Soliton Soltuions of Nonlinear Evolutions Equation by using the Extended Exp $(-\varphi(\xi))$ Expansion Method

Mostafa M. A. Khater
Mansoura University
Department of Mathematics,
Faculty of Science,
35516 Mansoura, Egypt

Emad H. M. Zahran
Benha University
Department of mathematical
and physical engineering,
College of engineering Shubra

ABSTRACT

In this research, we employ the extended $\exp(-\varphi(\xi))$expansion method for the first time to obtain the exact and solitary wave solutions of the (3+1)-Dimensional Yu-Toda-Sasa-Fukuyama Equation. We obtain the wide range of exact and solitary wave solutions of distinct physical structure.

Keywords

Extended $\exp(-\varphi(\xi))$-expansion method, The (3+1)-Dimensional Yu-Toda-Sasa-Fukuyama equation, Traveling wave solutions, Solitary wave solutions.

1. INTRODUCTION

Most of complicated phenomena in physics and different branches of applied science (plasma, fluid mechanics, optical fibers, solid state physics, chemical kinetics and geochemistry phenomena...etc.) can be represented by nonlinear partial differential equations (NLPDEs.) which are widely used to describe them.The nonlinear partial differential equations of mathematical physics are major subjects in physical science [1]. Investigation the exact solutions of NLPDEs. help us to understand the nonlinear physical phenomena. Recently, many solutions methods have been introduced to get the exact solutions of NLPDEs. For example, extended Jacobian Elliptic Function Expansion Method [2], the modified simple equation method[3], the tanh method [4], extended tanh - method [5]-[7], sine - cosine method [8]-[10], homogeneous balance method [11, 12],F-expansion method [13]-[15], exp-function method [16, 17], trigonometric function series method [18], $(\frac{G'}{G})-$ expansion method [19]-[22], Jacobi elliptic function method [23]-[26], The $\exp(-\varphi(\xi))$-expansion method[27]-[29] and so on.
In this article we propose a new method to get the exact traveling wave solutions and the solitary wave solutions of the (3+1)-Dimensional Yu-Toda-Sasa-Fukuyama Equatio which is a widely used model for investigation the dynamics of solitons and nonlinear wave in areas such as fluid dynamics, plasma physics and weakly dispersive media by using the extended $\exp(-\varphi(\xi))$-expansion method.
The rest of this paper is organized as follows: In Section 2, we give the description of The extended $\exp(-\varphi(\xi))$-expansion method In Section 3, we use this method to find the exact solutions of the nonlinear evolution equations pointed out above. In Section 4, conclusions are given.

2. DESCRIPTION OF METHOD

Consider the following nonlinear evolution equation in the polynomial form

$$F(u, u_t, u_x, u_{tt}, u_{xx}, ...) = 0, \tag{1}$$

in which the highest order derivatives and nonlinear terms are involved. In the following,we give the main steps of this method
Step 1. We use the wave transformation

$$u(x, y, z, t) = u(\xi), \qquad \xi = x + y + z - \omega t, \tag{2}$$

where c is a positive constant, to reduce Eq. (1)to the following ODE:

$$P(u, u', u'', u''', ...) = 0, \tag{3}$$

where P is a polynomial in $u(\xi)$ and its total derivatives,while $' = \frac{d}{d\xi}{}'$.
Step 2. Suppose that the solution of ODE (3) can be expressed by a polynomial in $exp(-\varphi(\xi))$ as follows

$$u(\xi) = \sum_{i=-m}^{m} a_i \left(exp\left(-\varphi\left(\xi\right)\right)\right)^i, \tag{4}$$

Since a_m $(0 \leq m \leq n)$ are constants to be determined, such that $a_m or a_{-m} \neq 0$.
the positive integer m can be determined by considering the homogenous balance between the highest order derivatives and nonlinear terms appearing in Eq. (3). Moreover precisely, we define the degree of $u\left(\xi\right)$ as $D\left(u\left(\xi\right)\right) = m$, which gives rise to degree of other expression as follows:

$$D\left(\frac{d^q u}{d\xi^q}\right) = n + q,\ D\left(u^p\left(\frac{d^q u}{d\xi^q}\right)^s\right) = np + s\left(n + q\right).$$

Therefore, we can find the value of m in Eq. (3), where $\varphi = \varphi(\xi)$ satisfies the ODE in the form

$$\varphi'(\xi) = exp\left(-\varphi\left(\xi\right)\right) + \mu exp\left(\varphi\left(\xi\right)\right) + \lambda, \tag{5}$$

the solutions of ODE (3) are
when $\lambda^2 - 4\mu > 0, \mu \neq 0$,

$$\varphi(\xi) = ln \left(\frac{-\sqrt{\lambda^2 - 4\mu}\, tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\,(\xi + C_1)\right) - \lambda}{2\mu} \right), \tag{6}$$

and

$$\varphi(\xi) = ln \left(\frac{-\sqrt{\lambda^2 - 4\mu}\, coth\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\,(\xi + C_1)\right) - \lambda}{2\mu} \right), \tag{7}$$

when $\lambda^2 - 4\mu > 0, \mu = 0$,

$$\varphi(\xi) = -ln \left(\frac{\lambda}{exp\left(\lambda\left(\xi + C_1\right)\right) - 1} \right), \tag{8}$$

when $\lambda^2 - 4\mu = 0, \mu \neq 0, \lambda \neq 0$,

$$\varphi(\xi) = ln \left(-\frac{2\left(\lambda\left(\xi + C_1\right) + 2\right)}{\lambda^2\left(\xi + C_1\right)} \right), \tag{9}$$

when $\lambda^2 - 4\mu = 0, \mu = 0, \lambda = 0$,

$$\varphi(\xi) = ln\left(\xi + C_1\right), \tag{10}$$

when $\lambda^2 - 4\mu < 0$,

$$\varphi(\xi) = ln \left(\frac{\sqrt{4\mu - \lambda^2}\, tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\,(\xi + C_1)\right) - \lambda}{2\mu} \right), \tag{11}$$

and

$$\varphi(\xi) = ln \left(\frac{\sqrt{4\mu - \lambda^2}\, cot\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\,(\xi + C_1)\right) - \lambda}{2\mu} \right), \tag{12}$$

where $a_m, \ldots\ldots, \lambda, \mu$ are constants to be determined later,
Step 3. After we determine the index parameter m, we substitute Eq. (4) along Eq. (5) into Eq. (3) and collecting all the terms of the same power $exp\left(-m\varphi(\xi)\right)$, $m = 0, 1, 2, 3, ...$ and equating them to zero, we obtain a system of algebraic equations, which can be solved by Maple or Mathematica to get the values of a_i.
Step 4. substituting these values and the solutions of Eq. (5) into Eq. (3) we obtain the exact solutions of Eq. (3).

3. THE (3+1)-DIMENSIONAL YU-TODA-SASA-FUKUYAMA EQUATION

Let us consider the (3+1)-Dimensional Yu-Toda-Sasa-Fukuyama Equation [30, 31]

$$-4u_{xt} + u_{xxxz} + 4u_x u_{xz} + 2u_z u_{xx} + 3u_{yy} = 0. \tag{13}$$

This equation is a potential-type counterpart of the (3+1)-dimensional nonlinear equation

$$\left(-4v_t + \phi(v)v_z\right)_x + 3v_{yy} = 0, \phi = \partial_x^2 + 4v + 2v_x + 2v_x\partial_x^{-1}, \tag{14}$$

introduced by Zhang et al [30] and Hu et al [31], while making the (3+1)-dimensional generalization from (2+1)-dimensional Calogero-Bogoyavlenkii-Schiff equation [32]:

$$-4v_t + \phi(v)v_z = 0, \phi = \partial_x^2 + 4v + 2v_x\partial_x^{-1}, \tag{15}$$

as did for the KP equation from the KdV equation. Taking $v = u_x$ Eq.y2 transform into the potential-YTSF Eq. (13). By using the transformation (2) we get

$$u''' + 3u'^2 + (4\omega + 3)u' = 0. \tag{16}$$

Balancing U''' and u'^2 we get $m = 1$. So that we assume the solution be in the form

$$u(\xi) = a_{-1}exp(\varphi(\xi)) + a_0 + a_1 exp(-\varphi(\xi)). \tag{17}$$

Substituting Eq. (17) and its derivatives into Eq. (16) and collecting all term of the same power of $\exp(-i\varphi(\xi))$ where $(i = 0, \pm1, \pm2, \pm3, ...)$ and equating them to zero, we obtain system of algebraic equation by solving them by using Maple 16 we get

Case 1.

$$\omega = \mu - \frac{1}{4}\lambda^2 - \frac{3}{4}, a_0 = a_{-1} = 0, a_1 = 2.$$

The solutions of Eq. (16) according these values
When $\lambda^2 - 4\mu > 0, \mu \neq 0$,

$$u(\xi) = \frac{-4\mu}{\sqrt{\lambda^2 - 4\mu}\, tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\,(\xi + C_1)\right) + \lambda}, \tag{18}$$

and

$$u(\xi) = \frac{-4\mu}{\sqrt{\lambda^2 - 4\mu}\, coth\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\,(\xi + C_1)\right) + \lambda}. \tag{19}$$

When $\lambda^2 - 4\mu > 0, \mu = 0$,

$$u(\xi) = \frac{2\lambda}{exp\left(\lambda\left(\xi + C_1\right)\right) - 1}. \tag{20}$$

When $\lambda^2 - 4\mu = 0, \mu \neq 0, \lambda \neq 0$,

$$u(\xi) = \frac{-4\left(\lambda\left(\xi + C_1\right) + 2\right)}{\lambda^2\left(\xi + C_1\right)} \tag{21}$$

When $\lambda^2 - 4\mu = 0, \mu = 0, \lambda = 0$,

$$u(\xi) = \frac{2}{\xi + C_1}. \tag{22}$$

when $\lambda^2 - 4\mu < 0$,

$$u(\xi) = \frac{4\mu}{\sqrt{\lambda^2 - 4\mu}\, tan\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\,(\xi + C_1)\right) - \lambda}, \tag{23}$$

$$u(\xi) = \frac{4\mu}{\sqrt{\lambda^2 - 4\mu}\, cot\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\,(\xi + C_1)\right) - \lambda}, \tag{24}$$

Case 2.

$$\mu = \frac{3}{4} + \omega, \lambda = a_0 = a_{-1} = 0, a_1 = 2.$$

The solutions of Eq. (16) according these values
When $\lambda^2 - 4\mu > 0, \mu \neq 0, \mu < 0$

$$u(\xi) = \frac{-4\mu}{\sqrt{\lambda^2 - 4\mu}\, tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\xi + C_1\right)\right) + \lambda}, \tag{25}$$

and

$$u(\xi) = \frac{-4\mu}{\sqrt{\lambda^2 - 4\mu}\, coth\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\xi + C_1\right)\right) + \lambda}. \tag{26}$$

when $\lambda^2 - 4\mu < 0, \mu > 0$

$$u(\xi) = \frac{4\mu}{\sqrt{\lambda^2 - 4\mu}\, tan\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\xi + C_1\right)\right) - \lambda}, \tag{27}$$

$$u(\xi) = \frac{4\mu}{\sqrt{\lambda^2 - 4\mu}\, cot\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\xi + C_1\right)\right) - \lambda}, \tag{28}$$

Case 3.

$$\mu = \frac{3}{4} + \frac{1}{4}\lambda^2 + \omega, a_0 = a_1 = 0, a_{-1} = \frac{-3}{2} - \frac{\lambda^2}{2} - 2\,\omega.$$

The solutions of Eq. (16) according these values
When $\lambda^2 - 4\mu > 0, \mu \neq 0,$

$$u(\xi) = \left(\frac{-3}{2} - \frac{\lambda^2}{2} - 2\,\omega\right) A, \tag{29}$$

and

$$u(\xi) = \left(\frac{-3}{2} - \frac{\lambda^2}{2} - 2\,\omega\right) B, \tag{30}$$

When $\lambda^2 - 4\mu > 0, \mu = 0,$

$$u(\xi) = \left(\frac{-3}{2} - \frac{\lambda^2}{2} - 2\,\omega\right) \frac{exp\left(\lambda\left(\xi + C_1\right)\right) - 1}{\lambda}. \tag{31}$$

When $\lambda^2 - 4\mu = 0, \mu \neq 0, \lambda \neq 0,$

$$u(\xi) = \left(\frac{-3}{2} - \frac{\lambda^2}{2} - 2\,\omega\right) \frac{\lambda^2\left(\xi + C_1\right)}{2\left(\lambda\left(\xi + C_1\right) + 2\right)}. \tag{32}$$

When $\lambda^2 - 4\mu = 0, \mu = 0, \lambda = 0,$

$$u(\xi) = \left(\frac{-3}{2} - \frac{\lambda^2}{2} - 2\,\omega\right)\left(\xi + C_1\right). \tag{33}$$

when $\lambda^2 - 4\mu < 0,$

$$u(\xi) = \left(\frac{-3}{2} - \frac{\lambda^2}{2} - 2\,\omega\right) C, \tag{34}$$

$$u(\xi) = \left(\frac{-3}{2} - \frac{\lambda^2}{2} - 2\,\omega\right) D. \tag{35}$$

Case 4.

$$\omega = 4\,\mu - \frac{3}{4}, a_0 = \lambda = 0, a_{-1} = -2\,\mu, a_1 = 2.$$

The solutions of Eq. (16) according these values
When $\lambda^2 - 4\mu > 0, \mu \neq 0, \mu < 0$

$$u(\xi) = \frac{A}{-2\mu} - \frac{+4\mu}{\sqrt{\lambda^2 - 4\mu}\, tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\xi + C_1\right)\right) + \lambda}, \tag{36}$$

and

$$u(\xi) = \frac{B}{-2\mu} - \frac{4\mu}{\sqrt{\lambda^2 - 4\mu}\, coth\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\xi + C_1\right)\right) + \lambda}, \tag{37}$$

when $\lambda^2 - 4\mu < 0, \mu > 0$

$$u(\xi) = \frac{C}{-2\mu} - \frac{4\mu}{\sqrt{\lambda^2 - 4\mu}\, tan\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\xi + C_1\right)\right) + \lambda}, \tag{38}$$

$$u(\xi) = \frac{D}{-2\mu} - \frac{4\mu}{\sqrt{\lambda^2 - 4\mu}\, cot\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\xi + C_1\right)\right) + \lambda}, \tag{39}$$

where

$$A = \left(\frac{-\sqrt{4\mu - \lambda^2}\, tanh\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\ \left(\xi + C_1\right)\right) - \lambda}{2\mu}\right) \tag{40}$$

$$B = \left(\frac{-\sqrt{4\mu - \lambda^2}\, coth\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\ \left(\xi + C_1\right)\right) - \lambda}{2\mu}\right) \tag{41}$$

$$C = \left(\frac{-\sqrt{4\mu - \lambda^2}\, tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\ \left(\xi + C_1\right)\right) - \lambda}{2\mu}\right) \tag{42}$$

$$D = \left(\frac{-\sqrt{4\mu - \lambda^2}\, cot\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\ \left(\xi + C_1\right)\right) - \lambda}{2\mu}\right) \tag{43}$$

4. CONCLUSION

In this research, we introduce a new technique namely the extended $\exp(-\varphi(\xi))$-expansion method for the first time to finding the exact and solitary wave solutions the (3+1)-Dimensional Yu-Toda-Sasa-Fukuyama equation which play an important role in mathematical physics. From the suggested method we found that it introduce 24 solitary wave solutions according to 4 cases and these solutions give a wide range and more accurate than that obtained by other methods such as tanh method, $\left(\frac{G'}{G}\right)$-expansion method, the $\exp(-\varphi(\xi))$-expansion method and modified simple equation method. These solutions give more interpretation for the physical properties to the equation studied.

5. COMPETING INTERESTS

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors. The author did not have any competing interests in this research.

6. AUTHOR'S CONTRIBUTIONS

All parts contained in the research carried out by the researcher through hard work and a review of the various references and contributions in the field of mathematics and the physical Applied

7. ACKNOWLEDGMENT

The author thanks the referees for their suggestions and comments.

8. REFERENCES

[1] M. J. Ablowitz, H. Segur, Solitions and Inverse Scattering Transform, SIAM, Philadelphia 1981.

[2] Emad H. M. Zahran and Mostafa M.A. Khater, Exact Traveling Wave Solutions for the System of Shallow Water Wave Equations and Modified Liouville Equation Using Extended Jacobian Elliptic Function Expansion Method. American Journal of Computational Mathematics $(AJCM)$ Vol.4 No.5 2014.

[3] Emad H. M. Zahran and Mostafa M.A. Khater, The modified simple equation method and its applications for solving some nonlinear evolutions equations in mathematical physics. J$\ddot{o}$kull journal- Vol. 64. Issue 5 - May 2014.

[4] A. M. Wazwaz, The tanh method for travelling wave solutions of nonlinear equations, Appl. Math. Comput., 154 (2004) 714-723.

[5] Mostafa M. A. Khater, Emad H. M. Zahran, New solitary wave solution of the generalized Hirota-Satsuma couple KdV system, International Journal of Scientific & Engineering Research, Volume 6, Issue 8, August (2015).

[6] Mahmoud A.E. Abdelrahman and Mostafa M.A. Khater, Traveling Solitary Wave Solutions for the Symmetric Regularized Long-Wave Equation, JOURNAL OF ADVANCES IN MATHEMATICS, Vol .11, No.8, 5520- 5528 (December 0 7 , 2015).

[7] Mostafa M.A. Khater and Emad H. M. Zahran, Modified extended tanh function method and its applications to the Bogoyavlenskii equation, Applied Mathematical Modelling, 40, 1769-1775 (2016).

[8] A. M. Wazwaz, Exact solutions to the double sinh-Gordon equation by the tanh method and a variable separated ODE. method, Comput. Math. Appl., 50 (2005) 1685-1696.

[9] A. M. Wazwaz, A sine-cosine method for handling nonlinear wave equations, Math. Comput. Modelling, 40 (2004) 499-508.

[10] C. Yan, A simple transformation for nonlinear waves, Phys. Lett. A 224 (1996) 77-84.

[11] E. Fan, H.Zhang, A note on the homogeneous balance method, Phys. Lett. A 246 (1998) 403-406.

[12] M. L. Wang, Exct solutions for a compound KdV-Burgers equation, Phys. Lett. A 213 (1996) 279-287.

[13] M. A. Abdou, The extended F-expansion method and its application for a class of nonlinear evolution equations, Chaos Solitons Fractals, 31 (2007) 95-104.

[14] Y. J. Ren, H. Q. Zhang, A generalized F-expansion method to find abundant families of Jacobi elliptic function solutions of the (2+1)-dimensional Nizhnik-Novikov-Veselov equation, Chaos Solitons Fractals, 27 (2006) 959-979.

[15] J. L. Zhang, M. L. Wang, Y. M. Wang, Z. D. Fang, The improved F-expansion method and its applications, Phys.Lett.A 350 (2006) 103-109.

[16] J. H. He, X. H. Wu, Exp-function method for nonlinear wave equations, Chaos Solitons Fractals 30 (2006) 700-708.

[17] H. Aminikhad, H. Moosaei, M. Hajipour, Exact solutions for nonlinear partial differential equations via Exp-function method, Numer. Methods Partial Differ. Equations, 26 (2009) 1427-1433.

[18] Z. Y. Zhang, New exact traveling wave solutions for the nonlinear Klein-Gordon equation, Turk. J. Phys., 32 (2008) 235-240.

[19] M. L. Wang, J. L. Zhang, X. Z. Li, The $(\frac{G'}{G})$- expansion method and travelling wave solutions of nonlinear evolutions equations in mathematical physics, Phys. Lett. A 372 (2008) 417-423.

[20] S. Zhang, J. L. Tong, W.Wang, A generalized $(\frac{G'}{G})$- expansion method for the mKdv equation with variable coefficients, Phys. Lett. A 372 (2008) 2254-2257.

[21] Mostafa M. A. Khater, On the New Solitary Wave Solution of the Generalized Hirota-Satsuma Couple KdV System, GJSFR-A Volume 15 Issue 4 Version 1.0 (2015).

[22] Emad H. M. Zahran and Mostafa M.A. Khater, Exact solutions to some nonlinear evolution equations by using $\left(\frac{G'}{G}\right)$-expansion method, J$\ddot{o}$kull journal- Vol. 64. Issue 5 - May 2014.

[23] Mahmoud A.E. Abdelrahman and Mostafa M.A. Khater, Traveling Wave Solutions For The Couple Boiti Leon-Pempinelli System By Using Extended Jacobian Elliptic Function Expansion Method, JOURNAL OF ADVANCES IN PHYSICS Vol. 11, No. 3, 3134-3138 (December 0 7 , 2015).

[24] Emad H. M. Zahran & Mostafa M. A. Khater, Extended Jacobian Elliptic Function Expansion Method and Its Applications in Biology. Applied Mathematics, 6, 1174- 1181 (2015).

[25] S. Liu, Z. Fu, S. Liu, Q.Zhao, Jacobi elliptic function expansion method and periodic wave solutions of nonlinear wave equations, Phys. Lett. A 289 (2001) 69-74.

[26] Mahmoud A.E. Abdelrahman and Mostafa M.A. Khater, the Exp $(\Phi(\xi))$ - Expansion Method and its Application for Solving Nonlinear Evolution Equations. International Journal of Science and Research (IJSR) ISSN (Online): 2319-7064, Volume 4 Issue 2, 2143-2146. February (2015).

[27] Mahmoud A.E. Abdelrahman, Emad H. M. Zahran and Mostafa M.A. Khater, Exact Traveling Wave Solutions for Power law and Kerr law non Linearity Using the Exp$(\Phi(\xi))$-expansion Method, Volume 14 Iss. 4 Version 1.0, 2014.

[28] Mahmoud A.E. Abdelrahman, Emad H. M. Zahran and Mostafa M.A. Khater, the exp $(\Phi(\xi))$ -Expansion Method and Its Application for Solving Nonlinear Evolution Equations. International Journal of Modern Nonlinear Theory and Application, 4, 37-47(2015).

[29] MA Khater, Mostafa. "Extended exp $(\Phi(\xi))$-Expansion method for Solving the Generalized Hirota-Satsuma Coupled KdV System." Global Journal of Science Frontier Research 15.7 (2015).

[30] Zhang, Sheng, and Hong-Qing Zhang. "A transformed rational function method for (3+ 1)-dimensional potential YuTodaSasaFukuyama equation." Pramana 76.4 (2011): 561-571.

[31] Hu, Yangjie, Hanlin Chen, and Zhengde Dai. ”New kink multi-soliton solutions for the (3+ 1)-dimensional potential-YuTodaSasaFukuyama equation.” Applied Mathematics and Computation 234 (2014): 548-556.

[32] S. J. Yu, K. Toda, N. Sasa and T. Fukuyama, N-soliton solutions to Bogoyavknskii-Schiff equation and a guest for the soliton solutions in (3 + 1)-dimensions, J. Phys. A: Math. And Gene. 31 (1998) 3337-3347.