$.
$\{q_4\} \in I[q_2, b]$ and $\{q_1\} \in I[q_3, b]$. The required pairs are $\{(q_1, q_4)\}$.
Already, $M[1, 4] > 0$. Therefore, leave the pair.
The fuzzy matrix and NewPair list becomes,

$$M[i,j] = \begin{pmatrix} 0 & 0.2 & 0.3 & 0.5 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$NewPair = \{(q_3, q_4), (q_1, q_2)\}$.
Delete the pair (q_3, q_4). From I find all pairs $(q_i, q_j), i < j$.
$\{q_1, q_3, q_4\} \in I[q_3, a]$ and $\{q_2\} \in I[q_4, a]$. The required pairs are $\{(q_1, q_2), (q_2, q_3), (q_2, q_4)\}$.
Already, $M[1, 2] > 0$ and $M[2, 3] > 0$.
Therefore, leave the pairs (q_1, q_2) and (q_2, q_3).
Consider the pair (q_2, q_4).
$M[2, 4] = Min\{f_M(q_2, aa, q_3), f_M(q_4, aa, q_3)\}$
$= Min\{0.2, 0.3\} = 0.2$.
Add the pair (q_2, q_4) in the NewPair list.
Therefore, the fuzzy matrix and NewPair list becomes,

$$M[i,j] = \begin{pmatrix} 0 & 0.2 & 0.3 & 0.5 \\ 0 & 0 & 0.1 & 0.2 \\ 0 & 0 & 0 & 0.3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$NewPair = \{(q_1, q_2), (q_2, q_4)\}$.
Delete the pair (q_1, q_2). From I find all pairs $(q_i, q_j), i < j$.
$\{q_2, q_3\} \in I[q_1, b]$ and $\{q_4\} \in I[q_2, b]$.
The required pairs are $\{(q_2, q_4), (q_3, q_4)\}$.
Already, $M[2, 4] > 0$ and $M[3, 4] > 0$.
Therefore, leave the pairs (q_2, q_4) and (q_3, q_4).
The fuzzy matrix and NewPair list becomes,

$$M[i,j] = \begin{pmatrix} 0 & 0.2 & 0.3 & 0.5 \\ 0 & 0 & 0.1 & 0.2 \\ 0 & 0 & 0 & 0.3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$NewPair = \{(q_2, q_4)\}$.
Delete the last pair (q_2, q_4). From NewPair list and no such (q_i, q_j) exists for the pair (q_2, q_4) in I.
Therefore, the fuzzy matrix and NewPair list becomes,

$$M[i,j] = \begin{pmatrix} 0 & 0.2 & 0.3 & 0.5 \\ 0 & 0 & 0.1 & 0.2 \\ 0 & 0 & 0 & 0.3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$NewPair = \{\epsilon\}$.
Now all entries in upper part of $M[i, j] > 0$.
Hence, the fuzzy automaton is directable.

6. CONCLUSION

In this paper, we introduce a method for testing the directability of a fuzzy automaton and provide an algorithm to find whether a fuzzy automaton is directable or not using the mergeability relation of the states. Also, we provide a necessary and sufficient condition for a fuzzy automaton to be directable and congruence of a fuzzy automaton to be directing.

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