Medical Image Denoising based on Log-Gabor Wavelet Dictionary and K-SVD Algorithm

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ABSTRACT
Medical image denoising is the main step in medical diagnosis, which removes the noise without affecting relevant features of the image. There are many algorithms that can be used to reduce the noise such as: threshold and the sparse representation. The K-SVD is one of the most popular sparse representation algorithms, which is depend on Orthogonal Matching Pursuit (OMP) and Discrete Cosine Transform (DCT) dictionary. In this paper, an algorithm for image denoising was designed to develop K-SVD by using Regularized Orthogonal Matching Pursuit (ROMP) over log Gabor wavelet adaptive dictionary. To evaluate the performance of the proposed techniques, the results were compared with the results of DCT and Gabor wavelet dictionary. The numerical results show that the performance of our algorithm is more efficient in medical image denoising.

Keywords
Sparse representation (SR), K-SVD, log-Gabor wavelet dictionary, regularized orthogonal matching pursuit and orthogonal matching pursuit.

1. INTRODUCTION
In the recent years, image denoising had more attention because, it was considered as a preprocessing step in many applications such as: feature extraction, image segmentation and medical image applications [1]. In medical applications, the process of imaging acquisition can be affected by noises and artifacts. This will lead to degrading the contrast resolution of images and making medical images difficult to interpret. The denoising of medical image algorithms is needed to achieve best diagnosis without affecting relevant features of the image [1-3]. There are many types of denoising depend on different criteria such as: threshold or statistical measures. In the threshold methods, the coefficients that are lower than a certain threshold set to zero and higher than this threshold remains constant [4-6]. The threshold methods are effective in image denoising, but some of relevant features in the image lost.

Also, there are many algorithms depend on statistical measures are used to reduce the noise such as: The median filter is effective in eliminating noise, where it replaces the middle pixel in the window with the median value of its neighbors [7]. Also, maximum-likelihood filter and Bayesian denoising method is used to remove the noise models and adopt the robust parametric estimation approaches [8, 9]. However, these filters suffer from some drawbacks such as: they can remove the relevant feature from the image and also the blurring problem. To solve these drawbacks, the sparse representation (SR) was used [10]. There are a lot of advantages to use the SR such as: it will require less space on the computer and less time in computation when the value and position of entire needs to be recorded. So the SR was used in many applications such as: feature extraction [11, 12], denoising [13, 14], inpainting [15, 16], dimension reduction in image processing [17, 18].

There are many algorithms that have proposed to obtain the SR such as: The OMP algorithm is still one of the most effective sparse decomposition algorithms [19, 20]. However, the OMP is a static algorithm in which, if the initial dictionary is not perfect, then the solution of OMP is not good, and to solve this problem ROMP method is used. The K-SVD algorithm is an iterative algorithm for updating the dictionary. It was used to find the best SR [13, 21]. There are many algorithms that are developed K-SVD, for example: in [22, 23] Gabor wavelet dictionary it was used and it was found that this dictionary is better than a DCT dictionary for image denoising.

The objective of this paper is to develop K-SVD algorithm by using log-Gabor wavelet to avoid the problem in other dictionaries and also the ROMP was used to solve the drawbacks in OMP. Finally, the results of this method were compared with other methods based on different medical images.

The rest of this paper is organized as follows: Section 2 presents the sparse representation. Section 3 illustrates orthogonal matching pursuit algorithm. Section 4 introduces a description of K-SVD algorithm. In section 5, the description of log-Gabor wavelet dictionary equations is presented. Section 6 discusses regularized orthogonal matching pursuit algorithm. Finally, the numerical results of our algorithm in image denoising is discussed and compared with other methods in section 7.

2. SPARSE REPRESENTATION
Sparse representation of images tends to find a set of atoms $d_i \in R^n$ that form the dictionary $D \in R^{n\times K}$ and it is defined as [13]:

$$y_i = Dx_i \qquad (1)$$

Where x_i is sparse representation of y_i. However, the exact solution is difficult to be found, so equation (1) can be rewritten as:

$$\min_{\mathrm{D},x}\|\mathrm{y}-\mathrm{D}x\|_2 \quad \mathrm{s.t} \quad \|x\|_0 < L \qquad (2)$$

Where L is a threshold that controls the sparseness of the vector x and $\|.\|_0$ is l_o-norm that is defined as the number of non-zero entries. Another formula of the above problem is:

$$\min_{\mathrm{D},x}\|x\|_0 \quad \mathrm{s.t} \quad \|\mathrm{y}-\mathrm{D}x\| < \varepsilon, \varepsilon \in \mathrm{R} \qquad (3)$$

There are many algorithms that can be used to compute the SR such as: Basis Pursuit (BP) [24] that replaces the l_o-norm with the l_1-norm and this makes the solution SR better. In other words, the Matching Pursuit (MP) algorithm selects only one atom for iteration [25]. However, the MP is time computational and to solve this, OMP is used as illustrated in next section [19].

3. ORTHOGONAL MATCHING PURSUIT

Orthogonal matching pursuit (OMP) is an iterative algorithm that selects at each step the atom, which is most correlated with the current residuals. At each iteration the locally optimum solution is calculated. OMP algorithm assumes that residual vector is equal to the vector that is required to be approximated (i.e. $r_0 = y$). OMP algorithm can ensure that no zero components are selected. The algorithm attempts to reduce the error in representation for iteration. This is done by selecting the best atom from the dictionary that has the largest correlation with residual. The final OMP algorithm is given in algorithm 1[22].

Algorithm 1 OMP
Input: Dictionary D, stopping criterion ε and y. ***Output:*** Approximation vector c. 1. Initial $r_0 = y, t = 0$ and Index set $V_0 = \emptyset$ 2. Let $v_t = i$ where d_i gives the solution of $max < r_t, d_k >$ 3. Update the set V_t with v_t: V_t=$V_{t-1} \cup \{v_t\}$ 4. Solve the least-squares problem $min_{c \in C^{V_t}} \|\| y - \sum_{j=1}^{t} c(v_j) d_{v_j} \|\|2$ 5. Calculate the new residual using c, r_t=$r_{t-1} - \sum_{j=1}^{t} c(v_j) d_{v_j}$ 6. Set $t \leftarrow t + 1$ 7. Check stopping criterion if the criterion has not been satisfied then return to Step 2.

4. K-SVD ALGORITHM

In this section, the K-SVD algorithm is exposed for training of dictionaries. This algorithm can be flexible and also operates together with ROMP algorithm. First, dictionary D was fixed and the best coefficient matrix X that can be found. Coefficient matrix X can be founded by using ROMP method. The second stage is conducted to look for a better dictionary. This procedure updates one column at a time, fixing all columns in D other than one, d_k and finding a new d_k and new values for its coefficients that will reduce the MSE (mean square error). Sparse representation formulation was extended to include the entire set of observed images denoted simply by the set Y= $\{y_i|_i \in [1, K], y_i \in R^n\}$ as [21]:

$$\min_{D,X} ||Y - DX||_F^2 \quad \text{s.t} \quad \forall\, i, \quad || \quad x_i||_0 \leq T_0 \tag{4}$$

The penalty term can be rewritten as:

$$||Y - DX||_F^2 = ||\text{Y-} \sum_{j=1}^{k} d_j\, x_T^j||_F^2 = || \left(Y - \sum_{j \neq k} d_j x_T^j\right) - d_k x_T^k||_F^2 = ||E_k - d_k X_T^k||_F^2 \tag{5}$$

The SVD finds the closest rank-1 of the matrix (in Frobenius norm) that approximates E_k , and this will effectively minimize the error as defined in (5). w_k Represents band of indices pointing to $\{y_j\}$ that use the atom d_k , i.e., where $X_T^k(i)$ is actually nonzero. Thus

$$w_k = \{\, i \,|1 \leq i \leq K, X_T^k(i) \neq 0\} \tag{6}$$

The singular value was used to update d_k and x_T^j based on E_k^R , where E_k was decomposed to $U\Delta V^T$. The perfect solution for d_k was defined as the first column of U, and the coefficient vector X_T^k as the first column of V multiplied by $\Delta(1,1)$.

However, The K-SVD depends on both OMP and DCT, where OMP algorithm has some drawbacks such as: it calculated local optimum at each iteration and time computation so, to solve these problems ROMP was used. Log Gabor Wavelet was used to avoid the problems that appear from other dictionaries. K-SVD can be represented as [21]:

Algorithm 2 K-SVD
Task: Find the best dictionary to present the data samples $\{y_i\}_{i=1}^N$ as sparse composition, by solving $min_{D,X}\{\|\|Y - DX\|\|_F^2\}$ Subject to $\forall\, i, \|\|x_i\|\|_0 < T_0$. Initialization: set the dictionary matrix $D^{(0)} \in R^{n*k}$ from with l^2normalized columns. Set $J = 1$ Repeat until convergence (stopping rule): -Sparse coding stage: x_i =OMP (y_i, D) // $min_{x_i}\{\|\|y_i - Dx_i\|\|_F^2\}$ Subject to $\|\|x_i\|\|_0$<T_0, -Codebook update stage: For each column $k = 1 \ldots K\ in\ D^{(J-1)}$update it by 1. Defining the group of examples that use this atom,$w_k = \{i\|1 \leq i \leq N, x_T^k(i) \neq 0\}$ 2. Computing the overall computation error matrix $E_k, by E_k = Y - \sum_{j \neq k} d_j\, x_T^j$. 3. Restrict E_k by choosing only the columns corresponding to w_k,and obtain E_k^R. 4. Apply SVD decomposition $E_k^r = U\Delta V^T$. Choose the updated dictionary column d_k to be the first column of U.update the coefficient vector x_R^k to be the first column of V multiplied by$\Delta(1,1)$. Set $J = J + 1$

5. LOG GABOR WAVELET DICTIONARY

There are many types of dictionaries such as: DCT, Gabor and wavelets but exhibit major drawbacks. The log-Gabor wavelet dictionary was used to allow exactor construction and strengthen the excellent mathematical properties, for example: the higher frequency bands are covered by narrowly localized oriented filters. Also the set of filters, uniformly covers the Fourier domain, including the highest and lowest frequencies [27].

Log-Gabor wavelets get Gaussian transfer functions when viewed on the logarithmic frequency scale, however, with linear scale. This transfer perform will be asymmetric, specifically, it offers extensive tails at the higher frequency, which leads to a more efficient encoding for natural images than ordinary Gabor functions [28]. Log-Gabor wavelets don't have a DC component, and can be divided into two components: radial and angular filters and it was defined as [29]:

$$G\ (r_0,\alpha_0) = G\ (r_0)\ G\ (\alpha_0) = exp\left(-\frac{(log(r/r_0))^2}{2(log(\frac{\sigma_r}{r_0}))^2}\right) exp\left(-\frac{(\alpha - \alpha_0)^2}{2\sigma^2_{\alpha}}\right) \tag{7}$$

Where r_0 is the filter's center frequency, α_0 is orientation angle, σ_r is the radial standard deviation and σ_α is angular standard deviation. To get continual appearance rate filtration, the term σ_r/r_0 should be held continuously related to a lot of r_0.

6. REGULARIZED ORTHOGONAL MATCHING PURSUIT

ROMP algorithm uses more than one vector at each iteration [26] to form the solution and doesn't use a preset threshold ROMP algorithm. Where, in ROMP-LG the basis of Log Gabor Wavelet was used to form the atoms of dictionary. ROMP-LG was replaced by OMP. ROMP can be represented as [31]:

Algorithm 3 ROMP

Input:* *Dictionary D, vector y
Output: Approximation vector x
1. Initialize $r_0 = y$, t =0 and index set $V = \emptyset$
2. Identify Choose a set $J$of the n biggest coordinates in the vector u=$D^T r_t$, (or all of its nonzero coordinates if this set is smaller)
3. Regularize Find among all subsets $J_0 \subset J$ the maximum$\left\|u|_{J_0}\right\|_2$.
Where J_0 is define $i, j \in J_0$ if $||u_i|| \leq 2||u_j||$and $u_i \in u|_{j_0}$ if $k \in J_0$

4. Update the index set by V=V ∪ $J_0$and residual by
$x = \underset{c \in R^v}{min} ||y - Dc||_2$
$r_{t+1} = y - Dx$
Check stopping criterion if the criterion has not been satisfied then return to step 2

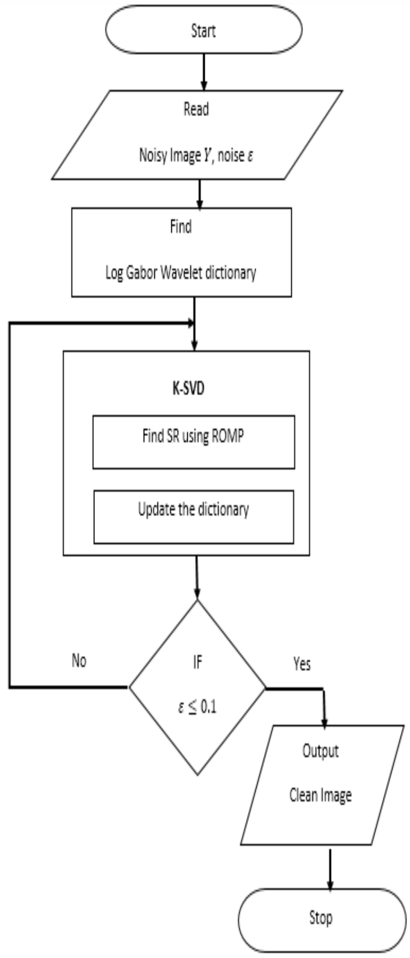

Figure 1: shows the algorithm flowchart.

This flowchart diagram can be illustrated as follows: Noisy image Y and error tolerance ε considered inputs to this flowchart. Formula (7) was used to calculate the log-Gabor wavelet. Column vector $d_{i,j} \in D$ is defined as the log Gabor wavelet feature vector. The Log Gabor Wavelet was used to form the atoms of dictionary. From the previous statement, vector which is defined as $d_{i,1}, d_{i,2}, \dots, d_{i,n} \in D$ could be gotten $d_i = (d_{i,1}, d_{i,2}, \dots, d_{i,n})^T$ then utilized to be represented in one column of over-complete dictionary D. Using the above$d_i = (d_{i,1}, d_{i,2}, \dots, d_{i,n})^T$, the Over complete dictionary$D = \{d_i, i = 1,2, \dots, k\}$ was obtained. Then, K-SVD algorithm is used to train this type of over complete redundancy dictionary. Now the optimized over complete redundancy dictionary D could be found. Testing a small sample T that can be considered as a linear combination of training samples, is defined as $\mathrm{Y} = \mathrm{d}_{1,1}\ x_1 + \dots + \mathrm{d}_{\mathrm{K},\mathrm{n}_\mathrm{k}}\ x_\mathrm{n}$, sparse vector x is calculated through the use of ROMP method $x\ = (x_1, x_2, \dots, x_\mathrm{n})^\mathrm{T}$, where $\mathrm{n} = \mathrm{n}_1 + \mathrm{n}_2 + \dots + \mathrm{n}_\mathrm{k}$. Consider the noise could be written as $\mathrm{Y} = \mathrm{D}\,x\ + \varepsilon$, where ε is defined as an optional error tolerance. Then, Solve problem $\mathrm{Y} = \mathrm{D}x + \varepsilon$, the same as $\min_{\mathrm{D},x} \|\mathrm{y} - \mathrm{D}x\|_2\ \mathrm{s.t}\ \|x\|_0 < \varepsilon, \varepsilon \in R$. If the error tolerance $\varepsilon \leq 0.1$ the image will be clean unless $\varepsilon \leq 0.1$returns to K-SVD method.

7. NUMERICAL RESULTS

To evaluate the performance of our algorithm, it was compared with other algorithms such as: KSVD based on Gabor wavelet [22], DCT [13], log-Gabor [23]. All experiments are performed on a PC running Windows 7 64bit and 4G RAM. The experiments are based on some benchmark MRI images dataset from [30]. The performance of the proposed algorithm was quantified across different noise levels. For each noise level, the average performance was calculated for each algorithm over 10 runs. To measure the performance of algorithms the Peak Signal to Noise Ratio (PSNR) was used and it was defined as:

$$PSNR = 20log_{10}\left(\frac{255}{MSE}\right) \quad (8)$$

The results are illustrated in table 1 and figures 2-4. In tables 1 the K-SVD (based on ROMP) using different type of dictionaries and our algorithm are performed, in which the best results are highlighted (in bold red). From this table the log Gabor wavelet is a better one, and then log Gabor is better than others, where the original images and the image corrupted by white Gaussian noise are shown in Figures 2-4 (a) and Figures 2-4 (b) respectively. The denoising results obtained by log Gabor, our algorithm, Gabor, Gabor wavelet, DCT and wavelet are illustrated in Figures 2-4 (c)-(h) respectively.

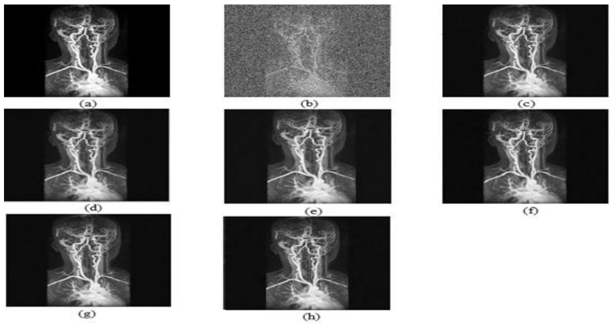

Figure 2: (a) Original image, (b) noised image, denoised image by using dictionary (c) log Gabor, (d) our algorithm, (e) Gabor, (f) Gabor wavelet, (g) DCT, (h) wavelet.

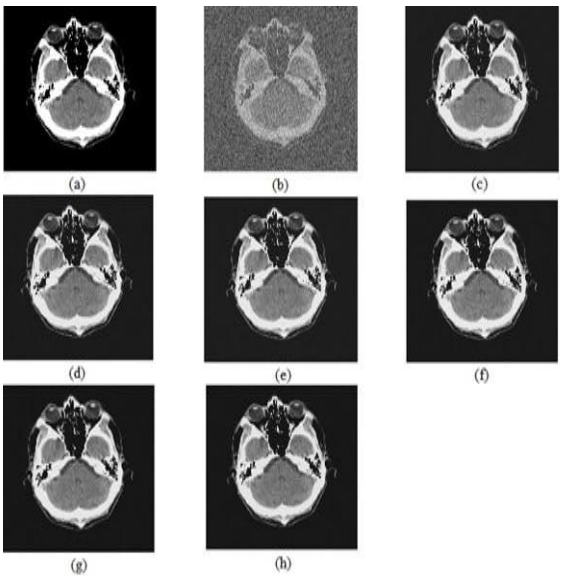

Figure 3: (a) Original image, (b) noised image, denoised image by using dictionary (c) log Gabor, (d) our algorithm, (e) Gabor, (f) Gabor wavelet, (g) DCT, (h) wavelet.

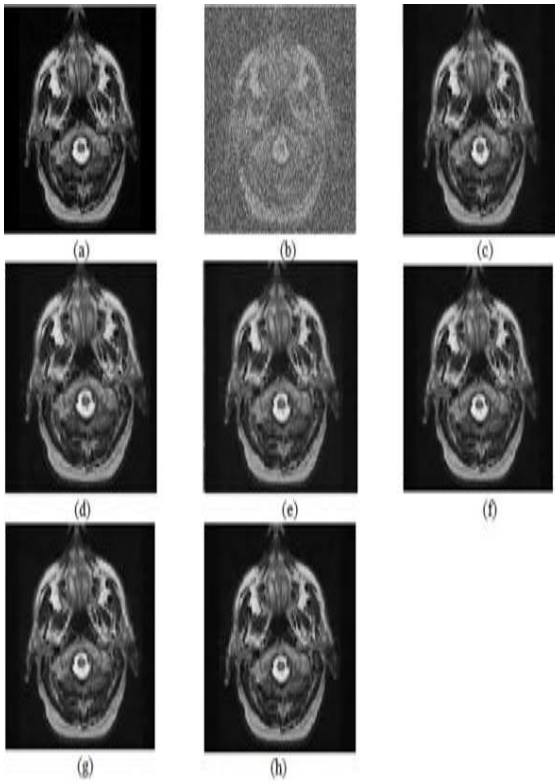

Figure 4: (a) Original image, (b) noised image, denoised image by using dictionary (c) log Gabor, (d) our algorithm, (e) Gabor, (f) Gabor wavelet, (g) DCT, (h) wavelet.

Table 1: Shows comparison between the K-SVD that developed by (log Gabor- wavelet-DCT) with different values of sigma and our algorithm.

	Log Gabor	*our algorithm*	**Gabor**	**Gabor Wavelet**	**DCT**	**Wavelet**
Sigma =10	**37.4159**	37.3606	37.3855	37.3354	37.361	37.3406
	37.4381	37.4007	37.3806	37.3656	37.4121	37.3145
	33.3333	33.3405	33.262	33.2786	**33.4933**	33.2378
Sigma =20	33.6904	**33.772**	33.5372	33.646	33.533	33.6054
	33.6049	**33.7202**	33.6738	33.5916	33.6372	33.6774
	29.7943	29.7095	29.5993	29.6772	29.7062	29.6747
Sigma =30	31.4444	**31.4917**	31.4095	31.3366	31.3598	31.476
	31.4269	**31.5437**	31.4331	31.3441	31.3346	31.4545
	27.9112	27.8412	27.7845	27.7573	27.8101	27.8398
Sigma =40	**29.9162**	29.8831	29.8153	29.7989	29.7453	29.8433
	29.8327	**29.9242**	29.8011	29.7985	29.8582	29.8464
	26.6486	26.6058	26.2738	26.4632	26.4581	26.3299
Sigma =50	**28.6574**	28.5789	28.4951	28.5258	28.4943	28.4174
	28.5492	**28.5652**	28.5339	28.5576	28.511	28.5018
	25.4815	25.4642	25.2901	25.3413	25.3022	25.3688

Sigma =60	**27.56**	27.5123	27.4365	27.3981	27.5517	27.3948
	27.4606	**27.5272**	27.4303	27.4551	27.4388	27.3524
	24.5113	24.3603	24.3883	24.1102	**24.5442**	24.3605
Sigma =70	26.4708	26.5494	26.5134	26.5107	**26.677**	26.4961
	26.6673	26.5955	26.5775	26.329	26.576	26.4631
	23.6017	**23.8458**	23.5327	23.7779	23.7328	23.5219
Sigma =80	25.7499	**25.8772**	25.7586	25.6786	25.8754	25.6776
	25.5956	25.7852	25.6575	25.7074	**25.8176**	25.7072
	23.1469	22.9121	23.0781	22.9121	**22.9579**	23.0941
Sigma =90	**25.096**	25.0736	25.0778	25.0834	25.0713	25.0891
	25.0171	**25.6692**	25.0211	24.9827	25.2024	25.0149
	22.4846	**22.6856**	22.4142	22.465	22.4917	22.4023
Average	28.4632	28.5035	28.3911	28.3788	28.4427	28.389

8. CONCLUSION

In this paper, image denoising has been discussed using log Gabor wavelet dictionary to remove noise from medical images. Some methods based on sparse and redundant representations of images over learned dictionaries are proposed. PSNR was used to measure the performance of the different techniques. The numerical results in table 1 and figures 2-4 show that, log Gabor wavelet and ROMP method are able to outperform some of these methods in all results, taking into account the average and the best solutions found. The comparison results demonstrate that log Gabor wavelet dictionary and ROMP method provides the best results.

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