A Fuzzy Technique for Solving Rough Interval Multiobjective Transportation Problem

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ABSTRACT
In this paper the rough interval multiobjective transportation problem (RIMOTP) is presented and its solution procedure is introduced. The concept of solving the interval multiobjective transportation problem is applied for solving RIMOTP. So, The rough interval in the objective function and the constrains, is represented by three different models and such models are solved by using fuzzy programming technique based on the right limit, the center and the half-width of each rough interval using possibly region. Numerical examples are provided to illustrate the solution procedure of three possible types of the original problem.

General Terms
Fuzzy Programming Technique, Transportation Problem, rough interval multiobjective transportation

Keywords
Rough interval, multiobjective programming, transportation problem, fuzzy programming technique, Pareto optimal solution

1. INTRODUCTION
The theory of rough sets is presented by Pawlak [12]. The rough programming is discussed by many authors [5, 9, and 15]. Abd El-Wahed [1, 7] presented a fuzzy programming approach to determine the optimal compromise solution of a multiobjective transportation problem. A fuzzy technique is used to solve the multiobjective transportation problems with interval cost [4]. Ammar et al [3] proposed a method to solve rough interval multiobjective transportation problem based on weighting method and separation method. Mohanty and Dash [8] presented the uncertainty distribution to solve rough transportation problem. El-Sisy [5] presented the duality of multiobjective rough convex programming problem under uncertainty. Xu and Tao [14] introduced a class of rough multiobjective programming problem. Bit et al [2] developed a procedure applying fuzzy programming technique for solving the multi-criteria decision making transportation problem.

In this paper, the concept of solving the conventional interval linear programming problem combined with the fuzzy technique is used to solve the rough interval multiobjective transportation problem. Such technique is used to deal with three different types of rough interval multiobjective transportation problem. The remaining of the paper unfolds as follows: In Section 2, rough interval definition is presented. In section 3, Multiobjective Transportation Problem is illustrated. Section 4, The Proposed Approach is devoted to numerical examples for the illustration. .

2. ROUGH INTERVAL
Definition 1: Let w denote a closed and bounded set of numbers. A rough interval $W^R = [w^L : w^U]$ is defined as an interval with known lower $w^L = [w^a,\ w^b]$ and upper bound $w^U = [w^c,\ w^d]$ but unknown distribution information for w. and $w^L \subseteq w^U$. When $w^L = w^U$; W^R becomes a conventional interval, i.e. $W^R = w^L = w^U$ [13].

Definition 2: Let $* \in \{+, -, \times, \div\}$ be a binary operation on rough interval W^R and Z^R when W^R , $Z^R \geq 0$ we have w^L, w^U, Z^L and ZU are conventional intervals, the above operations can be further transferred to the following functions if letting $w^L = [w^a,\ w^b]$, $w^U = [w^c,\ w^d]$, $Z^L = [Z^a,\ Z^b]$, $Z^U = [Z^c,\ Z^d]$ where w^a , w^b, w^c, w^d ,Z^a, Z^b, Z^c and Zd are deterministic numbers denoting the lower and upper bounds of w^L, w^U, Z^L and Z^U

$$W^R + Z^R = ([w^a + Z^a, w^b + Z^b] : [w^c + Z^c, w^d + Z^d])$$
$$W^R - Z^R = ([w^a - Z^a, w^b - Z^b] : [w^c - Z^c, w^d - Z^d])$$
$$W^R \times Z^R = ([w^a \times Z^a, w^b \times Z^b] : [w^c \times Z^c, w^d \times Z^d])$$
$$W^R \div Z^R = ([w^a \div Z^a, w^b \div Z^b] : [w^c \div Z^c, w^d \div Z^d])$$

where Z^a, Z^b, Z^c and $Z^d \neq 0$ in $\{\div\}$ binary operation[6].

3. MULTIOBJECTIVE TRANSPORTATION PROBLEM
Multiobjective Transportation Problem (MOTP) can be classified into three different types based on the certainty of its coefficient. The traditional MOTP where all the coefficients in the objective function and the entire constrains are deterministic values. The second type is the interval multiobjective transportation problem (IMOTP) where the coefficients of the model represented by interval values. The third type is rough interval multiobjective transportation problem (RIMOTP) where the coefficients of the model are represented by rough interval values. The following section will illustrate the mathematical model of each type.

3.1 The Deterministic Multiobjective Transportation Problem
The Deterministic multiobjective transportation problem can be defined as: Suppose that [8] there are m sources and n

destinations. Let a_i be the number of supply units available at sources i $(i=1, 2,\dots, m)$ and let bj the number of demands units required at destination j $(j=1, 2,\dots,n)$. Let cij represents the unit transportation cost for transportation the units from source i to destination j. The target is to determine the number of units to be transported from source i to destination j, so that the total transportation cost is minimum. Let x_{ij} be the decision variable which denotes the number of units shipped from sources i to destination j.

$$\left.\begin{aligned} &Min\ f^{k}(x)=\sum_{i=1}^{m}\sum_{j=1}^{n}c_{Lij}^{k}x_{ij}\ \ where\ k=1,2,\dots,K\\ &subject\ to\\ &\sum_{j=1}^{n}x_{ij}=a_i\ ,\ i=1,2,\dots,m\\ &\sum_{i=1}^{m}x_{ij}=b_j\ ,\ j=1,2,\dots,n\\ &\sum_{i=1}^{m}a_i=\sum_{j=1}^{n}b_j\ ,i=1,2,\dots,m,\ j=1,2,\dots,n\\ &x_{ij}\ \geq\ 0 \end{aligned}\right\}\quad (1)$$

The weighting method [2], ε-constrained [5], and fuzzy technique [16] can be used to solve model (1).

3.2 Interval Multiobjective Transportation Problem

The formulation of interval multiobjective transportation (IMOTP) problem of minimizing interval cost of K^{th} objectives function under constrains of interval sources $[a_i^L : a_i^U]$ and interval demands $[b_j^L : b_j^U]$ where L, U are the lower and upper of each conventional interval [4]:

$$\left.\begin{aligned} &Min\ f^{k}(x)=\sum_{i=1}^{m}\sum_{j=1}^{n}[c_{ij}^{kL},c_{ij}^{kU}]x_{ij}\ \ where\ k=1,2,\dots,K\\ &subject\ to\\ &\sum_{j=1}^{n}x_{ij}=[a_i^L,\ a_i^U]\ ,\ i=1,2,\dots,m\\ &\sum_{i=1}^{m}x_{ij}=[b_j^L,\ b_j^U]\ ,\ j=1,2,\dots,n\\ &\sum_{i=1}^{m}[a_i^L,\ a_i^U]=\sum_{j=1}^{n}[b_j^L,\ b_j^U]\ ,i=1,2,\dots,m,\ j=1,2,\dots,n\\ &x_{ij}\ \geq\ 0 \end{aligned}\right\}\quad (2)$$

Where $[c_{ij}^{kL} : c_{ij}^{kU}]$: the lower and upper cost of good which to be transported from the source i to destination j in objective function k.

There are special types of IMOTP can be derived from its general form. In the first type, the objective functions' parameters are denoted by conventional intervals while the remaining parameters of the model (the supplies capacities and destination demands) are deterministic. This type can be represented as in (3)

$$\left.\begin{aligned} &Min\ f^{k}(x)=\sum_{i=1}^{m}\sum_{j=1}^{n}[c_{Lij}^{k},c_{Uij}^{k}]x_{ij}\ \ where\ k=1,2,\dots,K\\ &subject\ to\\ &\sum_{j=1}^{n}x_{ij}=a_i\ ,\ i=1,2,\dots,m\\ &\sum_{i=1}^{m}x_{ij}=b_j\ ,\ j=1,2,\dots,n\\ &\sum_{i=1}^{m}a_i=\sum_{j=1}^{n}b_j\ ,i=1,2,\dots,m,\ j=1,2,\dots,n\\ &x_{ij}\ \geq\ 0\ \ \forall i,j,i=1,2,\dots,m,\ j=1,2,\dots,n \end{aligned}\right\}\quad (3)$$

In the second type, the objective functions' parameters are deterministic while the supplies capacities and destination demands are denoted by conventional intervals. The linear model of such problem can be presented as in (4)

$$\left.\begin{aligned} &Min\ f^{k}(x)=\sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}^{k}x_{ij}\ \ where\ k=1,2,\dots,K\\ &subject\ to\\ &\sum_{j=1}^{n}x_{ij}=[a_i^L,\ a_i^R]\ ,\ i=1,2,\dots,m\\ &\sum_{i=1}^{m}x_{ij}=[b_j^L,\ b_j^R]\ ,\ j=1,2,\dots,n\\ &\sum_{i=1}^{m}[a_i^L,\ a_i^R]=\sum_{j=1}^{n}[b_j^L,\ b_j^R]\ ,i=1,2,\dots,m,\ j=1,2,\dots,n\\ &x_{ij}\ \geq\ 0 \end{aligned}\right\}\quad (4)$$

In order to solve the IMOTP, the separation method [3] is used. The original model can be represented by two different models which are the lower bound approximation model and the upper bound approximation model and solve each model separately. After that the solutions of both models are collected together to represent the interval solution for the problem. While the deterministic solution is targeted the fuzzy technique is applied to deal with the problem [11].

3.3 Rough Interval Multiobjective Transportation Problem

The general rough interval multiobjective transportation problem (RIMOTP) can be stated by (5) as follows:

$$\left.\begin{aligned} &Min\ f^{k}(x)=\sum_{i=1}^{m}\sum_{j=1}^{n}[[c_{ij}^{ak},c_{ij}^{bk}]:[c_{ij}^{ck},c_{ij}^{dk}]]x_{ij}\quad where\ k=1,2,\dots K\\ &subject\ to\\ &\sum_{j=1}^{n}x_{ij}=[[a_i^a,a_i^b]:[a_i^c,a_i^d]]\quad i=1,2,\dots m\\ &\sum_{i=1}^{m}x_{ij}=[[b_j^a,b_j^b]:[b_j^c,b_j^d]]\quad j=1,2,\dots n\\ &\sum_{j=1}^{n}[[a_i^a,a_i^b]:[a_i^c,a_i^d]]=\sum_{i=1}^{m}[[b_j^a,b_j^b]:[b_j^c,b_j^d]]\\ &x_{ij}\geq 0 \end{aligned}\right\}\quad (5)$$

Where the supply, demand, and objective functions are donated by rough intervals $a_i^{RI} = [[a_i^a, a_i^b]:[a_i^c, a_i^d]]$, $b_j^{RI} = [[b_j^a, b_j^b]:[b_j^c, b_j^d]]$ and, $f^{RI(k)} =$

$[[f^{ak}, f^{bk}]:[f^{ck}, f^{dk}]]$ respectively and $k = 1, 2, \ldots, K$ is the number of objectives.

The rough intervals $[[c_{ij}^{ak}, cb_{ij}^{bk}]:[c_{ij}^{ck}, c_{ij}^{dk}]]\ k = 1, 2, \ldots, K$ are denoted the uncertain costs for the transportation problem. The source parameter lies between lower approximations interval *LAI* $[a_i^a, a_i^b]$ (surly) and upper approximation interval *UAI* $[a_i^c, a_i^d]$(possibly). Similarly, destination parameter lies between lower approximation *LAI* $[b_j^a, b_j^b]$ (surly) and upper approximation *UAI*$[b_j^c, b_j^d]$ (possibly)

Two special types of (RIMOTP) can be derived from the general form. In the first type

$$\left.\begin{aligned} & Min\ f^k = \sum_{i=1}^{m}\sum_{j=1}^{n}[[c_{ij}^{ak}, c_{ij}^{bk}]:[c_{ij}^{ck}, c_{ij}^{dk}]]x_{ij} \quad where\ k = 1, 2, \ldots K \\ & subject\ to \\ & \qquad \sum_{j=1}^{n} x_{ij} = a_i \quad , i = 1, 2, \ldots, m \\ & \qquad \sum_{i=1}^{m} x_{ij} = b_j \quad , j = 1, 2, \ldots, n \\ & \qquad \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \\ & \qquad x_{ij} \geq 0 \end{aligned}\right\} \quad (6)$$

The objective functions' parameters are denoted by rough interval while the remaining parameters of the model (the supplies capacities and destination demands) are deterministic values as in (6).

In the second type, the objective functions' parameters are deterministic while the supplies capacities and destination demands are denoted by rough intervals as in (7).

$$\left.\begin{aligned} & Min\ f^k(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^k x_{ij} \quad where\ k = 1, 2, \ldots K \\ & subject\ to \\ & \qquad \sum_{j=1}^{n} x_{ij} = [[a_i^a, a_i^b]:[a_i^c, a_i^d]] \quad i = 1, 2, \ldots, m \\ & \qquad \sum_{i=1}^{m} x_{ij} = [[b_j^a, b_j^b]:[b_j^c, b_j^d]] \quad j = 1, 2, \ldots, n \\ & \qquad \sum_{j=1}^{n}[[a_i^a, a_i^b]:[a_i^c, a_i^d]] = \sum_{i=1}^{m}[[b_j^a, b_j^b]:[b_j^c, b_j^d]] \\ & \qquad x_{ij} \geq 0 \end{aligned}\right\} \quad (7)$$

Definition 3 (Pareto surely-feasible solution) A solution is defined as surely-feasible solution if x belongs to the lower approximation of the feasible set.

Definition 4 (Pareto possibly-feasible solution): A solution is defined as possibly-feasible solution if x belongs to the upper approximation of the feasible set

4. THE PROPRSED APPROACH

The proposed approach is built based on the concept of:

Step 1: Convert the rough objective into deterministic objective as in section 4.1.

Step 2: Convert the rough constraints into deterministic constraints as in section 4.2.

Step 3: Construct the payoff Table by solving each objective function individually under the set of constrains and calculate the values of the other objectives at the resulted solution.

Step 4: Define the maximum and the minimum value of each objective function from the payoff Table.

Step 5: Construct the membership function of each objective function using (11) ,(12) , (13) and (14).

Step 6: Construct the linear model defined by (15).

Step 7: Solve the linear model resulted from Step 6

4.1 Converting Rough Interval Objective to Deterministic Objective

In order to solve the conventional interval multiobjectve programming many Authors converted it to 2 separate models and solve it separately and collecting the solution of the two in on solution [11]. Based on such concept, the rough interval multiobjective problem can be converted into a linear multiobjectve programming as in (8).

$$\left.\begin{aligned} & Min \quad f_{ULRI}^k(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{kb} x_{ij} \\ & Min \quad f_{CNLRI}^k(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{kCNL} x_{ij} \\ & Min \quad f_{UURI}^k(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{kd} x_{ij} \\ & Min \quad f_{CNURI}^k(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{kCNU} x_{ij} \\ & where\ k = 1, 2, \ldots, K \\ & f_{ULRI}^k = \sum_{i=1}^{m}\sum_{j=1}^{n}(wL^k + cL^{kcnL})\mathrm{X}_{ij} = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^b x_{ij} \quad , \\ & f_{CNLRI}^k = \sum_{i=1}^{m}\sum_{j=1}^{n}((a+b)\div 2)X_{ij} \quad , \\ & \mathrm{W}L^k = (\mathrm{b}-\mathrm{a})\div 2 \quad and\ C^{kCNL} = (\mathrm{b}+\mathrm{a})\div 2 \\ & f_{UURI}^k = \sum_{i=1}^{m}\sum_{j=1}^{n}(wU^k + cU^{kCNU})\mathrm{X}_{ij} = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^d x_{ij} \ , \\ & f_{CNURI}^k = \sum_{i=1}^{m}\sum_{j=1}^{n}((c+d)\div 2)X_{ij} \quad , \\ & \mathrm{W}U^k = (\mathrm{d}-c)\div 2 \quad and\ C^{kCNU} = (\mathrm{d}+c)\div 2 \end{aligned}\right\} \quad (8)$$

Where:

f^K_{ULRI}**:** the objective of the upper for lower approximation rough interval

f^K_{CNLRI}: the objective of the center for lower approximation rough interval

f^K_{UURI}**:** the objective of the upper for upper approximation rough interval

f^K_{CNURI}: the objective of the center for upper approximation rough interval

4.2 Converting Rough Interval Constraint to Deterministic Constraint

Consider the following multiobjective transportation problem as illustrated in (*9.a-9.f*):

$$Min\ f^{RI(k)}(x)=\sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}^{RI(K)}x_{ij},\ where\ RI=a,b,c,d,\ k=1,2,...,K \quad (9.a)$$

subject to

$$\sum_{j=1}^{n}x_{ij}\le a_i^b,\ \sum_{j=1}^{n}x_{ij}\ge a_i^a,\ i=1,2,...,m \quad (9.b)$$

$$\sum_{j=1}^{n}x_{ij}\le a_i^d,\ \sum_{j=1}^{n}x_{ij}\ge a_i^c,\ i=1,2,...,m \quad (9.c)$$

$$\sum_{i=1}^{m}x_{ij}\le b_j^b,\ \sum_{i=1}^{m}x_{ij}\ge b_j^a,\ j=1,2,...,n \quad (9.d)$$

$$\sum_{i=1}^{m}x_{ij}\le b_j^d,\ \sum_{i=1}^{m}x_{ij}\ge b_j^c,\ j=1,2,...,n \quad (9.e)$$

$$\sum_{i=1}^{m}a_i^a=\sum_{j=1}^{n}b_j^a,\ \sum_{i=1}^{m}a_i^b=\sum_{j=1}^{n}b_j^b,\ \sum_{i=1}^{m}a_i^c=\sum_{j=1}^{n}b_j^c,\ \sum_{i=1}^{m}a_i^d=\sum_{j=1}^{n}b_j^d \quad (9.f)$$

$$x_{ij}\ \ge\ 0.$$

By studying model (9) we can find that constraints (9.b) are dominated by constraints (9.c). Also, constraints (9.d) are dominated by constraints (9.e). So, the model (9) can be rewritten as in (10).i.e possibly region.

$$\left.\begin{array}{l} Min\ f^{RI(k)}=\sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}^{RI(K)}x_{ij},where\ RI=a,b,c,d,\ k=1,2,...,K \\ subject\ to \\ \sum_{j=1}^{n}x_{ij}\le a_i^d,\ \sum_{j=1}^{n}x_{ij}\ge a_i^c,\ i=1,2,...,m \\ \sum_{j=1}^{n}x_{ij}\le b_j^d,\ \sum_{j=1}^{n}x_{ij}\ge b_j^c,\ j=1,2,...,n \\ \sum_{i=1}^{m}a_i^c=\sum_{j=1}^{n}b_j^c,\ i=1,2,...,m,\ j=1,2,...,n \\ \sum_{i=1}^{m}a_i^d=\sum_{j=1}^{n}b_j^d,\ i=1,2,...,m,\ j=1,2,...,n \\ x_{ij}\ge 0\ \forall i,j. \end{array}\right\} \quad (10)$$

4.3 Membership Function

Assuming that membership functions are linear, the linear membership for minimization of f_{ULRI}^{K}, f_{CNLRI}^{K} f_{UURI}^{K}, and f_{CNURI}^{K} are given by (11), (12), (13), and (14) respectively [16].

$$\mu_k\left(f_{ULRI}^k\right)=\begin{cases}1 & if\quad f_{ULRI}^k\le Lf_{ULRI}^k\\ 1-\dfrac{f_{ULRI}^k-Lf_{ULRI}^k}{Uf_{ULRI}^k-Lf_{ULRI}^k} & if\quad Lf_{ULRI}^k<f_{ULRI}^k<Uf_{ULRI}^k\\ 0 & if\quad f_{ULRI}^k\ge Uf_{ULRI}^k\end{cases} \quad (11)$$

$$\mu_k\left(f_{CNLRI}^k\right)=\begin{cases}1 & if\quad f_{CNLRI}^k\le Lf_{CNLRI}^k\\ 1-\dfrac{f_{CNLRI}^k-Lf_{CNLRI}^k}{Uf_{CNLRI}^k-Lf_{CNLRI}^k} & if\quad Lf_{CNLRI}^k<f_{CNLRI}^k<Uf_{CNLRI}^k\\ 0 & if\quad f_{CNLRI}^k\ge Uf_{CNLRI}^k\end{cases} \quad (12)$$

$$\mu_k\left(f_{UURI}^k\right)=\begin{cases}1 & if\quad f_{UURI}^k\le Lf_{UURI}^k\\ 1-\dfrac{f_{UURI}^k-Lf_{UURI}^k}{Uf_{UURI}^k-Lf_{UURI}^k} & if\quad Lf_{UURI}^k<f_{UURI}^k<Uf_{UURI}^k\\ 0 & if\quad f_{UURI}^k\ge Uf_{UURI}^k\end{cases} \quad (13)$$

$$\mu_k\left(f_{CNURI}^k\right)=\begin{cases}1 & if\quad f_{CNURI}^k\le Lf_{CNURI}^k\\ 1-\dfrac{f_{CNURI}^k-Lf_{CNURI}^k}{Uf_{CNURI}^k-Lf_{CNURI}^k} & if\quad Lf_{CNURI}^k<f_{CNURI}^k<Uf_{CNURI}^k\\ 0 & if\quad f_{CNURI}^k\ge Uf_{CNURI}^k\end{cases} \quad (14)$$

$$Max\ \mu;$$

subject to

$$\begin{array}{l} f_{ULRI}^k+\mu(Uf_{ULRI}^k-Lf_{ULRI}^k)\ \le Uf_{ULRI}^k \\ f_{CNLRI}^k+\mu(Uf_{CNLRI}^k-Lf_{CNLRI}^k)\le Uf_{CNLRI}^k \\ f_{UURI}^k+\mu(Uf_{UURI}^k-Lf_{UURI}^k)\ \le Uf_{UURI}^k \\ f_{CNURI}^k+\mu(Uf_{CNURI}^k-Lf_{CNURI}^k)\ \le Uf_{CNURI}^k \\ \sum_{j=1}^{n}x_{ij}\le a_i^d,\ \sum_{j=1}^{n}x_{ij}\ge a_i^c,\ i=1,2,...,m \\ \sum_{j=1}^{n}x_{ij}\le b_j^d,\ \sum_{j=1}^{n}x_{ij}\ge b_j^c,\ j=1,2,...,n \\ \sum_{i=1}^{m}a_i^c=\sum_{j=1}^{n}b_j^c,\ i=1,2,...,m,\ j=1,2,...,n \\ \sum_{i=1}^{m}a_i^d=\sum_{j=1}^{n}b_j^d,\ i=1,2,...,m,\ j=1,2,...,n \\ x_{ij}\ge 0\ \forall i,j,\mu\ge 0 \end{array} \quad (15)$$

5. ILLUSTRATIVE EXAMPLES

To solve the 3 examples(roughness in objective function only ,roughness in feasible region only and fully roughness) beasd on the the right limit, the center and the half-width of each rough interval using possibly region. And all solutions are deterministic.

In order to illustrate the solution of the first one which cost of problem given as rough and the DM is quite sure of the quantities exist in demands and capacities of supplies. i.e. the roughness exist only in the objective function while the constraints are deterministic.

Example 1

$$\min \quad f^{1} = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{RI1} x_{ij}$$

$$\min \quad f^{2} = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{RI2} x_{ij}$$

subject to

$$\sum_{j=1}^{4} x_{1j} = 8 \ , \sum_{j=1}^{4} x_{2j} = 19, \sum_{j=1}^{4} x_{3j} = 17$$

$$\sum_{i=1}^{3} x_{i1} = 11 \ , \sum_{i=1}^{3} x_{i2} = 3 \ , \sum_{i=1}^{3} x_{i3} = 14 \ , \sum_{i=1}^{3} x_{i4} = 16$$

$$x_{ij} \geq 0$$

Where the C_{ij}^{RI1} and C_{ij}^{RI2}

$$C_{ij}^{RI1} = \begin{pmatrix} [[6,7]:[5,9]] & [[6,7]:[5,9]] & [[5,7]:[4,8]] & [[6,7]:[5,9]] \\ [[9,11]:[8,12]] & [[15,17]:[14,18]] & [[16,17]:[14,19]] & [[18,21]:[17,22]] \\ [[11,13]:[10,14]] & [[3,5]:[2,6]] & [[9,11]:[8,12]] & [[3,4]:[1,5]] \end{pmatrix}$$

$$C_{ij}^{RI2} = \begin{pmatrix} [[7,9]:[6,10]] & [[10,11]:[8,12]] & [[11,12]:[10,13]] & [[4,6]:[3,6]] \\ [[6,8]:[5,9]] & [[9,10]:[7,11]] & [[5,7]:[4,8]] & [[7,9]:[6,11]] \\ [[8,10]:[7,11]] & [[13,15]:[12,16]] & [[8,10]:[7,11]] & [[0,11]:[8,12]] \end{pmatrix}$$

In order to solve such example we construct the model as in (8) based on the bounded of the rough interval and canter as follows:

$$\begin{aligned} Min \quad f_{ULRI}^{1}(x) &= 7x_{11} + 7x_{12} + 7x_{13} + 7x_{14} + 11x_{21} + 17x_{22} \\ &+ 17x_{23} + 21x_{24} + 13x_{31} + 5x_{32} + 11x_{33} + 4x_{34} \\ Min \quad f_{CNLRI}^{1}(x) &= 6.5x_{11} + 6.5x_{12} + 6x_{13} + 6.5x_{14} + 10x_{21} + 16x_{22} \\ &+ 16.5x_{23} + 19.5x_{24} + 12x_{31} + 4x_{32} + 10x_{33} + 3.5x_{34} \\ Min \quad f_{UURI}^{1}(x) &= 9x_{11} + 9x_{12} + 8x_{13} + 9x_{14} + 10x_{21} + 18x_{22} + 19x_{23} \\ &+ 22x_{24} + 14x_{31} + 6x_{32} + 12x_{33} + 5x_{34} \\ Min \quad f_{CNURI}^{1}(x) &= 7x_{11} + 7x_{12} + 6x_{13} + 7x_{14} + 10x_{21} + 16x_{22} \\ &+ 16.5x_{23} + 19.5x_{24} + 12x_{31} + 4x_{32} + 10x_{33} + 3x_{34} \end{aligned}$$

$$\begin{aligned} Min \quad f_{ULRI}^{2}(x) &= 9x_{11} + 11x_{12} + 12x_{13} + 6x_{14} + 8x_{21} + 10x_{22} \\ &+ 7x_{23} + 9x_{24} + 10x_{31} + 15x_{32} + 10x_{33} + 11x_{34} \\ Min \quad f_{CNLRI}^{2}(x) &= 8x_{11} + 10.5x_{12} + 11.5x_{13} + 5x_{14} + 7x_{21} + 9.5x_{22} \\ &+ 6x_{23} + 8x_{24} + 8x_{31} + 14x_{32} + 9x_{33} + 10.5x_{34} \\ Min \quad f_{UURI}^{2}(x) &= 10x_{11} + 12x_{12} + 13x_{13} + 6x_{14} + 9x_{21} + 11x_{22} \\ &+ 8x_{23} + 11x_{24} + 11x_{31} + 16x_{32} + 11x_{33} + 12x_{34} \\ Min \quad f_{CNURI}^{2}(x) &= 8x_{11} + 10x_{12} + 11.5x_{13} + 4.5x_{14} + 7x_{21} + 9x_{22} \\ &+ 6x_{23} + 8.5x_{24} + 9x_{31} + 14x_{32} + 9x_{33} + 10x_{34} \end{aligned}$$

subject to

$$\begin{aligned} & x_{11} + x_{12} + x_{13} + x_{14} = 8 \\ & x_{21} + x_{22} + x_{23} + x_{24} = 19 \\ & x_{31} + x_{32} + x_{33} + x_{34} = 17 \\ & x_{11} + x_{21} + x_{31} = 11 \\ & x_{12} + x_{22} + x_{32} = 3 \\ & x_{13} + x_{23} + x_{33} = 14 \\ & x_{14} + x_{24} + x_{34} = 16 \\ & x_{ij} \geq 0 \quad \forall i, \forall j \ , \end{aligned}$$

In order to solve the above model we have to solve each objective function separately and calculate the remaining objectives at the resulted as presented on Table 1.

As the 1, 2, 3 steps, the solution each single objective transportation problem where

$$X = (x_{11}, x_{12}, x_{13}, x_{14}, x_{21}, x_{22}, x_{23}, x_{24}, x_{31}, x_{32}, x_{33}, x_{34})$$

$$X^{ULRI1} = (0, 2, 6, 0, 11, 0, 8, 0, 0, 1, 0, 16)$$

$$X^{CNLRI1} = (0, 0, 8, 0, 11, 2, 6, 0, 0, 1, 0, 16)$$

$$X^{UURI1} = (0, 0, 8, 0, 11, 2, 6, 0, 0, 1, 0, 16)$$

$$X^{CNURI1} = (0, 0, 8, 0, 11, 2, 6, 0, 0, 1, 0, 16)$$

$$X^{ULRI2} = (0, 0, 0, 8, 2, 3, 14, 0, 9, 0, 0, 8)$$

$$X^{CNLRI2} = (0, 0, 0, 8, 0, 3, 14, 2, 11, 0, 0, 6)$$

$$X^{UURI2} = (0, 0, 0, 8, 2, 3, 14, 0, 9, 0, 0, 8)$$

$$X^{CNURI2} = (0, 0, 0, 8, 2, 3, 14, 0, 9, 0, 0, 8)$$

Step 4: Find the best lower and the worst upper for each objective (payoff Table) see table 1

Table 1: Payoff Table of Example 1

	f_{ULRI}^{1}	f_{CNLRI}^{1}	f_{UURI}^{1}	f_{CNURI}^{1}	f_{ULRI}^{2}	f_{CNLRI}^{2}	f_{UURI}^{2}	f_{CNURI}^{2}
X^{ULRI1}	380	351	436	344	429	397	469	382
X^{CNLRI1}	382	349	410	341	437	406	473	397
X^{UURI1}	382	349	410	341	437	406	473	397
X^{CNURI1}	382	349	410	341	437	406	473	397
X^{ULRI2}	516	487	582	487	370	322.5	416	322

X^{CNLRI2}	554	523	620	524	370	319.5	408	323
X^{UURI2}	516	487	572	487	370	322.5	406	322
X^{CNURI2}	516	487	582	487	370	319.5	406	322
Upper bound	554	523	620	524	437	406	473	397
Lower bound	382	349	410	341	370	319.5	406	322
The difference	172	178	210	183	67	86.5	67	75

The linear model can be constructed based on the upper and lower bounds exist above as follows

$$\max \ \omega$$

subject to

$$7x_{11}+7x_{12}+7x_{13}+7x_{14}+11x_{21}+17x_{22}+17x_{23}$$
$$+21x_{24}+13x_{31}+5x_{32}+11x_{33}+4x_{34}+172\omega\le 554$$
$$6.5x_{11}+6.5x_{12}+6x_{13}+6.5x_{14}+10x_{21}+16x_{22}+16.5x_{23}$$
$$+19.5x_{24}+12x_{31}+4x_{32}+10x_{33}+3.5x_{34}+174\omega\le 523$$
$$9x_{11}+9x_{12}+8x_{13}+9x_{14}+10x_{21}+18x_{22}+19x_{23}$$
$$+22x_{24}+14x_{31}+6x_{32}+12x_{33}+5x_{34}+210\omega\le 620$$
$$7x_{11}+7x_{12}+6x_{13}+7x_{14}+10x_{21}+16x_{22}+16.5x_{23}$$
$$+19.5x_{24}+12x_{31}+4x_{32}+10x_{33}+3x_{34}+183\omega\le 524$$
$$9x_{11}+11x_{12}+12x_{13}+6x_{14}+8x_{21}+10x_{22}+7x_{23}$$
$$+9x_{24}+10x_{31}+15x_{32}+10x_{33}+11x_{34}+67\omega\le 473$$
$$8x_{11}+10.5x_{12}+11.5x_{13}+5x_{14}+7x_{21}+9.5x_{22}+6x_{23}$$
$$+8x_{24}+8x_{31}+14x_{32}+9x_{33}+10.5x_{34}+86.5\omega\le 406$$
$$10x_{11}+12x_{12}+13x_{13}+6x_{14}+9x_{21}+11x_{22}+8x_{23}$$
$$+11x_{24}+11x_{31}+16x_{32}+11x_{33}+12x_{34}+67\omega\le 473$$
$$8x_{11}+10x_{12}+11.5x_{13}+4.5x_{14}+7x_{21}+9x_{22}+6x_{23}$$
$$+8.5x_{24}+9x_{31}+14x_{32}+9x_{33}+10x_{34}+75\omega\le 397$$
$$x_{11}+x_{12}+x_{13}+x_{14}=8$$
$$x_{21}+x_{22}+x_{23}+x_{24}=19$$
$$x_{31}+x_{32}+x_{33}+x_{34}=17$$
$$x_{11}+x_{21}+x_{31}=11$$
$$x_{12}+x_{22}+x_{32}=3$$
$$x_{13}+x_{23}+x_{33}=14$$
$$x_{14}+x_{24}+x_{34}=16$$
$$x_{ij}\ge 0 \quad \forall i,\forall j ,$$

By solving the above model the Pareto optimal solution of the problem is obtained as follows

X=(0,0,0,8,11,1,7,0,0,2,7,8)

F^1*=[[367,432]:[308.491]]*, F^2*=[[304,383]:[251.419]]*

Example 2

$$Minimize\ f^{1}=\sum_{i=1}^{3}\sum_{j=1}^{4}c_{ij}^{1}x_{ij} \quad , f^{2}=\sum_{i=1}^{3}\sum_{j=1}^{4}c_{ij}^{2}x_{ij}$$

where c^1 *and* c^2 *are*

$$C^1=\begin{bmatrix}7&7&6&7\\10&16&17&20\\12&4&10&3\end{bmatrix} \quad , \quad C^2=\begin{bmatrix}8&10&11&5\\7&9&6&8\\9&14&9&10\end{bmatrix}$$

subject to

$$\sum_{j=1}^{4}x_{1j}=[[7,9]:[6,10]] \ , \ \sum_{j=1}^{4}x_{2j}=[[17,21]:[16,22]]$$
$$\sum_{j=1}^{4}x_{3j}=[[16,18]:[15,19]],$$
$$\sum_{i=1}^{3}x_{i1}=[[10,12]:[9,13]] \ , \ \sum_{i=1}^{3}x_{i2}=[[2,4]:[1,5]]$$
$$\sum_{i=1}^{3}x_{i3}=[[13,15]:[12,16]] \ , \ \sum_{i=1}^{3}x_{i4}=[[15,17]:[15,17]]$$
$$x_{ij}\ge 0$$

Convert the rough constraints into deterministic constraints

$$Min\ f^{1}(x)=7x_{11}+7x_{12}+6x_{13}+7x_{14}+10x_{21}+16x_{22}$$
$$+17x_{23}+20x_{24}+12x_{31}+4x_{32}+10x_{33}+3x_{34}$$
$$Min\ f^{2}(x)=8x_{11}+10x_{12}+11x_{13}+5x_{14}+7x_{21}+9x_{22}$$
$$+6x_{23}+8x_{24}+9x_{31}+14x_{32}+9x_{33}+10x_{34}$$

subject to

$$6\le x_{11}+x_{12}+x_{13}+x_{14}\le 10$$
$$16\le x_{21}+x_{22}+x_{23}+x_{24}\le 22$$
$$15\le x_{31}+x_{32}+x_{33}+x_{34}\le 19$$
$$9\le x_{11}+x_{21}+x_{31}\le 13$$
$$1\le x_{12}+x_{22}+x_{32}\le 5$$
$$12\le x_{13}+x_{23}+x_{33}\le 16$$
$$15\le x_{14}+x_{24}+x_{34}\le 17$$
$$x_{ij}\ge 0 \quad \forall i,\forall j ,$$

As the 1, 2, 3 steps, the solution of each single objective transportation problem

$$X^{1} = (0,0,8,0,12,0,5,0,0,2,0,15)$$
$$X^{2} = (0,0,0,7,0,2,13,2,10,0,0,6)$$

Find the best lower and the worst upper for each objective.

Table 2: Payoff Table example 2

	f^1	f^2
X^1	306	380
X^2	487	297
Upper	487	380
Lower	306	297
Difference	181	83

$$\max \quad \mu$$

$$\textit{subject to}$$

$$7x_{11} + 7x_{12} + 6x_{13} + 7x_{14} + 10x_{21} + 16x_{22} + 17x_{23}$$
$$+ 20x_{24} + 12x_{31} + 4x_{32} + 10x_{33} + 3x_{34} + 181\mu \le 487$$
$$8x_{11} + 10x_{12} + 11x_{13} + 5x_{14} + 7x_{21} + 9x_{22} + 6x_{23}$$
$$+ 8x_{24} + 9x_{31} + 14x_{32} + 9x_{33} + 10x_{34}\ 83\mu \le 380$$
$$6 \le x_{11} + x_{12} + x_{13} + x_{14} \le 10$$
$$16 \le x_{21} + x_{22} + x_{23} + x_{24} \le 22$$
$$15 \le x_{31} + x_{32} + x_{33} + x_{34} \le 19$$
$$9 \le x_{11} + x_{21} + x_{31} \le 13$$
$$1 \le x_{12} + x_{22} + x_{32} \le 5$$
$$12 \le x_{13} + x_{23} + x_{33} \le 16$$
$$15 \le x_{14} + x_{24} + x_{34} \le 17$$
$$x_{ij} \ge 0 \quad \forall i, j, \mu \ge 0$$

the Pareto optimal solution of the problem using fuzzy technique is obtained as:

X=(0,1,1,5,10,0,7,0,0,1,5,10) $F^1=356$, $F^2=317$

Example 3:

$$Min\ f^{1} = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{RI\,1} x_{ij}$$
$$Min\ f^{2} = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{RI\,2} x_{ij}$$

subject to

$$\sum_{j=1}^{4} x_{1j} = [[7,9]:[6,10]]\ ,\quad \sum_{j=1}^{4} x_{2j} = [[17,21]:[16,22]]$$
$$\sum_{j=1}^{4} x_{3j} = [[16,18]:[15,19]],$$
$$\sum_{i=1}^{3} x_{i1} = [[10,12]:[9,13]]\ ,\quad \sum_{i=1}^{3} x_{i2} = [[2,4]:[1,5]]$$
$$\sum_{i=1}^{3} x_{i3} = [[13,15]:[12,16]]\ ,\quad \sum_{i=1}^{3} x_{i4} = [[15,17]:[15,17]]$$
$$x_{ij} \ge 0$$

where $c^{RI\,1}$ *and* $c^{RI\,2}$ *are*

$$C_{ij}^{R1} = \begin{pmatrix} [[6,7]:[5,9]] & [[6,7]:[5,9]] & [[5,7]:[4,8]] & [[6,7]:[5,9]] \\ [[9,11]:[8,12]] & [[15,17]:[14,18]] & [[16,17]:[14,19]] & [[18,21]:[17,22]] \\ [[11,13]:[10,14]] & [[3,5]:[2,6]] & [[9,11]:[8,12]] & [[3,4]:[1,5]] \end{pmatrix}$$

$$C_{ij}^{R2} = \begin{pmatrix} [[7,9]:[6,10]] & [[10,11]:[8,12]] & [[11,12]:[10,13]] & [[4,6]:[3,6]] \\ [[6,8]:[5,9]] & [[9,10]:[7,11]] & [[5,7]:[4,8]] & [[7,9]:[6,11]] \\ [[8,10]:[7,11]] & [[13,15]:[12,16]] & [[8,10]:[7,11]] & [[0,11]:[8,12]] \end{pmatrix}$$

Convert the rough objective into deterministic objective

$$Min\ f_{ULRI}^{1}(x) = 7x_{11} + 7x_{12} + 7x_{13} + 7x_{14} + 11x_{21} + 17x_{22}$$
$$+ 17x_{23} + 21x_{24} + 13x_{31} + 5x_{32} + 11x_{33} + 4x_{34}$$
$$Min\ f_{CNLRI}^{1}(x) = 6.5x_{11} + 6.5x_{12} + 6x_{13} + 6.5x_{14} + 10x_{21} + 16x_{22}$$
$$+ 16.5x_{23} + 19.5x_{24} + 12x_{31} + 4x_{32} + 10x_{33} + 3.5x_{34}$$
$$Min\ f_{UURI}^{1}(x) = 9x_{11} + 9x_{12} + 8x_{13} + 9x_{14} + 10x_{21} + 18x_{22}$$
$$+ 19x_{23} + 22x_{24} + 14x_{31} + 6x_{32} + 12x_{33} + 5x_{34}$$
$$Min\ f_{CNURI}^{1}(x) = 7x_{11} + 7x_{12} + 6x_{13} + 7x_{14} + 10x_{21} + 16x_{22}$$
$$+ 16.5x_{23} + 19.5x_{24} + 12x_{31} + 4x_{32} + 10x_{33} + 3x_{34}$$
$$Min\ f_{ULRI}^{2}(x) = 9x_{11} + 11x_{12} + 12x_{13} + 6x_{14} + 8x_{21} + 10x_{22}$$
$$+ 7x_{23} + 9x_{24} + 10x_{31} + 15x_{32} + 10x_{33} + 11x_{34}$$
$$Min\ f_{CNLRI}^{2}(x) = 8x_{11} + 10.5x_{12} + 11.5x_{13} + 5x_{14} + 7x_{21} + 9.5x_{22}$$
$$+ 6x_{23} + 8x_{24} + 8x_{31} + 14x_{32} + 9x_{33} + 10.5x_{34}$$
$$Min\ f_{UURI}^{2}(x) = 10x_{11} + 12x_{12} + 13x_{13} + 6x_{14} + 9x_{21} + 11x_{22}$$
$$+ 8x_{23} + 11x_{24} + 11x_{31} + 16x_{32} + 11x_{33} + 12x_{34}$$
$$Min\ f_{CNURI}^{2}(x) = 8x_{11} + 10x_{12} + 11.5x_{13} + 4.5x_{14} + 7x_{21} + 9x_{22}$$
$$+ 6x_{23} + 8.5x_{24} + 9x_{31} + 14x_{32} + 9x_{33} + 10x_{34}$$

Convert the rough constraints into deterministic constraints

$$\textit{subject to}$$
$$6 \le x_{11} + x_{12} + x_{13} + x_{14} \le 10$$
$$16 \le x_{21} + x_{22} + x_{23} + x_{24} \le 22$$
$$15 \le x_{31} + x_{32} + x_{33} + x_{34} \le 19$$
$$9 \le x_{11} + x_{21} + x_{31} \le 13$$
$$1 \le x_{12} + x_{22} + x_{32} \le 5$$
$$12 \le x_{13} + x_{23} + x_{33} \le 16$$
$$15 \le x_{14} + x_{24} + x_{34} \le 17$$
$$x_{ij} \ge 0 \quad \forall i, j$$

As the 1, 2, 3 steps, the solution of each single objective transportation problem

$$X^{ULRI\,1} = (0,0,7,0,11,0,6,0,0,2,0,15)$$
$$X^{CNLRI\,1} = (0,0,8,0,12,0,5,0,0,2,0,15)$$
$$X^{UURI\,1} = (0,0,9,0,12,1,4,0,0,1,0,15)$$
$$X^{CNURI\,1} = (0,0,8,0,12,0,5,0,0,2,0,15)$$
$$X^{ULRL\,2} = (0,0,0,7,0,2,13,2,10,0,0,6)$$
$$X^{CNLRI\,2} = (0,0,0,7,0,2,13,2,10,0,0,6)$$
$$X^{UURI\,2} = (0,0,0,7,2,2,13,0,8,0,0,8)$$
$$X^{CNURI\,2} = (0,0,0,7,2,2,13,0,8,0,0,8)$$

Find the best lower and the worst upper for each objective by solving the classical multiobjective transportation problem.

Table 3: Payoff Table Example 3

	f^1_{ULRI}	f^1_{CNLRI}	f^1_{UURI}	f^1_{CNURI}	f^2_{ULRI}	f^2_{CNLRI}	f^2_{UURI}	f^2_{CNURI}
X^{ULRI1}	342	311.5	367	304	416	382.5	443	368
X^{CNLRI1}	343	311	366	303.5	430	395.5	456	380
X^{UURI1}	345	312.5	365	305	431	397	455	380
X^{CNURI1}	382	311	366	303.5	430	395.5	456	380
X^{ULRI2}	500	472	556	472.5	337	291	472	294.5
X^{CNLRI2}	500	472	556	472.5	337	291	472	294.5
X^{UURI2}	462	436	514	435.5	337	294	370	293.5
X^{CNURI2}	462	436	514	435.5	337	294	370	293.5
Upper bound	500	372	556	472.5	431	397	456	380
Lower bound	342	311	365	303.5	337	291	370	293.5
The difference	158	161	191	169	94	106	86	86.5

The linear model can be constructed Based on the upper and lower bounds exist above as follows:

$$\max \ \omega$$

subject to

$$7x_{11}+7x_{12}+7x_{13}+7x_{14}+11x_{21}+17x_{22}+17x_{23}$$
$$+21x_{24}+13x_{31}+5x_{32}+11x_{33}+4x_{34}+158\omega \le 500$$
$$6.5x_{11}+6.5x_{12}+6x_{13}+6.5x_{14}+10x_{21}+16x_{22}+16.5x_{23}$$
$$+19.5x_{24}+12x_{31}+4x_{32}+10x_{33}+3.5x_{34}+161\omega \le 372$$
$$9x_{11}+9x_{12}+8x_{13}+9x_{14}+10x_{21}+18x_{22}+19x_{23}$$
$$+22x_{24}+14x_{31}+6x_{32}+12x_{33}+5x_{34}+191\omega \le 556$$
$$7x_{11}+7x_{12}+6x_{13}+7x_{14}+10x_{21}+16x_{22}+16.5x_{23}$$
$$+19.5x_{24}+12x_{31}+4x_{32}+10x_{33}+3x_{34}+169\omega \le 472.5$$
$$9x_{11}+11x_{12}+12x_{13}+6x_{14}+8x_{21}+10x_{22}+7x_{23}$$
$$+9x_{24}+10x_{31}+15x_{32}+10x_{33}+11x_{34}+94\omega \le 431$$
$$8x_{11}+10.5x_{12}+11.5x_{13}+5x_{14}+7x_{21}+9.5x_{22}+6x_{23}$$
$$+8x_{24}+8x_{31}+14x_{32}+9x_{33}+10.5x_{34}+106\omega \le 397$$
$$10x_{11}+12x_{12}+13x_{13}+6x_{14}+9x_{21}+11x_{22}+8x_{23}$$
$$+11x_{24}+11x_{31}+16x_{32}+11x_{33}+12x_{34}+86\omega \le 456$$
$$8x_{11}+10x_{12}+11.5x_{13}+4.5x_{14}+7x_{21}+9x_{22}+6x_{23}$$
$$+8.5x_{24}+9x_{31}+14x_{32}+9x_{33}+10x_{34}+86.5\omega \le 380$$
$$6 \le x_{11}+x_{12}+x_{13}+x_{14} \le 10$$
$$16 \le x_{21}+x_{22}+x_{23}+x_{24} \le 22$$
$$15 \le x_{31}+x_{32}+x_{33}+x_{34} \le 19$$
$$9 \le x_{11}+x_{21}+x_{31} \le 13$$
$$1 \le x_{12}+x_{22}+x_{32} \le 5$$
$$12 \le x_{13}+x_{23}+x_{33} \le 16$$
$$15 \le x_{14}+x_{24}+x_{34} \le 17$$
$$x_{ij} \ge 0 \ , \ \forall i, j \ , \omega \ge 0$$

The Pareto optimal solution of the problem using fuzzy programming technique is obtained as following:

X=(0,0,6,1,10,1,6,0,0,1,1,14)

F^1= ([291,350]:[231.397), F^2=([310,389]:[275.428])

6. CONCLUSION

The different types of rough interval multiobjective transportation problem are introduced and the solution approach is presented. The concept of solving conventional interval programming combined with fuzzy programming is used to build the solution approach for RIMOTP. The proposed approach can be applied for solving different types of transportation problem such as rough interval fixed charge transportation problem, and rough interval solid transportation problem.

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