Investigations of Stack- driven Airflow through Rectangular Cross- Ventilated Building with Two Openings using Analytic Technique

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ABSTRACT
Natural ventilation of building provides improvement of internal comfort and air quality conditions leading to a significant reduction of cooling energy consumption. Design of natural ventilation systems for many types of building is based on buoyancy forces. However, external wind flow can have significant effects on stack- driven natural ventilation. Investigation of stack- driven airflow through rectangular cross- ventilated building with two openings using analytic technique was presented. Equations governing air flow are utilized. Approximation of reduced gravity is invoked. A solution of the model Equations for stack- driven airflow through rectangular openings with uniform interior temperature was obtained. The solution predicts the following; dimensionless- velocity-, temperature profiles together with volumetric airflow and mass transfer were obtained and evaluated numerically for several sets of values of the parameters; such as effective thermal coefficient (θ_0), Prandtl number (Pr), and discharge coefficient (c_d).

Keywords
Velocity-, temperature profile; volumetric; mass- transfer;

1. INTRODUCTION
Generally, air flow distributions in buildings are considered to be as a result of the knowledge of the exact air supply to a building. This is necessary to determine its thermal performance and the concentration of the indoor pollutants. The exchange of air can be achieved either by mechanical means (Mechanical ventilation) or through the large opening of the building envelope (Natural ventilation). Exchanges between external ambience and interior space of buildings caused by flows that are driven by wind or by temperature differences are the foundation of natural ventilation process. However, natural ventilation is being pursued by humans, who are increasingly spending more time indoors, to extend the possibilities of living in uncongenial or squally conditions etc. The improvements of the quality of the interior space both in its attractiveness, spaciousness, luminosity, and more importantly its proper natural ventilation are major concerns for designers of modern structures. Air flow modeling gives Architectures and Engineers the luxury to consider several design options in the minimum amount of time. As a result, the final design is not based on a tentative approach but on a professional design process considering several options and selecting the best design. Air flow models are used to simulate the rates of incoming and outgoing air flows for a domain with known leakages under given weather and shielding conditions. Air flow models can be divided into two main categories, single-zone models and multi-zone models. Single-zone models assume that the structure can be described by a single, well mixed zone. Many attempts to investigate this phenomenon have been made by some researchers**.** [1] Presented a simple basic theory of natural convection across openings in vertical partitions and generalized to include both heat and mass-transfer in a single-sided ventilated domain. Displacement ventilation (where the interior is stratified**)** was studied by [2], and the mixing ventilation (where the interior has uniform temperature) by [3]. [4] Presented a convective heat and mass-transfer through large openings, which plays an important role in the thermal behavior of domains. [5] considered building having two openings at different vertical level on opposite walls, the heights of the two openings are relatively small, and the areas of the top and bottom openings are A_t and A_b respectively. The study also considered an indoor source of heat E, and the wind force can assist or oppose the thermal buoyancy force, when the indoor temperature is uniform. [6] Considered natural ventilation in a full-scale building induced by combined wind and buoyancy forces. The overall objectives were to verify and validate a CFD model for the naturally ventilated buildings, collect high quality full-scale experimental data for CFD validation and formulate guidelines for modeling natural ventilation in design practice and a steady envelope flow model were applied to calculate mean ventilation rates. [7] Investigated the study on combination of natural ventilation methods. A test room of single-sided ventilation was equipped with a vertical vent. Ventilation rate through the openings was evaluated based on the air flow velocity measured at the surface area of the openings. The vertical vent was kept closed during the first run of the experiments then the same experiments repeated where the vent was in use. Based on the experimental results, the effects of the vertical vent on the ventilation rate were clarified and a model was suggested based on combination for the two ventilation methods. [8] Investigated air flow rate across a vertical opening induced by a thermal source in a room, various parameters were used in designing natural ventilation. [9] Considered wind-driven cross ventilation in building with small openings. Airflow process across vertical vents induced by stack- driven effect with an opposing flow in one of the openings was presented by [11]. [10] Presented and developed a simple mathematical model of stack ventilation flows in multi-compartment buildings, with a view to providing an intuitive understanding of the physical processes governing the movement of air and heat through naturally ventilated

buildings. In which, the solution for the velocity, temperature distributions and volumetric airflow, mass- transfer rates were obtained.

The main objective of this paper is to analytically investigate airflow process inside a rectangular cross- ventilated building with two openings on a vertical wall. This is the novel approach which will lead to better understanding of the phenomenon and help in optimizing the designs for better natural ventilation.

2. DOMAIN DESCRIPTION

The building considered, is un-stratified cross- ventilated rectangular building with two openings. In which the building has one upper and one lower rectangular opening. The upper opening has an area of $0.7m \times 1.0m$, while the lower opening is $0.7m \times 2.0m$. Dimension of the building is $5.3m \times 3.6m \times 2.8m$ with air as the connecting fluid. The domain envelops were separated from one another by a vertical rectangular openings of height y^* and width x_w, which is illustrated in Figure 1. The density of air in the building is maintained at ρ_0 with temperature at $\dot{\theta}^*$ and pressure P.

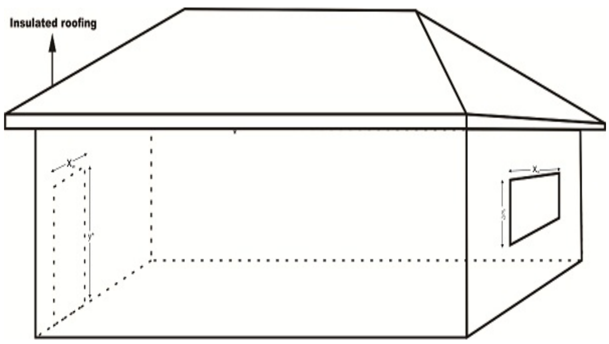

Figure 1: Diagram of un-stratified cross ventilated rectangular building with two openings.

3. RESEARCH METHODOLOGY

Flow chart of research methodology showing the process of obtaining results for airflow process in the building is illustrated in Figure 2.

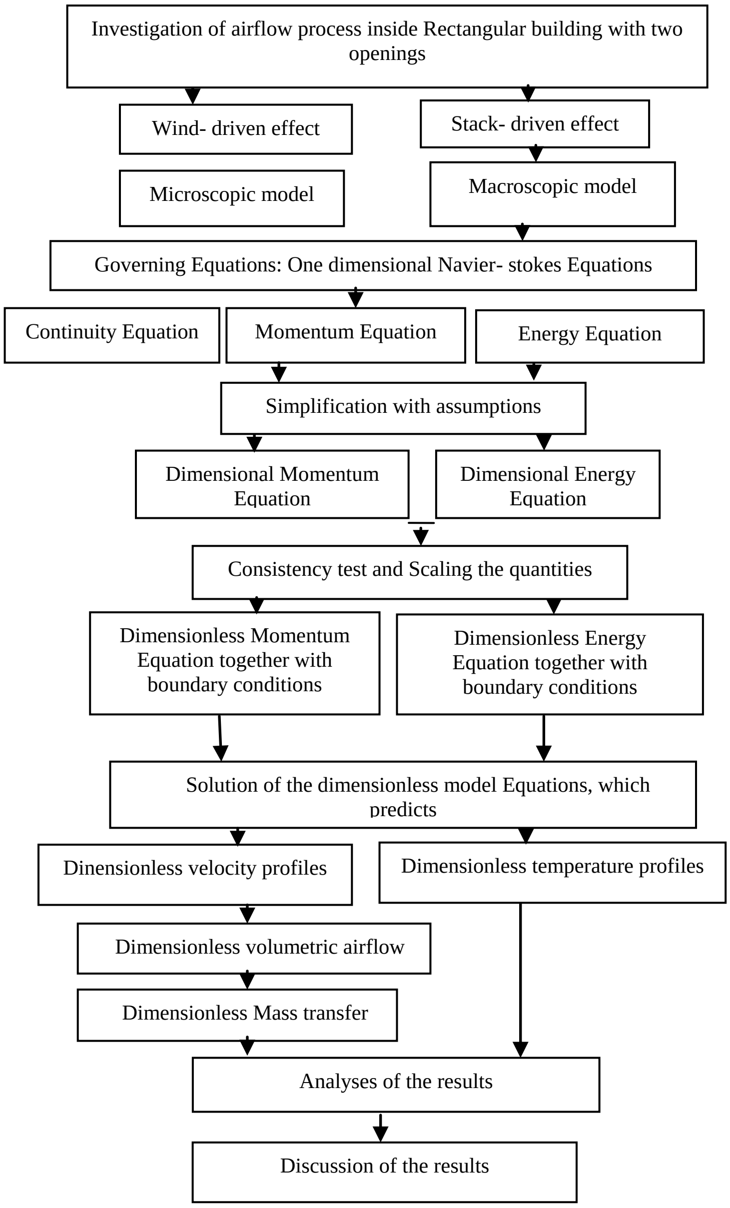

Figure 2: Graphical illustration of the research strategy

4. MODEL FORMULATION

Stack- driven airflow through rectangular openings in building is considered (see Figure 3 and 4). The flow is steady that depends on the height of the opening on the vertical walls. Airflow is assumed to be at low speed so that it will behave like incompressible fluid. Internal heat source is negligible. Boussinesq approximation is invoked. Navier Stokes Equations with appropriate boundary conditions described the problem. The model equations are written in a dimensionless form and solved analytically by means of variation of parameters.

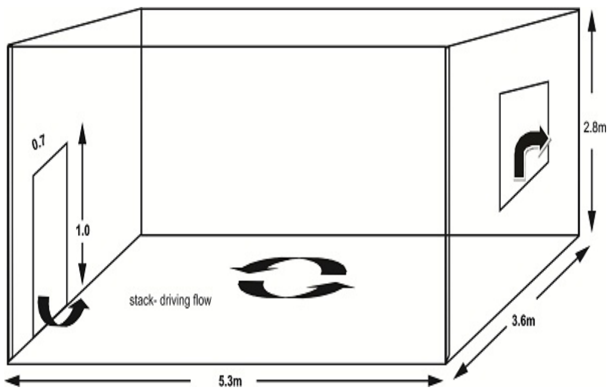

Fig 3: Schematic diagram of airflow process inside un-stratified cross ventilated rectangular building with two openings.

Governing Equations: One dimensional Navier-Stokes Equations are simplified by the above mentioned assumptions. In which Continuity Equation is satisfied identically. The convection motion is then driven by the stack- driven effect.

$$\beta\Delta\dot{\theta}g + \nu\frac{d^2U}{dy^2} = 0 \qquad (1)$$

$$\alpha\frac{d^2\dot{\theta}}{dy^2} = 0 \qquad (2)$$

Scaling y with y^*L, effective velocity U with $\frac{U^* g\beta\Delta\theta L^2}{\alpha}$, and introducing $\dot{\theta}$ with $\dot{\theta}^*\Delta\theta + \theta_0$, where $\Delta\dot{\theta} = \dot{\theta} - \theta_0$.

The dimensionless model Equations are,

$$Pr\frac{d^2U^*}{dy^{*2}} + \dot{\theta}^* = 0 \qquad (3)$$

$$\frac{d^2\dot{\theta}^*}{dy^{*2}} = 0 \qquad (4)$$

With dimensionless boundary conditions as;

$$0 \le y^* \le 1,\ U^*(0) = 0, U^*(1) = 0, \qquad (5)$$

$$\dot{\theta}^*(0) = -\theta_0,\ \dot{\theta}^*(1) = 1 - \theta_0, \qquad (6)$$

5. SOLUTION OF THE DIMENSIONLESS MODEL EQUATIONS

5.1 Temperature profiles

The solution of Equation (4), yields to,

$$\dot{\theta}^*(y^*) = C_1 + C_2y^*. \qquad (7)$$

The two constant C_1 and C_2 can be determined by prescribing the boundary conditions in Equation (6) for the temperature field, thus obtaining:

$$C_1 = -\theta_0, \text{ and } C_2 = 1 \qquad (8)$$

Equation (7), together with the boundary conditions (6), yields to temperature profiles as,

$$\dot{\theta}^*(y^*) = \dot{\theta}_2^*(y^*) = y^* - \theta_0. \qquad (9)$$

5.2 Velocity profiles

From Equation (3)

$$Pr\frac{d^2U^*}{dy^{*2}} = \dot{\theta}^* \qquad (10)$$

Putting Equation (9) into (10),

$$\frac{d^2U^*}{dy^{*2}} = -\frac{1}{Pr}(y^* - \theta_0)\ . \qquad (11)$$

Concerning Equation (11), one can start solving it from the homogeneous part:

$$\frac{d^2U^*}{dy^{*2}} = 0. \qquad (12)$$

Two independent solutions of Equation (11) are, u_c, u_p

$$U_C = C_3u_1 + C_4u_2$$

In which,

$$U_C = C_3 + C_4y^*. \qquad (13)$$

$$U_P = V_1y_1 + V_2y_2\ . \qquad (14)$$

In which,

$$U_P = V_1 + V_2y^* \qquad (15)$$

Where, $y_1 = 1, y_2 = y^*$

By employing the variation of parameters methods, one can write Wroskian of this solution as

$$W(y_1, y_2) = 1 \qquad (16)$$

Let,

$$R = -\frac{1}{Pr}(y^* - \theta_0) \qquad (17)$$

$$V_1 = \int\frac{-Ry_2}{w(y_1,y_2)}dy \text{ and } V_2 = \int\frac{Ry_1}{w(y_1,y_2)}dy \qquad (18)$$

Equation (15), can now be written as,

$$U_P = \frac{1}{Pr}\left(\frac{\theta_0y^{*2}}{2} - \frac{y^{*3}}{6}\right). \qquad (19)$$

Where,

$$V_1 = \frac{1}{Pr}\left(\frac{y^{*3}}{3} - \theta_0\frac{y^{*2}}{2}\right) \text{ and } V_2 = -\frac{1}{Pr}\left(\frac{y^{*2}}{2} - \theta_0y^*\right) \qquad (20)$$

The general solution of Equation (11) will be expected as,

$$U^*(y^*) = U_P + U_C \qquad (21)$$

Putting Equation (13), (19), into (21), one obtains

$$U^*(y^*) = C_3 + C_4y^* + \frac{1}{Pr}\left(\frac{\theta_0y^{*2}}{2} - \frac{y^{*3}}{6}\right). \qquad (22)$$

The two constant C_3, C_4 can be determined by prescribing the boundary conditions (5) for the velocity field, thus obtaining:

$$C_3 = 0 \qquad (23)$$

$$C_4 = -\frac{1}{Pr}\left(\frac{\theta_0}{2} - \frac{1}{6}\right) \qquad (24)$$

Therefore, the general solution of Equation (11), can be written as,

$$U^*(y^*) = U_2^*(y^*) = \frac{1}{Pr}\left(\frac{\theta_0y^{*2}}{2} - \frac{y^{*3}}{6} - \left(\frac{3\theta_0-1}{6}\right)y^*\right). \qquad (25)$$

5.3 Volumetric airflow

$$Q_2^*(y^*) = A^*{}_2c_d\int_{s=0}^{s=\frac{y^*}{2}}U^*(s)ds. \qquad (26)$$

Putting Equation (25) in (26), one obtains

$$Q_2^*(y^*) = A^*{}_2c_d\int_{s=0}^{s=\frac{y^*}{2}}\left[\frac{1}{Pr}\left(\frac{\theta_0s^2}{2} - \frac{s^3}{6} - \left(\frac{3\theta_0-1}{6}\right)s\right)\right]ds. \qquad (27)$$

By taking the integral in Equation (27), one obtain the volumetric airflow as,

$$Q_2^*(y^*) = \frac{A^*{}_2 c_d}{Pr}\left(\frac{\theta_0}{48}y^{*3} - \frac{1}{384}y^{*4} - \frac{(3\theta_0-1)}{48}y^{*2}\right). \quad (28)$$

5.4 Mass Transfer

$$m_2^*(y^*) = \rho_0 Q^*(y^*). \quad (29)$$

6. ASYMPTOTIC BEHAVIOR AND DISCUSSION OF THE RESULTS

In this section the main features of the solutions found in the previous section (5) will be discussed. This is done in order to see the effect of changes of parameters to the overall distributions, while keeping other operating conditions and parameters fixed, and ascertain the best one for optimal natural ventilation.

Physical interpretations of dimensionless velocity profiles for three incremental values of θ_0 and Pr is presented in Figure 4 and 5 in which as θ_0 and Pr increases the corresponding U_2^* decreases. The obvious features to be observed are in Figure 4 at $\theta_0 = 0.5$ the line of flow start with back flow in the domain. This down ward motion appears until the convective motion is reached. This fact depends on the different magnitude and sign of the buoyancy forces during the flow development. In Figure 5 almost all the lines of flow are uniformly distributed across the openings. Therefore, it is found that the best value of U_2^* for optimal natural ventilation in Figure 4, and 5 is when $\theta_0 = 0.1$ and $Pr = 0.710$.

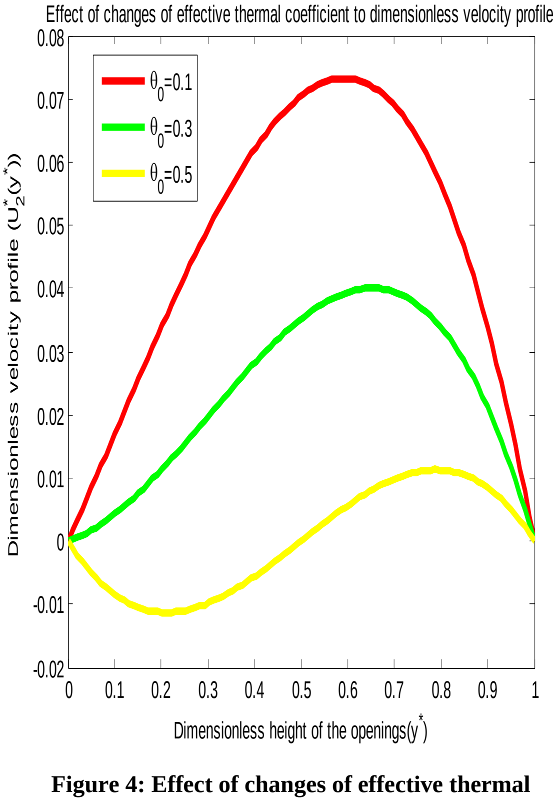

Figure 4: Effect of changes of effective thermal coefficient(θ_0) to dimensionless velocity profile ($U_2^*(y^*)$), for $Pr = 0.710, \theta_0 = 0.1\text{~}0.5$ and for $0 \le y^* \le 1$.

By substituting Equation (28) into (29), one obtain the mass transfer as,

$$m_2^*(y^*) = \frac{A^*{}_2 \rho_0 c_d}{Pr}\left(\frac{\theta_0}{48}y^{*3} - \frac{1}{384}y^{*4} - \frac{(3\theta_0-1)}{48}y^{*2}\right). \quad (30)$$

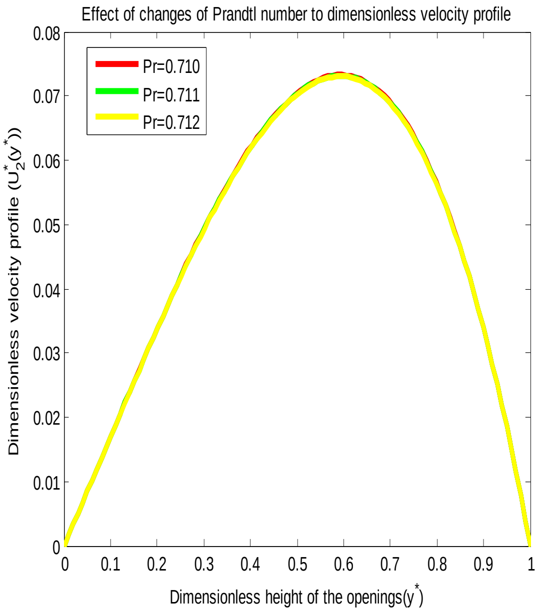

Figure 5: Effect of changes of Prandtl number(Pr) to dimensionless velocity profile ($U_2^*(y^*)$), for $\theta_0 = 0.1$, $Pr = 0.710\text{~}0.712$, and for $0 \le y^* \le 1$.

Physical interpretations of dimensionless volumetric airflow for three incremental values of θ_0, Pr and c_d are presented in Figure 6, 7, in which as θ_0 and Pr increases the corresponding Q_2^* decreases. The obvious features to be observed are in Figure 7 almost all lines of flow are uniformly distributed. And in Figure 8 as c_d increase the corresponding Q_2^* also increases. Therefore, it is found that the best value of Q_2^* for optimal natural ventilation in Figure 6, 7, and 8 is when $\theta_0 = 0.1, Pr = 0.710$ and $c_d = 0.75$.

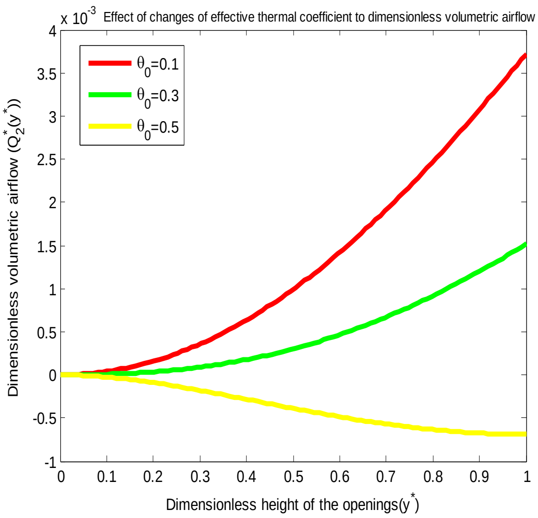

Figure 6: Effect of changes of effective thermal coefficient (θ_0) to dimensionless volumetric airflow ($Q_2^*(y^*)$), for $Pr = 0.710, c_d = 0.6, A_2^* = 0.3131, \theta_0 = 0.1\text{~}0.5$ and for $0 \le y^* \le 1$.

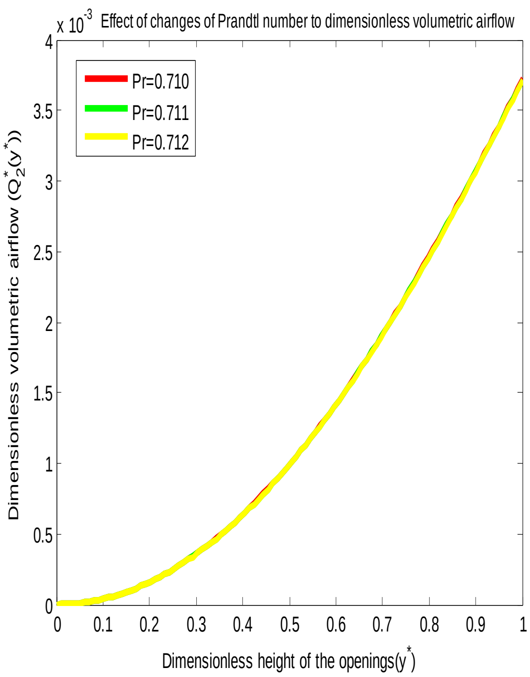

Figure 7: Effect of changes of effective thermal coefficient (θ_0) to dimensionless volumetric airflow $(Q_2^*(y^*))$, for $Pr = 0.710 \sim 0.712, c_d = 0.6, A_2^* = 0.3131, \theta_0 = 0.1$ and for $0 \le y^* \le 1$.

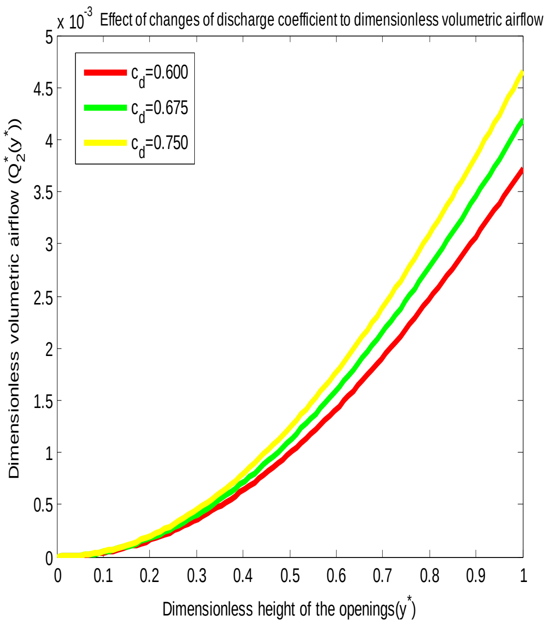

Figure 8: Effect of changes of discharge coefficient (c_d) to dimensionless volumetric airflow $(Q_2^*(y^*))$, for $Pr = 0.710, c_d = 0.6 \sim 0.75, A_2^* = 0.3131, \theta_0 = 0.1$ and for $0 \le y^* \le 1$.

Physical interpretations of dimensionless mass transfer for three incremental values of θ_0, Pr are presented in Figure 11, 10, in which as θ_0 and Pr increases the corresponding m_2^* decreases. The obvious features to be observed are in Figure 10 almost all lines of flow are uniformly distributed. And in Figure 9 as c_d increase the corresponding m_2^* also increase. Therefore, it is found that the best value of m_2^* for optimal natural ventilation in Figure 11, 10, and 9 is when $\theta_0 = 0.1, Pr = 0.710$ and $c_d = 0.75$.

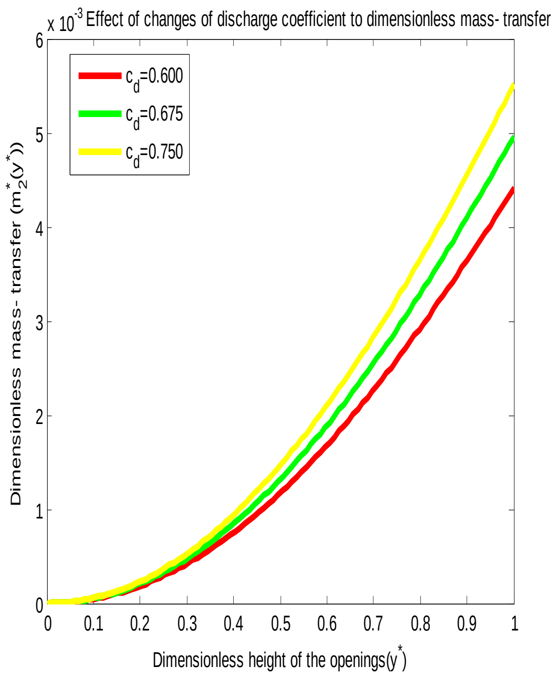

Figure 9: Effect of changes of discharge coefficient(c_d) to dimensionless mass transfer $(m_2^*(y^*))$, for $Pr = 0.710, c_d = 0.6 \sim 0.75, A_2^* = 0.3131, \theta_0 = 0.1, \rho_0 = 1.1849$ and for $0 \le y^* \le 1$.

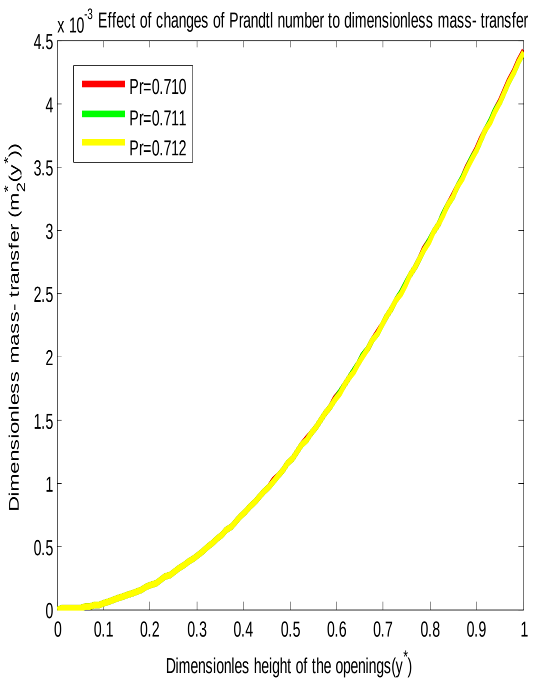

Figure 10: Effect of changes of Prandtl number(Pr) to dimensionless mass transfer $m_2^*(y^*)$, for $Pr = 0.710 \sim 0.712, c_d = 0.6, A_2^* = 0.3131, \theta_0 = 0.1, \rho_0 = 1.1849$ and for $0 \le y^* \le 1$.

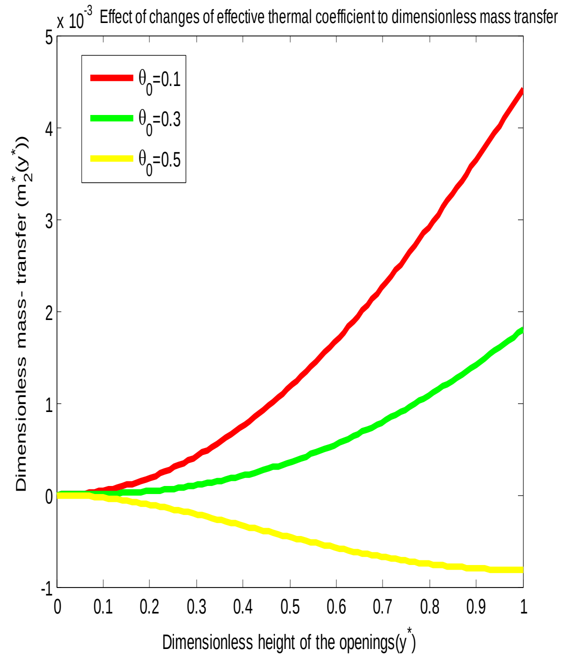

Figure 11: Effect of changes of effective thermal coefficient(θ_0) to dimensionless mass- transfer $m_2^*(y^*)$), for $Pr = 0.710, c_d = 0.6, A_2^* = 0.3131, \rho_0 = 1.1849$, $\theta_0 = 0.1\sim0.5$ and for $0 \leq y^* \leq 1$.

A Physical interpretation of dimensionless temperature profiles for three incremental values of θ_0 is presented in Figure 5.1 in which as θ_0 increase the corresponding $\dot{\theta}_2^*$ decreases. The obvious features to be observed is the lines of flow for temperature profiles across the openings are almost linearly the same Therefore, it is found that the best value of $\dot{\theta}_2^*$ for optimal natural ventilation is when $\theta_0 = 0.1$.

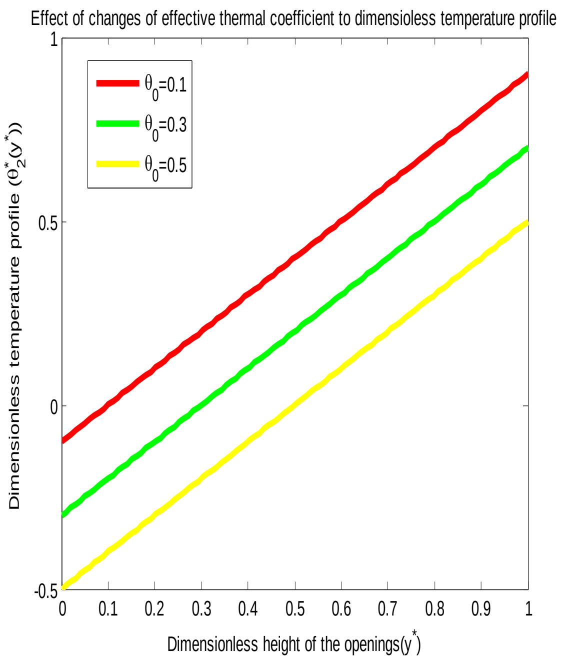

Figure 12 Effect of changes of effective thermal coefficient(θ_0) to dimensionless temperature profile $\dot{\theta}_2^*(y^*)$, for $\theta_0 = 0.1\sim0.5$ and for $0 \leq y^* \leq 1$

7. CONCLUSION

An investigation of stack- driven airflow through rectangular cross- ventilated building with two openings using analytic technique was presented. Parameters such as, effective thermal coefficient and Prandtl number were also introduced, which were believed to have significant effects on natural ventilation process in buildings. New models for stack driven airflow through cross- ventilated buildings with two opening were developed; analytical techniques were employed to obtain the possible solutions of the model Equations, which predicted velocity- and temperature profiles together with volumetric airflow and mass-transfer. Various parameters on air flow process were used to see the effect of changes of parameters to the overall flow distributions, and ascertain the best one for optimal natural ventilation. Therefore, expected objectives in the paper are achieved.

The paper lead to the following conclusions:

1. An increase in effective thermal coefficient θ_0 results in a decreases in temperature profiles $\dot{\theta}_2^*$ across the openings. The temperatures profiles $\dot{\theta}_2^*$ is more sensitive at lower values of effective thermal coefficient θ_0. Therefore, the main features to be observed is in the research is that the temperature profiles $\dot{\theta}_2^*$ was within comfortable conditions for lower values of θ_0.
2. The velocity U_2^* decreases more rapidly for higher values of effective thermal coefficient θ_0. An important observations of the study is that reverse flow are noted at the wall of $y^* = 0$ to almost 0.3 height with higher value of effective thermal coefficient $\theta_0 = 0.5$.
3. An increase in Prandtl number Pr results in a decrease in velocity profiles U_2^* across the openings. The main features to be observed are all the lines of flows are uniformly distributed.
4. The velocity profiles decreases more rapidly for higher values of effective thermal coefficient θ_0 and Prandtl number Pr, in comparison to lower values of θ_0 and Pr in U_2^* .
5. An increase in effective thermal coefficient θ_0 and Prandtl number Pr results in a decreases in volumetric airflows Q_2^*. The main features to be observed are all the lines of flows are uniformly distributed.
6. The volumetric airflows Q_2^* is more sensitive for higher values of discharge coefficients c_d, in comparison with lower values of c_d.
7. An increase in effective thermal coefficient θ_0 and Prandtl number Pr results in a decreases in mass transfer m_2^*. The main features to be observed are all the lines of flows are uniformly distributed.
8. The mass transfer m_2^* is more sensitive to higher values of discharge coefficients c_d, in comparison with lower values of c_d.
9. The greater vertical distance between the openings, and the greater temperature difference between the inside and the outside, the stronger is the effect of the buoyancy.

The model is only valid for cross- ventilated building with two openings at the same height.

To date there are few validated models for natural ventilation process on the basis of a theoretical approach. Lastly, the research findings will help in developing a better understanding of natural ventilation process and help researchers to gain more insights into the phenomenon and therefore come up with more models.

8. ACKNOWLEDGMENTS
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9. BIBLIOGRAPHY
9.1 Nomenclature
C_1, C_2, C_3, C_4 Coefficients;

A_2^* Total area of the openings;

x_w Width of the openings;

y Height of the openings;

y^* Dimensionless height of the openings;

C_d Discharge coefficient;

g Acceleration due to gravity;

c_d Discharge coefficient;

P Air pressure;

U Dimensional velocity profile;

$U^* = U_2^*$ Dimensionless velocity profiles;

m_2^* Dimensional mass – transfer;

Q_2^* Dimensional volumetric airflow;

$W(y_1, y_2)$ Wroskian;

y_1, y_2 Known functions obtained from complimentary Solution;

V_1, V_2 Known functions obtained from particular Solution;

U_C Complimentary solution;

U_p Particular solution;

$\frac{y^*}{2}$ Neutral height;

s Dummy variable;

L Length scale of the height of the opening;

9.1.1 Greek Symbols
α Thermal diffusivity;

ν Kinematic viscosity;

β Thermal expansion coefficient;

θ_0 Effective thermal coefficient;

$\Delta\dot{\theta}$ Dimensional change of air temperature;

$\dot{\theta}$ Dimensional temperature profile;

$\dot{\theta}^* = \dot{\theta}_2^*$ Dimensionless temperature profiles;

ρ_0 Uniform interior air density;

9.1.2 Dimensionless parameter
Pr Prandtl number;

9.1.3 Subscripts
w Width;

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