Studies the Reliability and Availability Characteristics of Two Different System under Preventive Maintenance

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ABSTRACT
The research studies the availability and reliability to two systems which different due to the effect of preventive maintenance (PM), a two system analyses a two state with two type failure. The rate of failure is exponential distribution but the rate of repair is general distribution. The system resolved by supplementary technique and Laplace transforms depend on Complex imagine roots. Several measures of availability and (MTTF) of system are obtained. we analysis graphically to watch the impress of several system parameters in mean time failure and availability.

Keywords
Availability, Reliability, (MTTF), Laplace transforms (L.T), supplementary technique, preventive maintenance (PM).

1. INTRODUCTION
Reliability study of the repair problem of device is importance in our lifetime where it is used widely in the manufacturing system. So the systems repairable study is an important component in reliability analysis.. Many authors study availability, reliability and (MTTF) under P.M. Like [1] deals with cost-benefit of a two- unit cold standby system with two-phase repair of the failed unit and preventive maintenance. The rate to failure and time to go PM are exponential while rate of repair and the time till go PM are general. [2] The author presents a two system having single unit in parallel which different because to the additional of the preventive maintenance. [3] Studied two parallel systems which every unit has two failure type where unit fails due to operating characteristics, so, the system go under preventive maintenance randomly. Rate of Failure are constant while the rate of PM & repair are general.[4] This paper deals with an aero plane model; namely, a two-unit (non-identical) parallel system with dual mode of failures which preventive maintenance happen at random epochs. [5] Deals with a 2-state repairable complex system of two failure types which solved by Laplace transform which rate of failure and repair of [type1, type2] are assumed as exponential distributed. [6] Study the availability and profit analysis of a repairable redundant 3-out-of-4 system with preventive [7] this paper show complex system where fail in n-mutually exclusive ways of total failure. [8] Study some reliability parameters of three states with failure environmental [9] the author tale about complex system consisting two subsystems where use supplementary technique & Laplace Transform (L.T). [10] Talk about the system reliability where transform the basic equations of the model into integro-differential eq. and solve it using Supplementary variables. This research show comparison between two systems where they different

because of the additional of PM. the two systems resolved by partial differential eq. & Laplace Transform where depend on Complex imagine roots then we show numerical results to analyze the impress of the various system parameters on reliability and system availability.

2. SYSTEM DESCRIPTION
The system is analyzed under following practical assumptions:

- The system unit contain of two state repairable units with two failure types.
- The system has two repair facilities, first for repairing type 1 & the second for type 2.
- After repairing a unit, it will work like a new state.

We suppose that preventive maintenance is provided to this system at random epochs when the system at state So. Through the P.M the system remains operating.

3. NOTATIONS AND SYSTEM STATES:

λ_1 & λ_2 Rate of failure of type 1& type 2

μ_1 & μ_2 Rate of repair of type 1 & type 2

μ_3 Constant rate for taking a unit into preventive maintenance

$\mu_1(x)$, $\mu_2(y)$: general repair of S_1, S_2 elapsed time of repair x and y.

$\mu_4(x)$: General repair end of PM

$p_j(\tau)$ Probability that the system is in state S_j at time t,j =0, 1, 2, 3

0→ normal state

1→ failed state of type1.

2→ failed state of type2.

3 → normal state and preventive maintenance.

$P_1(x,\tau)$ &$P_2(y,\tau)$ Probability that the system in state S_1&S_2at time τ, and under repair, elapsed time of repair is x& y

$P_3(x,\tau)$: Probability that the system in state S_3at time τ, and under Preventive Maintenance, elapsed time of repair is x

$p_j^*(s)$ Laplace transform (L.T) of $p_j(\tau)$

A (τ): functions of availability.

R (τ): functions of reliability.

MTTF: mean time failure.

Where Laplace transforms (L.T) of $p_j(\tau)$ is:

$p_j^*(s) = \int_0^\infty e^{-S\tau}\, p_j(\tau)$

There are a relation between repair rate $\mu_1(x)$, $\mu_2(y)$ and their cumulative functions F(x) , F(y), i.e.

$F_1(x) = \mu_1(x)\, e^{-\int_0^x \mu_1(x)\,dx}$

$F_2(y) = \mu_2(y)\, e^{-\int_0^y \mu 2(y)\,dy}$

$F_3(x) = \mu_4(x)\, e^{-\int_0^x \mu_4(x)\,dx}$

First system with P.M

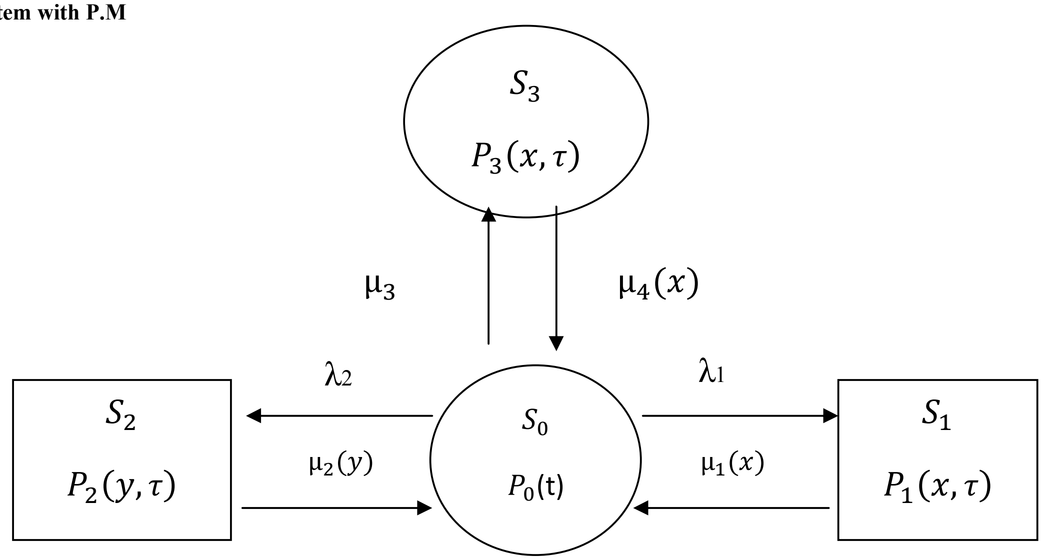

Figure1. System configuration diagram

4. MATHEMATICAL MODEL DESCRIPTION

This part showing the differential eq. for the system of figure (1) Transition states

$$\left(\frac{\partial}{\partial t} + \lambda_1 + \lambda_2 + \mu_3\right) P_0(\tau) = \int_0^\infty \mu_1(x)\, P_1(x,\tau)dx + \int_0^\infty \mu_2(y) P_2(y,\tau)\,dy + \int_0^\infty \mu_4(x)\, P_3(x,\tau)dx \quad (4\text{-}1)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_1(x)\right) P_1(x,\tau) = 0 \quad (4\text{-}2)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \mu_2(y)\right) P_2(y,\tau) = 0 \quad (4\text{-}3)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_4(x)\right) P_3(x,\tau) = 0 \quad (4\text{-}4)$$

Initial conditions:

$$P_j(0) = \begin{cases} 1 & \text{where } j = 0 \\ 0 & \text{else} \end{cases}$$

Boundary conditions

$$P_1(0,\tau) = \lambda_1\, P_0(\tau)$$

$$P_2(0,\tau) = \lambda_2\, P_0(\tau)$$

$$P_3(0,\tau) = \mu_3\, P_0(\tau)$$

5. MODEL SOLUTION

Use Laplace Transform (L.T) For Eq. (4-1) to (4-4) and Boundary conditions

$$(s + \lambda_1 + \lambda_2 + \mu_3)\, P_0^*(s) = 1 + \int_0^\infty \mu_1(x)\, P_1^*(x,s)dx \; \int_0^\infty \mu_2(y) P_2^*(y,s)\,dy + \int_0^\infty \mu_4(x)\, P_3^*(x,s)dx \quad (5\text{-}1)$$

$$\left(s + \frac{\partial}{\partial x} + \mu_1(x)\right) P_1^*(x,s) = 0 \quad (5\text{-}2)$$

$$\left(s + \frac{\partial}{\partial y} + \mu_2(y)\right) P_2^*(y,s) = 0 \quad (5\text{-}3)$$

$$\left(s + \frac{\partial}{\partial x} + \mu_4(x)\right) P_3^*(x,s) = 0 \quad (5\text{-}4)$$

With Boundary conditions

$$P_1^*(0,s) = \lambda_1\, P_0^*(s) \quad (5\text{-}5)$$

$$P_2^*(0,s) = \lambda_2\, P_0^*(s) \quad (5\text{-}6)$$

$$P_2^*(0,s) = \mu_3 P_0^*(s) \quad (5\text{-}7)$$

$$P_1^*(x,s) = P_1^*(0,s)\, e^{-sx - \int_0^x \mu_1(x)\,dx} \quad (5\text{-}8)$$

$$P_2^*(y,s) = P_2^*(0,s)\, e^{-sy - \int_0^y \mu_2(y)\,dy} \quad (5\text{-}9)$$

$$P_3^*(x,s) = P_3^*(0,s)\, e^{-sx - \int_0^x \mu_4(x)\,dx} \quad (5\text{-}10)$$

Again Integrating by parts equations (5-8)-(5-10) using (5-5)-(5-7)

$$P_1^*(s) = \int_0^\infty P_1^*(x,s)\, dx = P_1^*(0,s)\, s^{-1}\{1 - F_1^*(s)\}$$

$$\therefore P_1^*(s) = P_0^*(s)\; A_1(s) \quad (5\text{-}11)$$

$$\therefore P_2^*(s) = P_0^*(s)\; A_2(s) \quad (5\text{-}12)$$

$$\therefore P_3^*(s) = P_0^*(s)\; A_3(s) \quad (5\text{-}13)$$

Where

$A_1(s) = \lambda_1\, N_1(s)$

$A_2(s) = \lambda_2\, N_2(s)$

$A_3(s) = \mu_3\, N_3(s)$

$N_1(s) = s^{-1}\{1 - F_1^*(s)\}$

$N_2(s) = s^{-1}\{1 - F_2^*(s)\}$

$N_3(s) = s^{-1}\{1 - F_3^*(s)\}$

And

$F_1^*(s) = \int_0^\infty \mu_1(x)\ e^{-sx - \int_0^x \mu_1(x)dx}dx$

$F_2^*(s) = \int_0^\infty \mu_2(y)\ e^{-sy - \int_0^y \mu_2(y)dy}dy$

$F_3^*(s) = \int_0^\infty \mu_4(x)\ e^{-sx - \int_0^x \mu_4(x)dx}dx$

Also we have from equations (5-8)-(5-10)

using equations (5-5)-(5-7)

$\int_0^\infty P_1^*(x,s)\mu_1(x)\ dx = \lambda_1 P_0^*(s) F_1^*(s)$ (5-14)

$\int_0^\infty P_2^*(y,s)\mu_2(y)dy = \lambda_2 P_0^*(s) F_2^*(s)$ (5-15)

$\int_0^\infty P_3^*(x,s)\mu_4(x)\ dx = \mu_3 P_0^*(s) F_3^*(s)$ (5-16)

Now from equations (5-14) & (5-15) and (5-16) in (5-1) we get

$$P_0^*(s) = \frac{1}{s+\lambda_1+\lambda_2+\mu_3-\lambda_1 F_1^*(s)-\lambda_2 F_2^*(s)-\mu_3 F_3^*(s)} = \frac{1}{A(s)} \quad (5\text{-}17)$$

Where

$A(s) = s + \lambda_1 + \lambda_2 + \mu_3 - \lambda_1 F_1^*(s) - \lambda_2 F_2^*(s) - \mu_3 F_3^*(s)$

6. EVALUATION DOWN AND UP STATE AVAILABILITY BY LAPLACE TRANSFORMS

The system probability in operable (up) and failed (down) state at time "τ "can be obtained by Laplace transform as:

$P_{up}^*(s) = P_0^*(s) + P_3^*(s)$ (6-1)

$P_{down}^*(s) = 1 - P_{up}^*(s)$ (6-2)

7. PARTICULAR CASE

In this section the up and down state availability, MTTF, the steady -state availability of the system have been evaluated, when repair times will be exponential distribution.

Setting

$F_1^*(s) = \frac{\mu_1}{s+\mu_1}$, $F_2^*(s) = \frac{\mu_2}{s+\mu_2}$,

$F_3^*(s) = \frac{\mu_4}{s+\mu_4}$

In equations (5-11)-(5-13),(5-17) we gets

$$P_0^*(s) = \frac{(s+\mu_1)(s+\mu_2)(s+\mu_4)}{(s+\lambda_1+\lambda_2+\mu_3)(s+\mu_1)(s+\mu_2)(s+\mu_4)-\lambda_1\mu_1(s+\mu_2)(s+\mu_4)-\lambda_2\mu_2(s+\mu_1)(s+\mu_4)-\mu_4\mu_3(s+\mu_1)(s+\mu_2)} \quad (7\text{-}1)$$

$$P_1^*(s) = \frac{\lambda_1(s+\mu_2)(s+\mu_4)}{(s+\lambda_1+\lambda_2+\mu_3)(s+\mu_1)(s+\mu_2)(s+\mu_4)-\lambda_1\mu_1(s+\mu_2)(s+\mu_4)-\lambda_2\mu_2(s+\mu_1)(s+\mu_4)-\mu_4\mu_3(s+\mu_1)(s+\mu_2)} \quad (7\text{-}2)$$

$$P_2^*(s) = \frac{\lambda 2(s+\mu_1)(s+\mu_4)}{(s+\lambda_1+\lambda_2+\mu_3)(s+\mu_1)(s+\mu_2)(s+\mu_4)-\lambda_1\mu_1(s+\mu_2)(s+\mu_4)-\lambda_2\mu_2(s+\mu_1)(s+\mu_4)-\mu_4\mu_3(s+\mu_1)(s+\mu_2)} \quad (7\text{-}3)$$

$$P_3^*(s) = \frac{\mu_3(s+\mu_1)(s+\mu_2)}{(s+\lambda_1+\lambda_2+\mu_3)(s+\mu_1)(s+\mu_2)(s+\mu_4)-\lambda_1\mu_1(s+\mu_2)(s+\mu_4)-\lambda_2\mu_2(s+\mu_1)(s+\mu_4)-\mu_4\mu_3(s+\mu_1)(s+\mu_2)} \quad (7\text{-}4)$$

From equation (7-1),(7-4), one may get

$P_{up}^*(s) = P_0^*(s) + P_3^*(s)$

$$P_{up}^*(s) = \frac{(s+\mu_1)(s+\mu_2)(s+\mu_4)+\mu_3(s+\mu_1)(s+\mu_2)}{(s+\lambda 1+\lambda 2+\mu_3)(s+\mu_1)(s+\mu_2)(s+\mu_4)-\lambda 1\mu_1(s+\mu_2)(s+\mu_4)-\lambda 2\mu_2(s+\mu_1)(s+\mu_4)-\mu_4\mu_3(s+\mu_1)(s+\mu_2)}$$

$$P_{up}^*(s) = \frac{s^3+As^2+BS+m}{s[s^3+a_1s^2+a_2S+a_3]}$$

Where

$a_1 = \mu_1 + \mu_2 + \lambda_1 + \lambda_2 + \mu_3 + \mu_4$

$a_2 = \mu_1\mu_2 + \mu_1\lambda_2 + \mu_2\lambda_1 + \mu_1\mu_3 + \mu_1\mu_4 + \mu_2\mu_3 + \mu_2\mu_4 + \mu_4\lambda_2 + \mu_4\lambda_1$

$a_3 = \mu_1\mu_2\mu_3 + \mu_1\mu_2\mu_4 + \mu_1\mu_4\lambda_2 + \mu_2\mu_4\lambda_1$

$A = \mu_1 + \mu_4 + \mu_2 + \mu_3$

$B = \mu_4\mu_2 + \mu_1\mu_4 + \mu_1\mu_2 + \mu_3\mu_2 + \mu_1\mu_3$

$m = \mu_4\mu_1\mu_2 + \mu_3\mu_1\mu_2$

7.1. Cubic equations roots have are 2 cases

7.1.1. First case

(D > 0) [1 root is real and 2 complex]

$$P_{up}^*(s) = \frac{s^3+As^2+BS+m}{s(s+A_1-W)(S+A_1+w_1-i\sqrt{3}v_1)(S+A_1+w_1+i\sqrt{3}v_1)}$$

Where

$q = \frac{3a_2 - a_1^2}{9}$, $r = \frac{9a_1a_2 - 2a_1^3 - 27a_3}{54}$

$D = q^3 + r^2$, $u = (r + \sqrt{D})^{\frac{1}{3}}$,

$\tau = (r - \sqrt{D})^{\frac{1}{3}}$

$w_1 = \frac{(u+\tau)}{2}$, $w = (u+\tau)$

$v_1 = \frac{(u-\tau)}{2}$, $A_1 = \frac{a_1}{3}$

Applying inverse Laplace Transform for the eq., we obtain

$$P(\tau) = \frac{m}{(A_1-w)(A_1^2+A_1w+w_1^2+3v_1^2)} + \frac{(-A_1+w)^3+A(-A_1+w)^2+B(-A_1+w)+m}{(-A_1+w)(9w_1^2+3v_1^2)}e^{(-A_1+W)\tau}$$

$$+ \left\{\frac{2[PX+3HT][\cos\sqrt{3}(v_1)t]-2\sqrt{3}\,[(HX)-(TP)](\sin\sqrt{3}(v_1)t)}{X^2+3T^2}\right\}e^{(-A_1-w_1)\tau}$$

Where,

$P = (-A_1^3 - 3A_1^2w_1 + 9A_1v_1^2 - w_1^3 + 9w_1v_1^2 - A_1w_1^2) + A(A_1^2 + A_1w - 3v_1^2 + w_1^2 + B(-A_1 - w_1) + m$

$H = (6A_1v_1w_1 - 3v_1^3 + 3w_1^2v_1 + 3A_1^2v_1 - AA_1v - Av_1w + Bv_1)$

$X = 3v^2 w + 6{v_1}^2 A_1$

$T = 3wv_1A_1 + 3{w_1}^2 v - 6{v_1}^3$

- Reliability System and availability

System availability:

Availability of the system can be get from the relation

$$A(\tau)= \frac{m}{(A_1 - w)({A_1}^2 + A_1 w + {w_1}^2 + 3{v_1}^2)}$$

$$+ \frac{(-A_1 + w)^3 + A(-A_1 + w)^2 + B(-A_1 + w) + m}{(-A_1 + w)(9{w_1}^2 + 3{v_1}^2)} e^{(-A_1 + W)\tau}$$

$+$

$$\left\{\frac{2[PX+3HT][\cos\sqrt{3}(v_1)t] - 2\sqrt{3}\,[(H\,X) - (TP)](\sin\sqrt{3}(v_1)t)}{X^2+3T^2}\right\} e^{(-A_1 - w_1)\tau}$$

So we can obtain the steady – state availability (A) from the following relation

$A = \lim_{t\to\infty} A(\tau)$

$$A = \frac{m}{(A_1 - w)({A_1}^2 + A_1 w + {w_1}^2 + 3{v_1}^2)}$$

- The mean time failure (MTTF):

Taking all repairs zero in (7-4), mean time to failure of the system is obtained as

$$MTTF = \lim_{s\to 0} P^*_{up}(s)$$

7.1.2. Second case $D < 0$ [All roots are real and unequal]

$$P^*_{up}(s) = \frac{s^3 + As^2 + BS + m}{s(s + A_1 - w_0)(S + A_1 - w_2)(S + A_1 - v_2)}$$

Where

$s_1 = w_0 - \frac{a_1}{3}$, $s_2 = w_2 - \frac{a_1}{3}$

$s_3 = v_2 - \frac{a_1}{3}$, $\theta = \cos^{-1}\frac{r}{\sqrt{-q^3}}$

$w_0 = 2\sqrt{-q}\cos(\frac{\theta}{3})$, $w_2 = 2\sqrt{-q}\cos(\frac{\theta}{3} + 120)$

$v_2 = 2\sqrt{-q}\cos(\frac{\theta}{3} + 240)$, $A_1 = \frac{a_1}{3}$

By apply inverse Laplace transform

$$P_{up}(\tau) = \frac{m}{(A_1 - w_0)({A_1}^2 - A_1 w_2 - A_1 v_2 + w_2 v_2)}$$

$$+ \frac{(-A_1 + w_0)^3 + A(-A_1 + w_0)^2 + B(-A_1 + w_0) + m}{(-A_1 + w_0)({w_0}^2 - w_0 w_2 - w_0 v_2 + w_2 v_2)} e^{(-A_1 + w_0)\tau}$$

$$+ \frac{(-A_1 + w_2)^3 + A(-A_1 + w_2)^2 + B(-A_1 + w_2) + m}{(-A_1 + w_2)({w_2}^2 - w_0 w_2 - w_2 v_2 + w_0 v_2)} e^{(-A_1 + w_2)\tau}$$

$$+ \frac{(-A_1 + v_2)^3 + A(-A_1 + v_2)^2 + B(-A_1 + v_2) + m}{(-A_1 + v_2)({v_2}^2 - w_0 v_2 - w_2 v_2 + w_0 w_2)} e^{(-A_1 + v_2)\tau}$$

- Reliability System and availability

System availability:

$$A(\tau) = \frac{m}{(A_1 - w_0)({A_1}^2 - A_1 w_2 - A_1 v_2 + w_2 v_2)}$$

$$+ \frac{(-A_1 + w_0)^3 + A(-A_1 + w_0)^2 + B(-A_1 + w_0) + m}{(-A_1 + w_0)({w_0}^2 - w_0 w_2 - w_0 v_2 + w_2 v_2)} e^{(-A_1 + w_0)\tau}$$

$$+ \frac{(-A_1 + w_2)^3 + A(-A_1 + w_2)^2 + B(-A_1 + w_2) + m}{(-A_1 + w_2)({w_2}^2 - w_0 w_2 - w_2 v_2 + w_0 v_2)} e^{(-A_1 + w_2)\tau}$$

$$+ \frac{(-A_1 + v_2)^3 + A(-A_1 + v_2)^2 + B(-A_1 + v_2) + m}{(-A_1 + v_2)({v_2}^2 - w_0 v_2 - w_2 v_2 + w_0 w_2)} e^{(-A_1 + v_2)\tau}$$

And the steady – state availability are:

$A = \lim_{t\to\infty} A(\tau)$

$$A = \frac{m}{(A_1 - w_0)({A_1}^2 - A_1 w_2 - A_1 v_2 + w_2 v_2)}$$

- The mean time failure (MTTF):

Taking all repairs zero in (7-4), mean time to failure of the system is obtained as

$$MTTF = \lim_{s\to 0} P^*_{up}(s)$$

$$MTTF = \lim_{s\to 0} \frac{(s+\mu_1)(s+\mu_2)(s+\mu_4) + \mu_3(s+\mu_1)(s+\mu_2)}{(s + \lambda_1 + \lambda_2 + \mu_3)(s+\mu_1)(s+\mu_2)(s+\mu_4) - \lambda_1\mu_1(s+\mu_2)(s+\mu_4) - \lambda_2\mu_2(s+\mu_1)(s+\mu_4) - \mu_4\mu_3(s+\mu_1)(s+\mu_2)}$$

$$MTTF = \frac{\mu_4 + \mu_3}{(\lambda_1 + \lambda_2)(\mu_4)}$$

System without PM

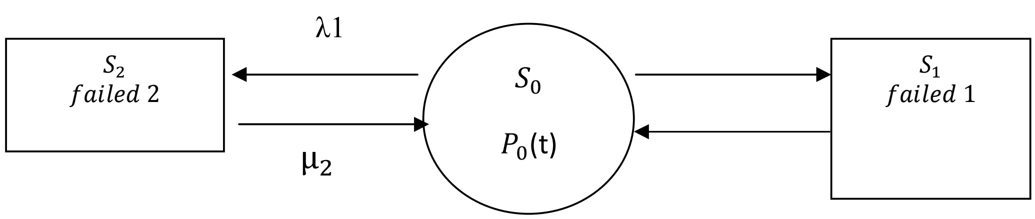

Figure2. System configuration diagram

8. MATHEMATICAL MODEL DESCRIPTION

This part showing the differential eq. for the system of Table (1) Transition states

$$(\frac{\partial}{\partial t} + \lambda_1 + \lambda_2)\, P_0(t) = \int_0^\infty \mu_1(x)\; P_1(x,t)dx + \int_0^\infty \mu_2(x) P_2(y,t)\, dy \quad (8\text{-}1)$$

$$(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_1\,(x))P_1(x,t) = 0 \quad (8\text{-}2)$$

$$(\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \mu_2\,(y))P_2(y,t) = 0 \quad (8\text{-}3)$$

Initial conditions:

$$P_j(0) = \begin{cases} 1 & \text{where } j = 0 \\ 0 & \text{else} \end{cases}$$

Boundary conditions

$P_1(0,t) = \lambda_1 P_0(t)$, $P_2(0,t) = \lambda_2 P_0(t)$

9. SOLUTION OF THE MODEL

By taking Laplace transform for (8-1) to (8-3) and Boundary conditions

$$(s+\lambda_1+\lambda_2)P_0^*(s) = 1+\int_0^\infty \mu_1(x)\,P_1^*(x,s)dx + \int_0^\infty \mu_2(y)P_2^*(y,s)\,dy \qquad (9\text{-}1)$$

$$(s+\frac{\partial}{\partial x}+\mu_1(x))P_1^*(x,s)=0 \qquad (9\text{-}2)$$

$$(s+\frac{\partial}{\partial y}+\mu_2(y))P_2^*(y,s)=0 \qquad (9\text{-}3)$$

With Boundary conditions

$$P_1^*(0,s)=\lambda_1 P_0^*(s) \qquad (9\text{-}4)$$

$$P_2^*(0,s)=\lambda_2 P_0^*(s) \qquad (9\text{-}5)$$

Integrating equations (9-2) & (9-3)

$$P_1^*(x,s)=P_1^*(0,s)\,e^{-sx-\int_0^x \mu_1(x)\,dx} \qquad (9\text{-}6)$$

$$P_2^*(y,s)=P_2^*(0,s)\,e^{-sy-\int_0^y \mu_2(y)\,dy} \qquad (9\text{-}7)$$

Again Integrating by parts equations (9-6) & (9-7) using (9-4) & (9-5)

$$P_1^*(s)=\int_0^\infty P_1^*(x,s)\,dx = P_1^*(0,s)\,s^{-1}\{1-F_1^*(s)\}$$

$$\therefore P_1^*(s)=P_0^*(s)\;A_1(s) \qquad (9\text{-}8)$$

$$\therefore P_2^*(s)=P_0^*(s)\;A_2(s) \qquad (9\text{-}9)$$

Where

$A_1(s)=\lambda_1 N_1(s)$

$A_2(s)=\lambda_2 N_2(s)$

$N_1(s)=s^{-1}\{1-F_1^*(s)\}$

$N_2(s)=s^{-1}\{1-F_2^*(s)\}$

And

$$F_1^*(s)=\int_0^\infty \mu_1(x)\;e^{-sx-\int_0^x \mu_1(x)\,dx}\,dx \;,$$

$$F_2^*(s)=\int_0^\infty \mu_2(y)\;e^{-sy-\int_0^y \mu_2(y)\,dy}\,dy$$

Also we have from equations (9-6) & (9-7) using equations (9-4) & (9-5)

$$\int_0^\infty P_1^*(x,s)\,\mu_1(x)\,dx = \lambda_1 P_0^*(s)\,F_1^*(s) \qquad (9\text{-}10)$$

$$\int_0^\infty P_2^*(y,s)\,\mu_2(y)dy = \lambda_2 P_0^*(s)\,F_2^*(s) \qquad (9\text{-}11)$$

Now from equations (9-10) & (9-11) in (8-1) we get

$$P_0^*(s)=\frac{1}{S+\lambda_1+\lambda_2-\lambda_1 F_1^*(s)-\lambda_2 F_2^*(s)}=\frac{1}{A(s)} \qquad (9\text{-}12)$$

Where

$A(s) = S+\lambda_1+\lambda_2-\lambda_1 F_1^*(s)-\lambda_2 F_2^*(s)$

10. EVALUATION DOWN AND UP STATE AVAILABILITY BY LAPLACE TRANSFORMS

The system probability in operable (up) and failed (down) state at time "τ "can be obtained by Laplace transform as:

$$P_{up}^*(s)=P_0^*(s) \qquad (10\text{-}1)$$

$$P_{down}^*(s)=1-P_{up}^*(s) \qquad (10\text{-}2)$$

11. PARTICULAR CASE

In this section the up and down state availability, MTTF, the steady -state availability of the system have been evaluated, when repair times will be exponential distribution.

$$F_1^*(s)=\frac{\mu_1}{s+\mu_1}\;,\qquad F_2^*(s)=\frac{\mu_2}{s+\mu_2}$$

$$P_0^*(s)=\frac{(s+\mu_1)(s+\mu_2)}{s(s^2+s\lambda_1+s\lambda_2+s\mu_1+s\mu_2+\lambda_1\mu_2+\lambda_2\mu_1+\mu_1\mu_2)} \qquad (11\text{-}1)$$

$$P_1^*(s)=\{\frac{\lambda_1(s+\mu_2)}{s(s^2+s\lambda_1+s\lambda_2+s\mu_1+s\mu_2+\lambda_1\mu_2+\lambda_2\mu_1+\mu_1\mu_2)}\} \qquad (11\text{-}2)$$

$$P_2^*(s)=\frac{\lambda_2(s+\mu_1)}{s(s^2+s\lambda_1+s\lambda_2+s\mu_1+s\mu_2+\lambda_1\mu_2+\lambda_2\mu_1+\mu_1\mu_2)} \qquad (11\text{-}3)$$

We know that

$$P_{up}^*(s)=P_0^*(s)$$

$$P_{up}^*(s)=\frac{(s+\mu_1)(s+\mu_2)}{s(s^2+s\lambda_1+s\lambda_2+s\mu_1+s\mu_2+\lambda_1\mu_2+\lambda_2\mu_1+\mu_1\mu_2)} \qquad (11\text{-}4)$$

$$P_{up}^*(s)=\frac{s^2+aS+b}{s[s^2+cS+d]}$$

Where

$b=\mu_1\mu_2$, $a=\mu_1+\mu_2$

$c=\mu_1+\mu_2+\lambda 1+\lambda 2$, $d=\mu_1\mu_2+\mu_1\lambda 2+\mu_2\lambda 1$

By using inverse of Laplace Transform (I.L.T) of eq., we obtain

$$p_{up}(t)=\frac{1}{d}\left\{b+e^{-\frac{1}{2}tc}\left[(d-b)\cosh\left(\frac{1}{2}t\sqrt{c^2-4d}\right)+\frac{(2ad-dc-cb)\sinh\left(\frac{1}{2}t\sqrt{c^2-4d}\right)}{\sqrt{c^2-4d}}\right]\right\}$$

- Reliability System and availability

System availability:

Availability of the system can be get from the relation

$$A_{up}(t)=\frac{1}{d}\left\{b+e^{-\frac{1}{2}tc}\left[(d-b)\cosh\left(\frac{1}{2}t\sqrt{c^2-4d}\right)+\frac{(2ad-dc-cb)\sinh\left(\frac{1}{2}t\sqrt{c^2-4d}\right)}{\sqrt{c^2-4d}}\right]\right\}$$

The steady – state availability can be obtained from the following relation

$$A=\lim_{t\to\infty} A_{up}(t)=\frac{\mu_1\mu_2}{\lambda_1\mu_2+\mu_1\mu_2+\lambda_2\mu_1}$$

- Mean time to system failure

Taking all repairs zero in (11-4), mean time to failure of the system is obtained as

$$MTTF=\lim_{s\to 0} P_{up}^*(s)$$

$$MTTF=\lim_{s\to 0}\frac{(s+\mu_1)(s+\mu_2)}{s(s^2+s\lambda_1+s\lambda_2+s\mu_1+s\mu_2+\lambda_1\mu_2+\lambda_2\mu_1+\mu_1\mu_2)}$$

$$MTTF=\frac{1}{(\lambda_1+\lambda_2)}$$

12. NUMERICAL EXAMPLE

To see the system behavior, we plot the steady -state availability for the models, against λ1keeping the other parameters fixed at

$\lambda_2 = 0.25$, $\mu_2=0.3$, $\mu_1=0.1$, $\mu_3=\mu_4=0.7$

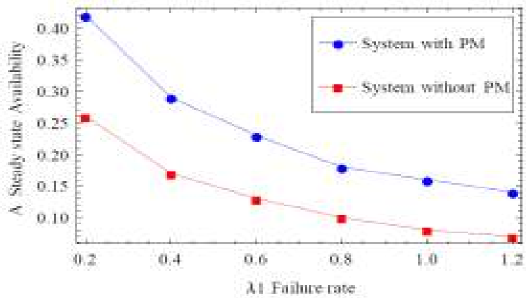

Fig 1.The Steady state Availability w.r.t. Failure Rate λ1

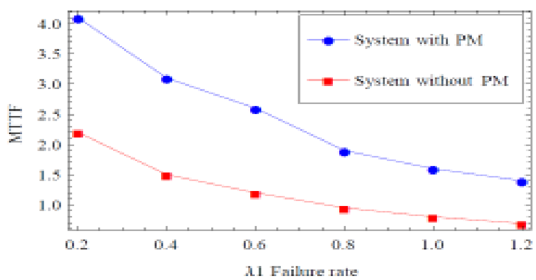

Fig 2.The mean time failure w.r.t. Failure Rate λ1

13. CONCLUSIONS

- We use computer software, to plot system availability and MTTF in figure 1 and 2 respectively. It is noted that A decrease as λ_1 increases and MTTF decrease as λ_1 increases also the system with (P.M) is better than the system without (P.M).

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