Relaxation and Retardation Effects on Free Convective Visco-Elastic Fluid Flow Past an Oscillating Plate

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ABSTRACT
An unsteady two dimensional free convective flow of visco-elastic fluid past a flat surface with heat and mass transfer has been investigated. The surface is oscillating with about a mean velocity U_0. Oscillating temperature and concentration about T_∞ and C_∞ respectively have been considered at the surface. The visco-elastic fluid flow is characterized by Oldroyd-B fluid model having two rheological parameters: relaxation time and retardation time. In the governing fluid flow, a magnetic field of uniform strength B_0 has been applied along the transverse direction to the surface. Governing equations of motion are solved analytically by using perturbation scheme. Analytical expressions for velocity profiles, shearing stress at the surface, temperature and concentration fields are obtained. Results are discussed graphically for various combinations of flow parameters involved in the solution. A special emphasis is given on the effects of relaxation and retardation times.

Keywords
Relaxation and retardation, Oldroyd-B fluid model, free convection, perturbation, shearing stress.

1. INTRODUCTION
The mechanism of visco-elastic fluid flow has attracted many scientists and researchers because of its uses in various industries. In visco-elastic fluid flow, energy is dissipated due to the presence of viscosity and elasticity restores the energy. Oldroyd [1, 2] has formulated a model characterizing the phenomena of visco-elastic fluid and it is named as Oldroyd fluid model. In oldroyd fluid model, the three rheological parameters are relaxation time, retardation time and co-efficient of viscosity. The constitutive equation of Oldroyd fluid model is given by

$$\sigma_{ij} = -p\delta_{ij} + \tau_{ij}$$

$$\left(1+\lambda_1\frac{d}{dt}\right)\tau_{ij} = 2\mu\left(1+\lambda_2\frac{d}{dt}\right)e_{ij} \qquad (1.1)$$

where, σ_{ij} is stress tesnor, p hydrostatic pressure, δ_{ij} kronecker delta, τ_{ij} viscous-stress tensor, λ_1 relaxation time, λ_2 retardation time, μ co-efficient of viscosity, e_{ij} is strain tensor and $\frac{d}{dt}$ is material derivative. $(\lambda_1 = 0, \lambda_2 = 0)$ characterizes Newtonian fluid, $(\lambda_1 = 0, \lambda_2 > 0)$ characterizes Second-grade fluid, $(\lambda_1 = 0, \lambda_2 < 0)$ characterizes Walters liquid and $(\lambda_1 \neq 0, \lambda_2 = 0)$ represents the Maxwell fluid model.

Rajagopal & Bhatnagar [3] have formulated the exact solutions for some simple flows of an Oldroyd-B fluid. An exact periodic solution of hydro-magnetic flow of an Oldroyd fluid in a channel has been obtained by Ray *et al*. [4]. Flow behaviour of Oldroyd fluid in presence / absence of magnetic field have been investigated by Hayat *et al*. [5, 6]. Hall effects on the unsteady hydromagnetic flows of an Oldroyd-B fluid have been analysed by Asghar *et al*. [7]. Ghosh [8] has studied Unsteady Hydro-Magnetic Flow of an Oldroyd Fluid through a Porous Channel with Oscillating Walls.

Gebhart and Pera [9] studied the problem of vertical convection flows resulting from combined buoyancy effects of thermal and mass diffusion. Heat and mass transfer flow problems of Newtonian or non-Newtonian fluid in presence of magnetic field with various physical properties have been investigated by Ericksen *et. al*. [10], Zueco and Ahmed [11], Chaudhary and Jain [12], Makinde [13], Choudhury and Dey [14, 15, 16] and Choudhury *et al*. [17].

The objective of the present study is to investigate the effects of relaxation and retardation parameter on free convective visco-elastic flow past an oscillating surface with heat and mass transfer in presence of transverse magnetic field.

2. MATHEMATICAL FORMULATION
An unsteady two dimensional free convective visco-elastic fluid flow characterized by Oldroyd model past an oscillating surface with heat and mass transfer has been investigated. Here x-axis is taken along the length of the plate and y-axis is perpendicular to it. The surface is oscillating with about a mean velocity U_0. A magnetic field of uniform strength B_0 is applied in the direction perpendicular to the fluid flow. Induced magnetic field is neglected as the magnetic Reynolds number is very small for weekly conducting system. Boussinesq approximation has been used. With the above assumptions, the equations of governing fluid motion are as follows:

Momentum equation:

$$\rho\left[\frac{\partial u'}{\partial t'}+\lambda_1\frac{\partial^2 u'}{\partial t'^2}\right] = \left[1+\lambda_1\frac{\partial}{\partial t'}\right]\left[g\beta(T'-T_\infty)+g\beta^*(C'-C_\infty)\right]+\mu\left[\frac{\partial^2 u'}{\partial y'^2}+\lambda_2\frac{\partial^3 u'}{\partial y'^2\partial t'}\right]-\left[1+\lambda_1\frac{\partial}{\partial t'}\right]\sigma B_0^2 u' \qquad (2.1)$$

Energy equation:

$$\frac{\partial T'}{\partial t'} = \frac{K}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} \qquad (2.2)$$

Energy equation for species concentration:

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} \qquad (2.3)$$

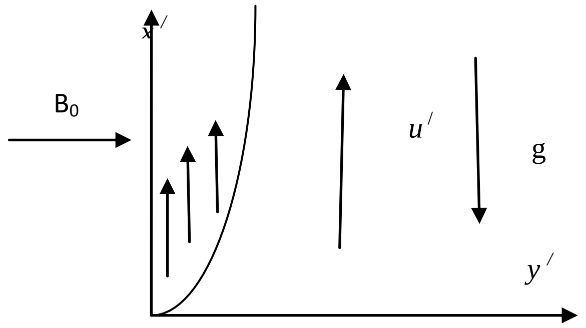

Figure 1: Geometry of the Problem

Here u' is the velocity along x'- axis, T' the temperature, T_∞ temperature of fluid far away from plate, C' concentration of fluid, C_∞ concentration of fluid away from the plate, g the acceleration due to gravity, ρ the density of the fluid, β and β^* are the coefficient of thermal and concentration expansion, D the thermal diffusivity, K the thermal conductivity, c_p the specific heat at constant pressure.

The corresponding boundary conditions are as follows:

$$y' = 0, u' = U_0 + \epsilon U_0 e^{i\omega' t'}, T' = T_\infty + \epsilon(T_w - T_\infty)e^{i\omega' t'}, C' = C_\infty + \epsilon(C_w - C_\infty)e^{i\omega' t'}$$

$$y' \to \infty, u' \to 0, T' \to T_\infty, C' \to C_\infty \quad (2.4)$$

Here T_w and C_w are temperature and concentration of fluid at the wall respectively

3. METHOD OF SOLUTION

To make the equations dimensionless, following non-dimensional quantities have been used into the equations (2.1) to (2.3),

$$y = \frac{U_0 y'}{\nu}, u = \frac{u'}{U_0}, t = \frac{t' U_0^2}{\nu}, \theta = \frac{T' - T_\infty}{T_w - T_\infty}, \phi = \frac{C' - C_\infty}{C_w - C_\infty}, a = \lambda_1 \frac{U_0^2}{\nu}, b = \lambda_2 \frac{U_0^2}{\nu}, M = \frac{\sigma B_0^2 \nu}{\rho U_0^2}$$

$$G_r = \frac{g\beta(T_w - T_\infty)}{\rho U_0^3}, G_m = \frac{g\beta^*(C_w - C_\infty)}{\rho U_0^3}, Sc = \frac{\nu}{D}, Pr = \frac{\mu c_p}{K} \quad (3.1)$$

Following set of dimensionless equations are obtained,

$$\frac{\partial u}{\partial t} + a\frac{\partial^2 u}{\partial t^2} = \left[1 + a\frac{\partial}{\partial t}\right][G_r\theta + G_m\phi] + \frac{\partial^2 u}{\partial y^2} + b\frac{\partial^3 u}{\partial y^2 \partial t} - \left[1 + a\frac{\partial}{\partial t}\right]Mu \quad (3.2)$$

$$Pr\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} \quad (3.3)$$

$$Sc\frac{\partial \phi}{\partial t} = \frac{\partial^2 \phi}{\partial y^2} \quad (3.4)$$

The relevant boundary conditions for solving the equations (3.2) and (3.3) are as follows:

$$y = 0, u = 1 + \epsilon e^{i\omega t}, \theta = \epsilon e^{i\omega t}, \phi = \epsilon e^{i\omega t} \ \& \ y \to \infty, u \to 0, \theta \to 0, \phi \to 0 \quad (3.5)$$

To solve the equations (3.2) – (3.4), perturbation technique has been adopted, where velocity, temperature and concentration in the neighbourhood of the surface is assumed as

$$u = u_0 + \varepsilon e^{i\omega t} u_1 + o(\varepsilon^2), \theta = \theta_0 + \varepsilon e^{i\omega t}\theta_1 + o(\varepsilon^2), \phi = \phi_0 + \varepsilon e^{i\omega t}\phi_1 + o(\varepsilon^2) \quad (3.6)$$

Using (3.6) in the above equations (3.2)- (3.4), and equating the co-efficient of ε, the zeroth order and first order equations are given as follows:

$$\theta_0'' = 0 \quad (3.7)$$

$$\phi_0'' = 0 \quad (3.8)$$

$$\theta_1'' + i\omega Pr\theta_1 = 0 \quad (3.9)$$

$$\phi_1'' + i\omega Sc\phi_1 = 0 \quad (3.10)$$

$$u_0'' - Mu_0 = -G_r\theta_0 - G_m\phi_0 \quad (3.11)$$

$$u_1''(1 + ib\omega) - u_1(i\omega - a\omega^2 + aMi\omega + M) = G_r(1 + ia\omega)\theta_1 + G_m(1 + ia\omega)\phi_1 \quad (3.12)$$

The relevant boundary conditions are:

$$y = 0, u_0 = 1, u_1 = 1, \theta_0 = 0, \theta_1 = 1, \phi_0 = 0, \phi_1 = 1$$
$$y \to \infty, u_0 \to 0, u_1 \to 0, \theta_0 \to 0, \theta_1 \to 0, \phi_0 \to 0,$$
$$\phi_1 \to 0 \quad (3.13)$$

4. RESULTS AND DISCUSSIONS

Solving the above equations, the velocity profile in the neighbourhood of the plate is

$$u = e^{-\sqrt{M}y} + \epsilon\cos(\omega t + \alpha) + i\epsilon\sin(\omega t + \alpha) \quad (4.1)$$

The shearing stress is represented by the first order differential equation,

$$\left(1 + a\frac{\partial}{\partial t}\right)\tau = \left(1 + b\frac{\partial}{\partial t}\right)\left(\frac{\partial u}{\partial y}\right) \quad (4.2)$$

where, τ is the dimensionless shearing stress and is given by

$$\tau = \frac{\tau'}{\rho u_0^2}$$

Solution of the differential equation (4.2) subject to the condition $\tau = 0$ at $y = 0$ is given by

$$\tau = 1 + \frac{\epsilon}{1 - a^2\omega^2}[\sin(\beta + \omega t) - i\cos(\beta + \omega t)] \quad (4.3)$$

The stream function of the governing fluid motion is obtained as

$$\psi = \frac{e^{-\sqrt{M}}}{-\sqrt{M}} + \epsilon[Xcos(\omega t) + Ysin(\omega t)] \quad (4.4)$$

Results are calculated for an arbitrary set of values of flow parameters present in the solution. The graphs of Velocity profiles are drawn against y for Hartmann number (M) (figure 2), relaxation parameter (figure 3) and retardation parameter (figure 4). From the figures, it is seen that, as it moves away from the surface, velocity is decreasing steadily from a fixed value at the surface. Application of magnetic field along the transverse direction generates Lorentz force and it decelerates the fluid motion governed by Oldroyd fluid model.

Two rheological parameters a and b characterize the relaxation and retardation parameters. Increase of relaxation ceases the stickiness of the system and effect of friction will be lesser, so mechanical energy will be maintained and as a result speed of the governing fluid motion increases (figure 3). On the other hand, during the growth of retardation parameter, effect of friction will be higher and as a consequence a decelerating trend is noticed in fluid motion (figure 4).

Shearing stress at the plate is drawn against time for various values other flow parameters involved in the solution.

It is seen that (fig 5) in the interval [0, 0.5] the increase of relaxation parameter subdues the magnitude of shearing stress. Physically it can be interpreted as, the growth of relaxation reduces the power of friction and as a result of that shearing stress decreases. But an opposite pattern is noticed in the interval [0.5, 1.1]. This periodical nature of shearing stress is observed as time increases.

Fig -6 represent the nature of shearing stress against time for various values of retardation parameter. Here also it is noticed the same periodic nature of shearing stress. The growth of retardation parameter makes the fluid thicker and as a result the shearing stress at the plate increases. This phenomenon is noticed in [0, 0.5], [1.1, 1.7] etc. but opposite nature is noticed in [0.5, 1.1] etc so on.

In this paper the figures of stream functions have been drawn against time t for various values of relaxation parameter and retardation parameter along with other flow parameters. The figures show that the stream function varies periodically with time. The growth of the relaxation parameter subdues the magnitude of the stream function in the intervals [0, 0.4], [1.1, 1.7] etc. but an opposite pattern is noticed in the intervals [0.4, 1.1], [1.7, 2] etc. The growth of the retardation parameter enhances the magnitude of the stream function in the intervals [0, 0.4], [1.1, 1.7] etc. but an opposite pattern is noticed in the interval [0.4, 1.1], [1.7, 2] etc so on.

5. CONCLUSIONS

Free convective hydromagnetic visco-elastic fluid flow characterized by Oldroyd model in presence of heat and mass transfer past an oscillating surface has been investigated. Some of the important are concluded as follows:

1. When relaxation parameter increases velocity of the fluid increases.
2. When retardation parameter increases velocity of the fluid decreases.
3. In the time period 0-0.5 and 1.1-1.7 shearing stress decreases with the increase in relaxation parameter and increases with the increases in retardation parameter.
4. In the time period 0.5-1.1 and 1.7-2 shearing stress increases with the increase in relaxation parameter and decreases with the increases in retardation parameter.
5. The stream function gives the flow pattern of the fluid flow.

Also, it can be concluded that the work may be extended in future by considering the effects of radiation, Joule heating and Soret Dufour effects in the heat and mass transfer problems.

6. GRAPHS

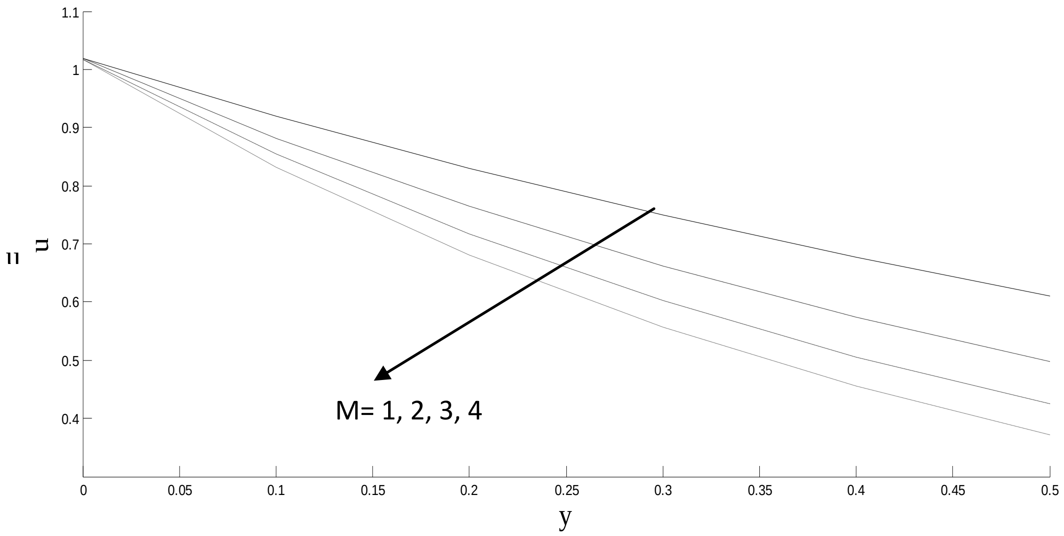

Figure 2: Velocity u against displacement y for a=0.1,b=0.1,Pr =3, Sc=1, ω=5,t=0.1,Gr=7,Gm=3,ε=0.01

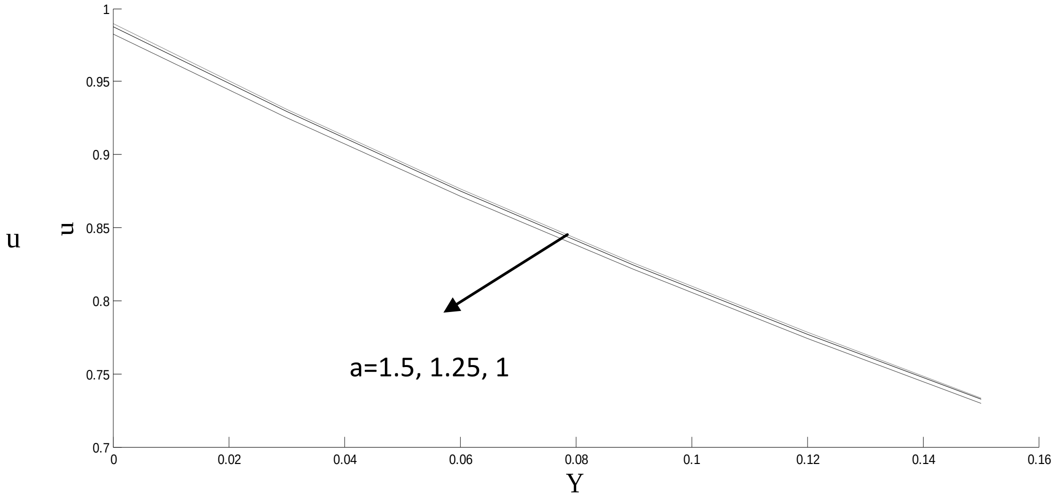

Figure 3: Velocity u against displacement y for b=0.1,M=4, Pr =3, Sc=1, ω=5, t=0.1, Gr=7,Gm=3, ε=0.01

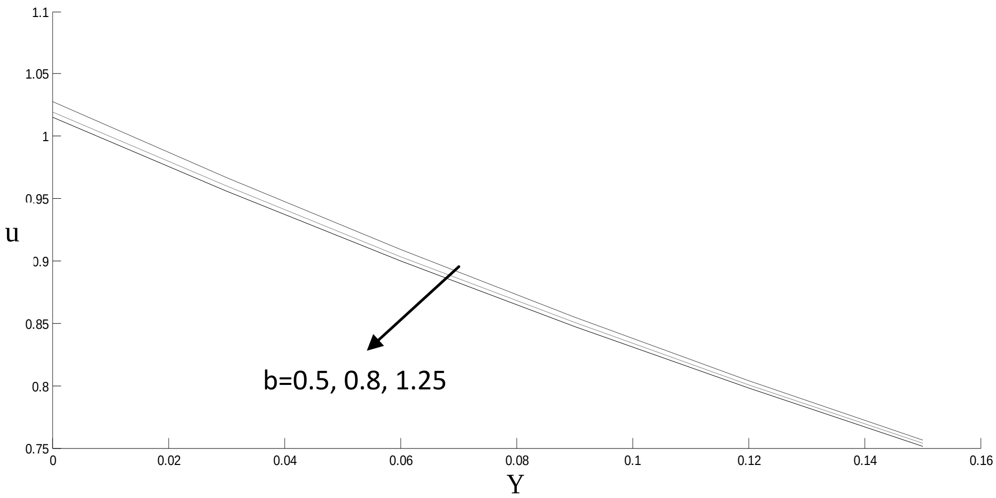

Figure 4: Velocity u against displacement y for a=1,M=4, Pr =3, Sc=1, ω=5, t=0.1, Gr=7,Gm=3, ε=0.01

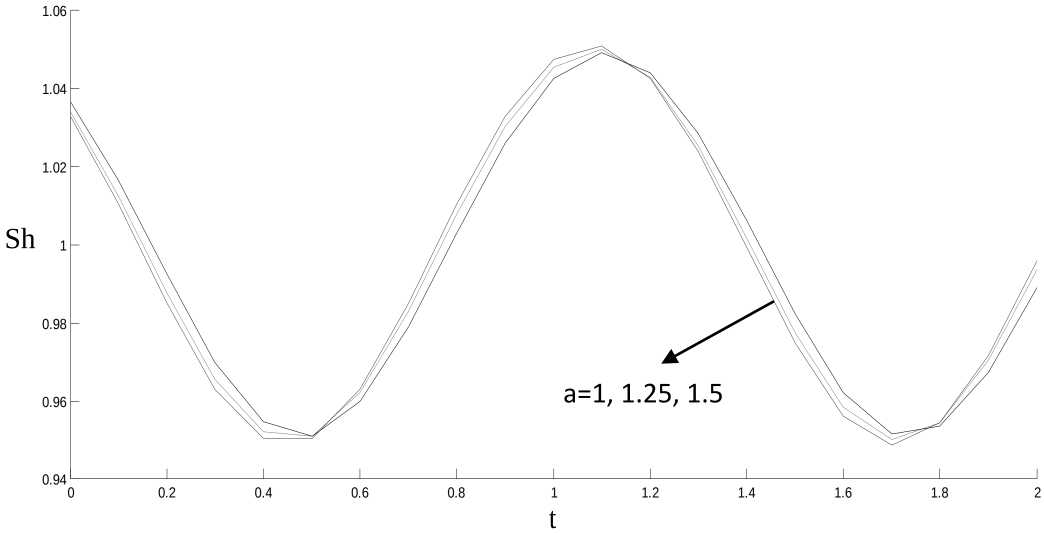

Figure 5: Shearing stress against time t for b=1, M=4, Pr =3, Sc=1, ω=5, Gr=7, Gm=3, ε=0.01

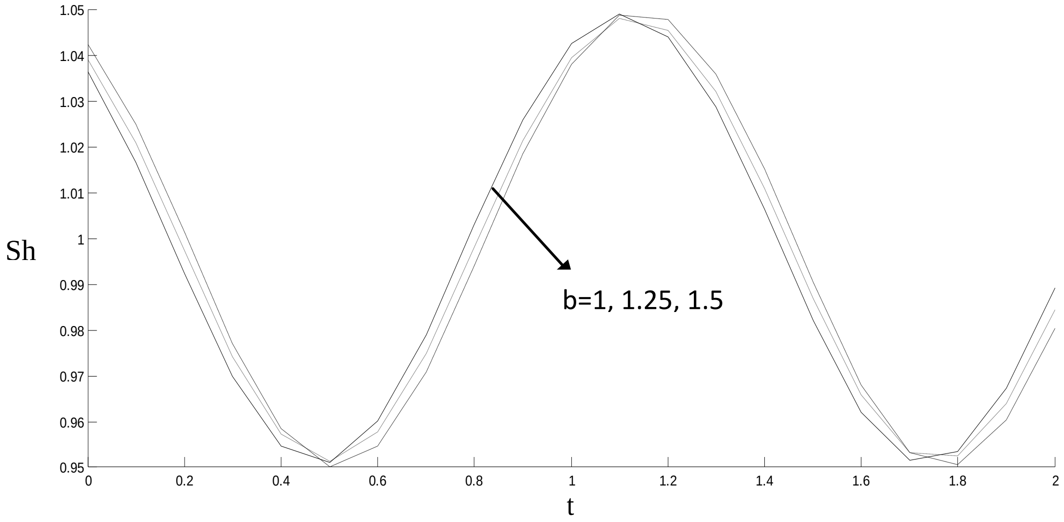

Figure 6: Shearing stress against time t for a=1, M=4, Pr =3, Sc=1, ω=5, Gr=7, Gm=3, ε=0.01

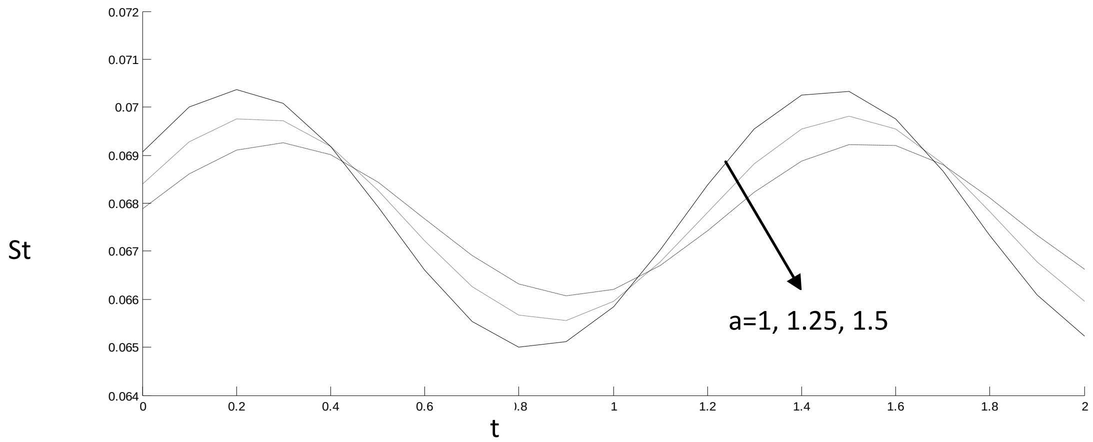

Figure 7: Stream function against t for b=1, M=4, Pr =3, Sc=1, ω=5,y=0.1 Gr=7, Gm=3, ε=0.01

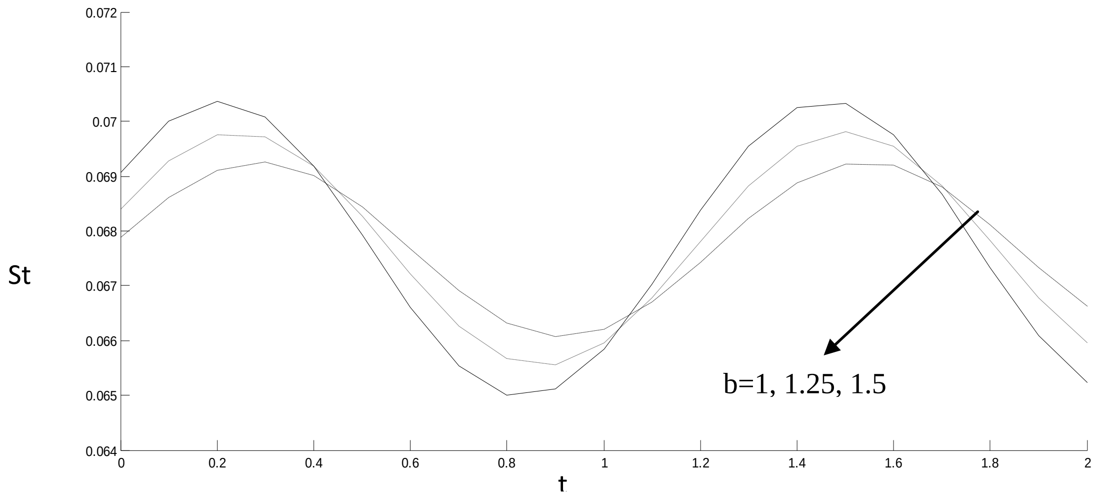

Figure 8: Stream function against t for a=1, M=4, Pr =3, Sc=1, ω=5,y=0.1 Gr=7, Gm=3, ε=0.01

7. REFERENCES

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8. APPENDIX CONSTANTS

$$\cos\alpha = C_{12a}e^{-A_{13}y}\cos A_{14}y + C_{12b}e^{-A_{13}y}\sin A_{14}y - A_9G_re^{-A_7y}\cos A_7y - A_9G_me^{-A_8y}\cos A_8y - A_{10}G_re^{-A_7y}\sin A_7y - A_{10}G_me^{-A_8y}\sin A_8y$$

$$\sin\alpha = -C_{12a}e^{-A_{13}y}\sin A_{14}y - C_{12b}e^{-A_{13}y}\cos A_{14}y + A_9G_re^{-A_7y}A_7y + A_9G_me^{-A_8y}\sin A_8y - A_{10}G_re^{-A_7y}\cos A_7y - A_{10}G_me^{-A_8y}\cos A_8y$$

$$\cos\beta = a\omega A_{15} - b\omega A_{15} - A_{16} - ab\omega^2 A_{16}\ ; \qquad \sin\beta = A_{15} + ab\omega^2 A_{15} + a\omega A_{16} - b\omega A_{16}$$

$$A_1 = M - a\omega^2\ ;\ A_2 = \omega + a\omega\ ;\ MA_3 = \frac{A_1 + A_2 b\omega}{1 + b^2\omega^2}\ ;\ A_4 = \frac{A_2 - A_1 b\omega}{1 + b^2\omega^2}$$

$$A_5 = \frac{1 + ab\omega^2}{1 + b^2\omega^2}\ ;\ A_6 = \frac{a\omega - b\omega}{1 + b^2\omega^2}\ ;\ A_7 = \sqrt{\frac{Pr\omega}{2}}\ ;\ A_8 = \sqrt{\frac{Sc\omega}{2}}$$

$$A_9 = \frac{A_3A_5 + A_4A_6 - 2A_6{A_7}^2}{{A_3}^2 + (A_4 - 2{A_7}^2)^2}\ ;\ A_{10} = \frac{A_3A_6 - A_5A_4 + 2A_5{A_7}^2}{{A_3}^2 + (A_4 - 2{A_7}^2)^2}$$

$$A_{11} = \frac{A_3A_5 + A_4A_6 - 2A_6{A_8}^2}{{A_3}^2 + (A_4 - 2{A_8}^2)^2}\ ;\ A_{12} = \frac{A_3A_6 - A_5A_4 + 2A_5{A_8}^2}{{A_3}^2 + (A_4 - 2{A_8}^2)^2}$$

$$A_{13} = \sqrt{1 + \sqrt{1 + {A_4}^2}}\ ;\ A_{14} = \frac{A_4}{\sqrt{2\left(1 + \sqrt{1 + {A_4}^2}\right)}}$$

$$A_{15} = -A_{13}C_{12a} + A_{14}C_{12b} + A_7A_9G_r - A_8A_9G_m - A_7A_{10}G_r - A_8A_{10}G_m$$

$$A_{16} = -A_{13}C_{12b} + A_{14}C_{12a} - A_7A_9G_r - A_8A_9G_m - A_7A_{10}G_r - A_8A_{10}G_m$$

$$C_1 = 0, C_2 = 0, C_3 = 0, C_4 = 1, C_5 = 0, C_6 = 0, C_7 = 0, C_8 = 1, C_9 = 0, C_{10} = 1, C_{11} = 0$$

$$C_{12a} = 1 + A_9G_r + A_9G_m\ ;\ C_{12b} = A_{10}G_r + A_{10}G_m\ ;\ C_{12} = C_{12a} + iC_{12b}$$

$$X = -C_{12a}\frac{e^{-A_{13}y}}{A_{13}} - C_{12b}\frac{e^{-A_{13}y}(A_{14}\cos(A_{14}y) + A_{13}\sin(A_{14}y))}{{A_{13}}^2 + {A_{14}}^2} - A_9G_r\frac{e^{-A_7y}(\sin(A_7y) - \cos(A_7y))}{2A_7} - A_9G_m\frac{e^{-A_8y}(\sin(A_8y) - \cos(A_8y))}{2A_8} + A_{10}G_r\frac{e^{-A_7y}(\cos(A_7y) + \sin(A_7y))}{2A_7} + A_{10}G_m\frac{e^{-A_8y}(\cos(A_8y) + \sin(A_8y))}{2A_8}$$

$$Y = C_{12a}\frac{e^{-A_{13}y}(A_{14}\cos(A_{14}y) + A_{13}\sin(A_{14}y))}{{A_{13}}^2 + {A_{14}}^2} + C_{12b}\frac{e^{-A_{13}y}(A_{14}\sin(A_{14}y) - A_{13}\cos(A_{14}y))}{{A_{13}}^2 + {A_{14}}^2} + A_9G_r\frac{e^{-A_7y}(\cos(A_7y) + \sin(A_7y))}{2A_7} + A_9G_m\frac{e^{-A_8y}(\cos(A_8y) + \sin(A_8y))}{2A_8} + A_{10}G_r\frac{e^{-A_7y}(\sin(A_7y) - \cos(A_7y))}{2A_7} + A_{10}G_m\frac{e^{-A_8y}(\sin(A_8y) - \cos(A_8y))}{2A_8}$$

NOMENCLATURE

σ_{ij} Stress tesnor.

p Hydrostatic pressure.

δ_{ij} Kronecker delta.

τ_{ij} Viscous-Stress tensor.

λ_1 Relaxation time.

λ_2 Retardation time.

μ Co-efficient of viscosity.

e_{ij} Strain tensor.

$\frac{d}{dt}$ Material derivative.

B_0 Uniform strength.

U_0 Mean velocity.

T' Temperature.

T_∞ Temperature of fluid far away from plate.

C' Concentration of fluid.

C_∞ Concentration of fluid away from the plate.

g Acceleration due to gravity.

ρ Density of the fluid.

β Coefficient of thermal expansion.

β^* Coefficient of concentration expansion.

D Thermal diffusivity.

K Thermal conductivity.

c_p Specific heat at constant pressure.

M Hartman number.

G_r Grashoff number for heat transfer.

G_m Grashoff number for mass transfer.

Sc Schmidt number.

Pr Prandtl number.