# Reaction-Diffusion System with Additional Source Term Applied to Image Restoration

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## ABSTRACT
Many noisy texture images require the enhancement of coherent structures in various applications. Traditional TV-based methods make the denoising fail when the data-fitting weight parameter $\lambda$ is strong, on the contrary, reducing the corresponding parameter $\lambda$ may distort the textures, and generate staircase artifacts. Therefore in order to smooth the image efficiently, the suitable $\lambda$ shall be adopted. Formally minimizing the TV-based energy functional yields the associated Euler-Lagrange equation, which can be seen as a reaction-diffusion system, in general the corresponding parameter $\lambda$ with respect to reaction term is always small to ensure sufficiently removal of the noise, and this feature has the contrast between coherent structures and the background decreased. Hence in this paper a reaction-diffusion system is investigated applied to image restoration with additional source term embedded into the system. Subsequently, this new model combines contrast enhancement with diffusion processes, so it may be more suitable for dealing with Gaussian white noise than the original models. The proposed method is assessed in terms of the theoretical and numerical properties changed by the source term. Finally, An experimental result is also given to demonstrate the efficiency of this kind of model.

## Keywords
Image restoration, Reaction-diffusion system, Source term, Texture enhancement

## 1. INTRODUCTION
In this paper, the problem of restoring images contaminated by additive Gaussian white noise is discussed. One way to restore such degraded images is to find solution of an evolution system corresponding to the Euler-Lagrange equation of the energy functional. Another approach is to directly address the diffusion equation system without introducing energy minimizing. No matter what method is adopted, one should consider some diffusion equations. To this end, a novel and effective model is proposed here which is to work directly on the partial differential equations. Suppose that a gray-scale image f(where f denotes gray-scale intensity)has been formed by adding Gaussian noise with standard deviation to a clean image $u$ :

$$f = u + n.$$

The task is to recover the clean image $u$ from the observed noisy image f. Clearly, without explicit knowledge of $n$, the recovery of u from f is not possible, total variation(TV)as frequently used prior is particularly impressive. The famous TV method first proposed by Rudin et al.[1] was to solve the following constrained minimization problem:

$$\arg\min \int_\Omega |\nabla u|,$$

$$\text{s.t.} \int_\Omega u\,dx = \int_\Omega f\,dx \text{ and } \int_\Omega |u-f|^2 = \sigma^2.$$

This problem shall naturally be converted into the following unconstrained variational problem:

$$\min E(u) = \int_\Omega |\nabla u| + \lambda \|u-f\|^2_{L^2(\Omega)}.$$

In the past decades TV and its variants were best studied (see [2],[3],[4],[5]). Despite the merits of the TV model, it faces some drawbacks, such as destroying the textures of images (see [6]). Interestingly in [8] this kind of attribute was richly applied to dental micro-CT image denoising. To overcome this drawback, Osher et al.[6] proposed the following model which combined the norm for oscillatory functions proposed by Meyer[7](involving the $H^{-1}$ norm) with the total variation minimization from the TV model:

$$\min E_{H^{-1}}(u) = \int_\Omega |\nabla u| + \lambda \|u-f\|^2_{H^{-1}(\Omega)}. \quad (1)$$

Minimizing the above variational problem can be formally converted into the following Euler-Lagrange equations:

$$\Delta^{-1}(f-u) = \frac{1}{2\lambda}\operatorname{div}(\nabla u/|\nabla u|), \quad (2)$$

$$\partial \operatorname{div}\left(\frac{\nabla u}{|\nabla u|}\right)/\partial\vec{n} = 0, \partial u/\partial \vec{n} = 0. \quad (3)$$

The above equations are of fourth order, and equivalent to

$$\begin{cases} -\Delta\omega + (f-u) = 0, & \text{in } \Omega, \\ -\operatorname{div}(\nabla u/|\nabla u|) + 2\lambda\omega = 0, & \text{in } \Omega, \\ \partial\omega/\partial\vec{n} = 0, \partial u/\partial\vec{n} = 0, & \text{on } \partial\Omega. \end{cases} \quad (4)$$

In [11], Elliott and McBeth discussed the well-posedness of a system which consisted of two coupled second order equations from (1).

Inspired by the ideas of (4), Guo et al.[12] introduced a new model of reaction-diffusion system:

$$\partial\omega/\partial t = \Delta\omega - (f-u), \text{ in } \Omega\times(0,T), \quad (5)$$

$$\partial u/\partial t = \operatorname{div}(\nabla u/|\nabla u|) - 2\lambda\omega, \text{ in } \Omega\times(0,T), \quad (6)$$

$$\omega(0,x)=0, \text{ in } \Omega, \quad (7)$$

$$u(0,x)=f, \text{ in } \Omega, \quad (8)$$

$$\partial\omega/\partial\vec{n}=0, \text{ on } \partial\Omega\times(0,T), \quad (9)$$

$$\partial u/\partial\vec{n} = 0, \text{on } \partial\Omega\times(0,T). \quad (10)$$

In [9], a novel image stylization system based on anisotropic reaction diffusion was proposed to facilitate pattern generation and stylized image design, and some filters formulated using nonlinear PDEs have obtained impressive results [10].

As for directly addressing the diffusion equation system Weickert [13] proposed a novel method called coherence-enhancing shock filters for enhancing the coherent flow-like structures in texture images. In [14], the authors proposed a forward-backward diffusion scheme for the enhancement. Moreover, Obara et al.[15] replaced the structure tensor with a phase congruency tensor, which facilitated the successful enhancement of noisy texture images. M Neri et al.[17] dealt with the simultaneous inpainting and denoising of a digital image using TV-based variational model.

All the models mentioned above remove the noise as much as possible under the condition that the diffusion processes shall ensure the satisfying of the data-fitting. Then the contrast is degraded because of the excessive diffusion process to eliminate the difference between texture structures and the background in most cases. Therefore in this paper, a novel reaction-diffusion system is proposed to deal with texture images, where a source term is embedded into the filtering model to enhance the contrast. To the best of our knowledge, a reaction-diffusion filtering system with an additonal source term has not been studied thoroughly. The experimental results demonstrate the effectiveness and efficiency of the proposed method compared with the original filtering approaches based on some typical images.

This paper is organized as follows. In Section 2, a novel coupled second order equations for removing Gaussian noise is proposed, and this model will be discussed in detail. Section 3 is devoted to discussing the notable effect caused by introducing the source term, and the way to solve the attendant effect. Numerical experiments are given in Section 4 to show the efficiency of the proposed system.

## 2. OUR PROPOSED MODEL

In this paper, the following reaction-diffusion system with the source term is proposed:

$$\partial \omega / \partial t = \Delta \omega - (f - u), \quad \text{in } \Omega \times (0, T), \tag{11}$$

$$\partial u / \partial t = g(|\nabla u|) \operatorname{div}\left(\frac{\nabla u}{|\nabla u|}\right) - 2\lambda\omega + \beta(u - \tilde{u}),$$

$$\text{in } \Omega \times (0, T), \tag{12}$$

$$u(0, x) = f, \ \text{in } \Omega, \tag{13}$$

$$\omega(0, x) = 0, \text{in } \Omega, \tag{14}$$

$$\partial u / \partial \tilde{n} = 0, \text{on } \partial\Omega \times (0, T), \tag{15}$$

$$\partial \omega / \partial \vec{n} = 0, \text{on } \partial\Omega \times (0, T), \tag{16}$$

where

$$g(|\nabla u|) = \frac{\alpha + |\nabla u|^2 / K^2}{1 + |\nabla u|^2 / K^2}, \tag{17}$$

$$\tilde{u} = \frac{1}{\text{mean }(\Omega)} \int_\Omega u \, dx. \tag{18}$$

In the model above, $g(|\nabla u|)$ as the weight parameter of the diffusion term, is a function of local image gradient magnitude. For $g(|\nabla u|)$ there are many different options, and this issue will be investigated in section 3. Within $g(|\nabla u|)$, $\alpha$ is an adjustable parameter to control the maximal influence lever of the regularization term, and K is the gradient magnitude threshold parameter, and $\tilde{u}$ denotes the mean value of u in the image domain, f is the original noisy image, and $\vec{n}$ represents the unit outward normal to the boundary, $\lambda$ and $\beta$ are positive constants, indicating the weight parameters of data-fitting term and source term respectively.

### 2.1 The Property of Noise Removal and Structure Enhancement

It is necessary to point out that the proposed model is based on the evolutionary form of the steady system (4). Different from the previous models, in the new system the first two equations from (11)-(12) interact with each other: from (11) $\omega$ is acquired by the evolutionary form, which derives from the introduction of $H^{-1}$ norm, to adjust data-fitting between u and f, then with the constraint of data-fitting term $\omega$ the smoother image u is achieved by (12). In other words the equation (11) has an exceptional fidelity term $\omega$ which is gotten from an evolutionary equation, thus an appropriate time scale T should be selected to achieve an appropriate data-fitting between u and f, and obtain an optimal result for noise removal and edge or texture preservation. Starting from the initial noisy image f and by running the system (11)-(18) the best image result can be chosen for the appropriate T.

From (17), it is clear that $g(|\nabla u|)$ depends on the gradient magnitude of u when running through the evolutionary process. In other words the weight of diffusion term at each location is determined by the local behavior of u. Obviously the value of the adaptive weight parameter of diffusion term is inversely proportional to the magnitude of the gradient $|\nabla u|$. Since the magnitude of gradient is weak within uniform or inner regions, the adaptive weight parameter $g(|\nabla u|)$ is close to $\alpha$ (the constant $\alpha$ is in general bigger than 1), it means that the weight parameter corresponding to the diffusion term is almost at the maximum $\alpha$, so this attribute fully has the inner region smoothed and the noise removed. Near boundaries the magnitude of the gradient $g(|\nabla u|)$ will be strong, thereby the value of the weight parameter of diffusion term approaches the minimum 1, and this makes the regularization term act as TV-based diffusion along edges.

Gang Dong et al.[16] explained the motivation for using a source term in the 1-D case by considering the well-known heat equation.

## 3. THE CHANGED PROPERTIES

In the proposed framework (11)-(18), a source term is embedded into the equation expressed by $\beta(u-\tilde{u})$. This section is devoted to discussing the effect caused by introducing the source term, and the way to deal with the attendant effect.

### 3.1 The Notable Effect Caused by Introducing The Source Term

For the sake of the enhancement of the recurring contrast between coherent structures and the background in the image domain, a source term is embedded into the filtering process based on reaction-diffusion system to handle texture images with additive Gaussian noise. In this subsection we consider the notable effect caused by introducing the source term. In the original model there only exist two parts consisting of a regularization term and a fidelity term in the right side of the evolutionary equation (6), but in the proposed model one more called the source term is embedded into the right side of the evolutionary equation(6), thereby to compare with the original model the effect of the regularization term is weakened in a certain degree in the diffusion process. Probably the most direct approach to work out this problem is to introduce a weight parameter of the regularization term to expand properly the influence of the regularization term, through this method the improved model can guarantee the adequate removal of noise while enhancing the contrast.

## 3.2 The Way to Solve The Attendant Effect

From the discussion of the last subsection 3.1, it is clear that in order to overcome the negative effect of the introduction of the source term, an effective approach is to add an adaptive weight parameter represented as $g(|\nabla u|)$ to the front of the regularization term just like what has been described in equation(12). $g(|\nabla u|)$ in (12) is called the adaptive weight parameter of diffusion term since it is the characteristic function of the gradient magnitude.

The function of the weight parameter $g(|\nabla u|)$ is chosen theoretically satisfying two conditions. One is $\lim_{s\to 0} g(s) = \alpha$ (where α as a constant must be bigger than 1), so that the weight of diffusion is bigger within uniform or inner regions, and the other one is $\lim_{s\to\infty} g(s) = 1$, so that it ensures TV-based diffusion to preserve edges.

Inspired by the edge stopping functions suggested by Perona and Malik [18], two adaptive weight functions $g_1(|\nabla u|)$ and $g_2(|\nabla u|)$ can be obtained satisfying two conditions mentioned above given in (19) & (20):

$$g_1(|\nabla u|) = (\alpha - 1)\exp\left[-\left(\frac{|\nabla u|}{K}\right)^2\right] + 1, \alpha > 1, \quad (19)$$

$$g_2(|\nabla u|) = \frac{\alpha + |\nabla u|^2/K^2}{1 + |\nabla u|^2/K^2}, \alpha > 1, \quad (20)$$

where K is the gradient magnitude threshold parameter that decides the amount of diffusion to take place.

Then based on the proposal of an edge stopping function called Tukey's biweight function given in Black et.al [19] another one is given in(21):

$$g_3(|\nabla u|) = \begin{cases} 0.5(\alpha - 1)\left[1 - \left(\frac{|\nabla u|}{S}\right)^2\right] + 1;\ \alpha > 1, |\nabla u| \le S \\ 1,\ otherwise \end{cases}$$
(21)

where

$$S = \sqrt{2}K$$

Weickert [20] proposed a novel edge stopping function, and some changes are made to it, then the resulting form is given in (22):

$$g_4(|\nabla u|) = \begin{cases} \alpha - (\alpha - 1)\exp(-3.31488 * K^8/|\nabla u|^8);\ |\nabla u| \ne 0 \\ \alpha;\ \text{otherwise} \end{cases} \quad (22)$$

## 4. NUMBERICAL EXPERIMENTS

In all of the experiments performed in this paper, the algorithm was tested using three typical images: Lena(512×512), Airfield(512 ×512) and Plane(512×512). By adopting the variable-controlling approach the effects of different parameters are analyzed for each weight function on the performance of the proposed model. The quality metrics such as MAE and PSNR are calculated and studied with respect to the changes of three parameters including α, K and β, all the values are those shown in table 1, table 2 and table 3. In order to get the optical pleasurable result and the maximum value of PSNR the optimum iteration times are chosen every time.

**Table 1. Numerical results regarding α for Lena image with Gaussian white noise of σ = 50 with K=45, $\lambda = 0.01$ and $\beta = 0.012$.**

| Quality Metrics | Weight Functions | α | | |
|---|---|---|---|---|
| | | 1.6 | 1.8 | 2.0 |
| MAE | $g_1$ | 7.8671 | 7.7093 | 7.6275 |
| | $g_2$ | 7.8272 | **7.6967** | **7.5428** |
| | $g_3$ | 8.2944 | 8.1057 | 7.9624 |
| | $g_4$ | **7.8243** | 7.6997 | 7.5934 |
| PSNR | $g_1$ | 27.2426 | 27.3428 | 27.3836 |
| | $g_2$ | 27.2870 | **27.3666** | **27.4751** |
| | $g_3$ | 26.9146 | 27.0606 | 27.1731 |
| | $g_4$ | **27.2876** | 27.3599 | 27.4015 |

**Table 2. Numerical results regarding K for Lena image with Gaussian white noise of σ = 50 with $\alpha = 1.8$, $\lambda = 0.01$ and $\beta = 0.012$.**

| Quality Metrics | Weight Functions | K | | |
|---|---|---|---|---|
| | | 30 | 50 | 70 |
| MAE | $g_1$ | 7.7698 | 7.7002 | 7.6892 |
| | $g_2$ | 7.7106 | 7.6959 | **7.6331** |
| | $g_3$ | 8.1938 | 8.0908 | 8.0490 |
| | $g_4$ | **7.7102** | **7.6845** | 7.6332 |
| PSNR | $g_1$ | 27.2849 | 27.3523 | 27.3667 |
| | $g_2$ | **27.3450** | **27.3777** | **27.4491** |
| | $g_3$ | 26.9730 | 27.0841 | 27.1389 |
| | $g_4$ | 27.3296 | 27.3619 | 27.4322 |

The analysis of the experimental data and results from table 1 and table 2 clearly indicates that with the increase of the values of α and K, the quality metrics PSNR and MAE perform better and better, i.e. the values of PSNR get bigger, and the values of MAE smaller. And in table 3 the experimental data illustrates the effect of parameter β for each weight function on the performance of the proposed model. Contrary to α and K, with the increase of value of β, the quality metrics PSNR and MAE go towards a bad direction.Moreover by carefully contrasting all the data in table 1, the model using weight function $g_2(|\nabla u|)$ is outperformed in most cases especially in terms of PSNR, which can be clearly seen from the data highlighted in boldface. Therefore from Fig.1 to Fig.4 typically the model using the weight function $g_2(|\nabla u|)$ is adopted for the research goals.

**Table 3. Numerical results regarding $\beta$ for Lena image with Gaussian white noise of $\sigma$ = 50 with $\alpha = 1.8$, K=45 and $\lambda = 0.01$.**

| Quality Metrics | Weight Functions | $\beta$ | | |
|---|---|---|---|---|
| | | 0.01 | 0.0125 | 0.015 |
| MAE | $g_1$ | 7.7040 | 7.7356 | 7.8719 |
| | $g_2$ | **7.6134** | 7.7065 | **7.8055** |
| | $g_3$ | 7.9717 | 8.1486 | 8.4567 |
| | $g_4$ | 7.6255 | **7.6998** | 7.8090 |
| PSNR | $g_1$ | 27.3828 | 27.3123 | 27.0885 |
| | $g_2$ | **27.4821** | **27.3449** | **27.1932** |
| | $g_3$ | 27.2371 | 27.0070 | 26.6217 |
| | $g_4$ | 27.4557 | 27.3368 | 27.1631 |

Though the increase of value of $\alpha$ would benefit the improvements of PSNR and MAE, it would also destroy some details and edges because of over-smoothing of textured regions, which can be seen in Fig.1. So in order to achieve a balance between noise removal and edge or texture protection, the suitable $\alpha$ should be adopted. In this paper the value of $\alpha$ is set equal to 1.8 or 2. From Fig.2 it can be found that though the variation of parameter K would change the quality metrics PSNR and MAE slightly, it may have little effects on the performance of the model visually. According to the experimental results of Fig.3 with the increasing value of $\beta$, the contrast between coherent structures and the background is enhanced obviously. However the negative influence of some noisy points are also amplified, and this may destroy the original information of the image. So an appropriate value of $\beta$ is vital to enhance the contrast while ensuring the the original information of image. $\beta$ is set equal to 0.012 in this paper.

**Fig 1: Denoising results of our proposed model using weight function $g_2$, Tese results are obtained by only changing the value of $\alpha$, from left to right $\alpha$=1.6, 2.2 and 2.8 respectively with the other two fixed parameters of K=45 and $\beta$=0.012.**

**Fig 2: Denoising results of our proposed model using weight function $g_2$. These results are obtained by only changing the value of K, from left to right K=45, 150 and 255 respectively with the other two fixed parameters of $\alpha$=1.8 and $\beta$=0.012.**

**Fig 3:Denoising results of our proposed model using weight function $g_2$. These results are obtained by only changing the value of β, from left to right β=0.01, 0.0125 and 0.015 respectively with the other two fixed parameters of α=1.8 and K=45.**

**Fig 4: Denoising results of different models. First column: images restored using the TV model($\tau$ = 0.10 and $\lambda$ = 0.01). Middle column: images restored using our proposed model(11)-(18)($\tau$ = 0.10, $\lambda$ = 0.01, α=2.0, K=45 and β=0.012). Right column: images restored using the model proposed by Guo et al.[12]($\tau$ = 0.10 and $\lambda$ = 0.01).**

In order to demonstrate the effectiveness of the proposed model the TV filter and the fundamental model proposed by Guo et al.[12] are also tested for comparative purposes.The results of the comparisons between our novel algorithm and other filters are shown in Fig.4 and table 4 for three typical images with Gaussian white noise of $\sigma = 50$. Clearly, the proposed method performed much better than the original model especially in terms of enhancement and iteration steps.

TV filter completed the removal of noise but the reduced contrast was clearly visible compared with the denoised results of the proposed model.

**Table 4. Denoised results of different models for three typical images with Gaussian white noise of σ = 50.**

| Metrics | Methods | Image | | |
|---|---|---|---|---|
| | | Lena | Airfield | Plane |
| MAE | TV | 8.0381 | 13.1757 | 8.2417 |
| | Guo | 8.2337 | 12.9325 | 8.4372 |
| | Ours | **7.5428** | **12.1330** | **7.5901** |
| PSNR | TV | 27.1250 | 23.2472 | 26.4581 |
| | Guo | 27.1950 | 23.4257 | 26.6145 |
| | Ours | **27.4751** | **23.6361** | **26.8783** |
| Iteration Number | TV | 800 | 533 | 731 |
| | Guo | 754 | **446** | 608 |
| | Ours | **481** | 488 | **475** |

## 5. CONCLUSION

In this study, a novel and efficient method based on reaction-diffusion system was proposed for handling noisy images with textures. Restoration of degraded texture images is a challenging task that affects various applications. The main features of the proposed method are as follows: (1) based on the reaction-diffusion system, the approach can effectively remove the noise while preserving small details and edges; (2) introducing a source term into the diffusion process allows for the contrast enhancement of textures; (3) combining the advantages of reaction-diffusion system with the use of a source term eliminates the main drawback of the original model; (4) the numerical experiment demonstrates the encouraging results because of their outperforming the original systems.

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