Radar Signals Compression using Singular Value Decomposition (SVD) Approach

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ABSTRACT
The ability to reduce noise in a received radar signal from its target largely indicates the accuracy of the radar system. Matched filters help to maximize the signal to noise ratio of radar systems. Matched Filters are commonly used in radars whose transmitted signals are known and are used as a replica to correlate with the received signal. The correlation is carried out in frequency domain applying Fourier Transforms. A Singular Value Decomposition (SVD) matrix filter is presented in place of the commonly used matched filter with the aim of leveraging the compression capability of the SVD algorithm as used in data compression. In this research, an FFT matched filter and an SVD matrix filter are simulated with an input chirp signal designed with LabVIEW. The results are compared based on their compression ratio, range resolution and signal to noise ratio. The SVD Matrix Filter from the simulation demonstrated a comparative improvement in compression ratio, range resolution, and signal to noise ratio over the Matched filter using the same parameters.

General Terms
Radar signal processing, singular value decomposition.

Keywords
Singular value decomposition, radar signals, signal-to-noise ratio, signal compression, radar.

1. INTRODUCTION
Radar technology has gone through a lot of development over the last 80 years starting from the late thirty's of the last century when radar was first invented for defence applications [1]. Radar as an acronym meaning radio Detection and Ranging, suggest the primary function of detecting and ranging target's positions using Radio Waves [2]. Radar refers to a system or techniques for determining the position, motion and characteristics of a remote object by radio waves reflected by that object [3]. By transmitting radiated energy into space, the echo signals (reflected signal) from the targets are received and processed [3]. Noise in signals influences the detection of the signal by the radar receiver, especially those occupying the same area in the frequency spectrum [4].

Adding a threshold at the output of the radar receiver forms the basis of detecting a radar signal. A signal will be considered to be noise if the amplitude of the receiver's output does not exceed that of the threshold [5]. The strength of the received echo determines the maximum detection range which is determined by a comparison of the received and transmitted signals which also shows the presence of targets although other information can also be obtained [5]. Skolnik, mentions range resolution and maximum range detection as two important factors to be noted [5].
Range resolution becomes better when the pulse width is narrower. Also, the quantity of energy in the pulse decreases if the pulse width is also decreased and hence the detection of the maximum range also reduces [5]. Pulse compression techniques are used to increase the sensitivity and resolution [5]. For detecting targets in noise, the Matched Filter is employed to increase the effective signal-to-noise ratio (SNR) [1].

The range resolution, pulse compression ratio and the signal to noise ratio are important issues which are of most important to engineers and radar experts. They are also core to the application of radar.

2. LITERATURE REVIEW
2.1 Signal to Noise Ratio
According to O'Donnell [6], signals from radars are affected by interferences like solar noise, galactic noise, man-made-noise and atmospheric noise. These are mostly evident in places where the signal levels are low. The NASA Science Mission Directorate, records that almost all electronic devices produces noise with different effects. The extra-terrestrial radiation sources also produce some levels of interference mostly in the Milky Way, the noise temperature and the absorption of electromagnetic radiation in the atmosphere [7].
The signal from the target at the output terminal of the radar competes with the sum of the noise [6]. At the receiver, the noise power is given by Equation 2.1 which consist of the product of boltzmann's constant (1.38 x 10^{-23} joules/^{o}k), k, the system noise temperature, T_n and the noise bandwidth of the receiver, B_n.

$$N = kB_nT_n \qquad \text{Equation 2.1}$$

In the stages of the radar signal receiver, the nose received is maximized since it cannot be separated from the backscattered signal [7]. The performance of a radar system is improved by either reducing the noise or increasing the radar signal [8]. SNR is the indication of a target being detected in range, can be explained in three related ways [6]. These are Noise Factor (F_n), Noise Figure and Noise Temperature.

2.2 Matched Filtering
As other radio waves, radar signal get interfered with noise in its two way trip from the transmitted antenna to the target and back to the antenna [9]. For engineers to derive meaning from the received signal, these noises must be reduced. The process of reducing the noise in the signal also increases the chance of noticing the actual signal which returned [9]. The matched filter, is the optimal solution [10].

Many models of matched filtering techniques are in use today. The choice of a Matched Filtering technique depends on the type of wave form, the processing and generation methods being employed in the radar system. Three of these models are Matched Filtering by Exciting the Coding Filter, Matched Filtering using Time Inverse of Echoes and Matched Filtering using Correlators [5]. There is also a pulse compression with FFT which takes advantage of a popular Digital Signal

Processing (DSP) technique for matched filtering involving a correlation of time domain signals [11]. The conversion of signals from time domain to the frequency domain is performed using Inverse Fast Fourier Transform (IFFT) and Fast Fourier Transform (FFT).

2.3 The Singular Value Decomposition (SVD) Matrix Filter

According to Hyun & Lee [12], many applications which uses high dimensioned data has benefited from the powerful computational ability of the SVD algorithm especially in science and engineering. Singular Value Decomposition (SVD) as a technique helps to derive parameters important to a given signal. It is applicable in ranked estimations, canonical correlation analysis and unconstrained linear least square [12]. Pan & Hamdi [13] adds that, the SVD is also applicable in real time signal processing especially in situations where the problem needs urgent attention and agrees to the real m-by-n (m x n) matrix factorization as being computationally intensive.

SVD provides three important benefits [14]. These are;

i. Exposing the relationships between original data items by transforming the correlated variables to uncorrelated data sets.

ii. Using fewer dimensions to get the best approximation of the data set using dimensions.

iii. As a procedure which identifies the dimensions of data points with most variations and orders them along these dimensions

SVD, have significantly impacted real life applications such as structural analysis, the design of intelligent systems, data mining and image compression [14]. The base idea of implementing SVD is the clear exposition of the substructure of the initial data units to a lower dimensional space and its descending order arrangement in variation [15]. Thus, making the like items in the data set look more alike and the unlike, deviating more from each other.

SVD has its base from linear algebra theorem which states that "a rectangular matrix M can be broken down into the product of three matrices. Thus, an orthogonal matrix U, a diagonal matrix Σ, and the transpose of an orthogonal matrix V" [12]. The approach of decomposition to matrix computations creates a platform of solving different types of problems and not to restrict the algorithm to particular problem [16].

From Figure 1, the matrices U, Σ and V are determinable if the rank of A is r and A is a $m \times n$ matrix. $\boldsymbol{U}$ represents a column-orthonormal matrix ($\boldsymbol{m} \times \boldsymbol{r}$): it has a unite vector in each of its columns and any two columns dot product is 0. $\boldsymbol{V}$ a column-orthonormal matrix $(\boldsymbol{n} \times \boldsymbol{r})$. Its transpose form is always used. Thus rows of $\boldsymbol{V^T}$ that are orthonormal.

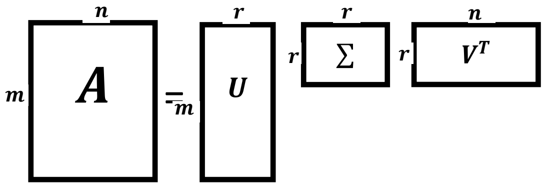

Figure 1: The Singular Value Decomposition

$\boldsymbol{\Sigma}$ represents a diagonal matrix with all elements not on the diagonal zero (0). These diagonal elements are called the Singular Values of **A.**

The SVD theorem is mathematically presented as;

$A_{mn} = U_{mr} \sum_{rr} V_{rn}^T$ Equation 2.2

From Equation 2-2, $\boldsymbol{U}$ constitute orthonormal eigenvectors of $\boldsymbol{AA^T}$, $\boldsymbol{V}$ also consist of orthonormal eigenvectors of $\boldsymbol{A^TA}$, and $\boldsymbol{\Sigma}$ represents a diagonal matrix containing the square roots of the eigenvalues from $\boldsymbol{U}$ or $\boldsymbol{V}$ in descending order. Eigenvectors of $\mathbf{AA^T}$ representing the left singular vectors of $\mathbf{A}$ are the first r columns of U covering the column space. Representing the right singular vectors of $\boldsymbol{A}$ covering the row space are the eigenvectors of A^TA *which is the first* $\boldsymbol{r}$ columns of $\boldsymbol{V}$. Focusing on the nonzero singular values of $\boldsymbol{r}$, $m\times r$, $r \times r$ and $r \times n$ *will* respectively be the effective dimensions of the SVD matrices U, $\boldsymbol{\Sigma}$ and V.

Stewart [16] states that, "the Singular Value Decomposition gives a basis for the row space of a matrix and is more reliable in determining rank". Retaining the k largest singular value or the first $\boldsymbol{k} \ll \boldsymbol{r}$ singular value of $\boldsymbol{\Sigma}$ from the original matrix $\boldsymbol{A}$ is the best low-ranked approximation provided by the SVD which is an important attribute.

From the relation, the latent relations that is present but not evident in the original representation can be captured. The original matrix's dimensionality can also be reduced because the entries of $\boldsymbol{\Sigma}$ are sorted. The diagonal matrix that is obtained after removing the unwanted (noise) matrix consideration is termed Σ_k. Matrices $\boldsymbol{U}$ and $\boldsymbol{V}$ are accordingly obtained by removing r – k columns to get $\boldsymbol{U_k}$ and $\boldsymbol{V_k}$ respectively. Matrix $\boldsymbol{A}$ can be reduced to $\boldsymbol{A_k}$ as the closest linear approximation with a reduced rank $\boldsymbol{k}$.

$A_k = U_k \times \sum_k \times V_k^T$ Equation 2.3

Taking the r-columns of $\boldsymbol{U}, \boldsymbol{\Sigma}$, and $\boldsymbol{V}$ as hidden concepts of matrix $\boldsymbol{A}$ helps to understand the offers of SVD and their importance. As concepts, $\boldsymbol{\Sigma}$ represent the strength of each concept, V connects $\boldsymbol{n}$ to concepts and $\boldsymbol{m}$ is connected by $\boldsymbol{U}$ to concepts.

2.4 Designing the Matrix Filter

The SVD matrix filter converts the received signal to a $m \times n$ matrix, reduced the noise value in the signal using SVD algorithm and the resultant signal convolve with the transmitted signal. The stages are as listed.

i. Convert signal to matrix

ii. Perform SVD operation on the signal to get U, $\sum$ and V matrices

iii. Find the inverse of the matrix $\boldsymbol{V}$ to get V^T

iv. Select the singular value of choice

v. Select the matrix size based on the selected singular value

vi. Multiply the selected matrices $U_k \sum_k V_k^T$

vii. Perform a convolution on the output of $U_k \sum_k V_k^T$ and the transmitted signals

Let $S(t)$ be the signal transmitted by the radar antennae and $N(t)$ as the added noise to the signal. If the elapsed time between the transmission and receiving the signal at time τ , then the resulting signal received is:

$S(\tau) = S(t) + N(t)$ Equation 2.4

The signal with the noise $S(\tau)$ converted to a matrix for the SVD operation which results in three matrices U, $\sum$ and V^T.The matrix $\sum$ represents the order of similarities or the characteristics found in the matrix with respect to the amount of noise *N(t)* introduce to the signal. The number of diagonal matrixes obtained depends on the rank of the matrix. If the matrix is of rank two, then there will be two singular values.

Selecting the singular value depends on the level of clarity needed. The highest indicates the presence of the highest characteristics and the lower, the lowest characteristics in the signal. If the K^{th} singular value is selected, then the K^{th} value becomes the reference point for the selection of other singular vectors for their multiplication. The rest of the singular value and vectors are ignored. In the case of this research, they are termed to be the noise added to the signal.

The resultant signal X_k results from the multiplication of the singular vectors and the singular values selected from the operation as shown in Equation 2.5.

$X_k = U_k \sum_k V_k^T$ Equation 2.5

From the Equation 2.5, X_k is the received matrix with a reduced noise. This matrix is converted to a signal and convolved with the transmitted signal *S(t)* as shown in Equation 2.6.

$R(t) = S(t) * X_k(t)$ Equation 2.6

Since the signal $X_k(t)$ from the singular value is much reduced in noise, the result of the filter is expected to have;

i. less calculations performed in the convolution
ii. more accurate detection stage
iii. noise in the antennae range approximated

3. METHODOLOGY

The research looked at improving the compression ratio, range resolutions and the SNR of a received to radar signal using SVD Matrix Filter. Using LabVIEW, a matched filter and SVD matrix filter were simulated with a Linear Frequency Modulated signal and Uniform White Noise as the interference as illustrated in Figure 3.1

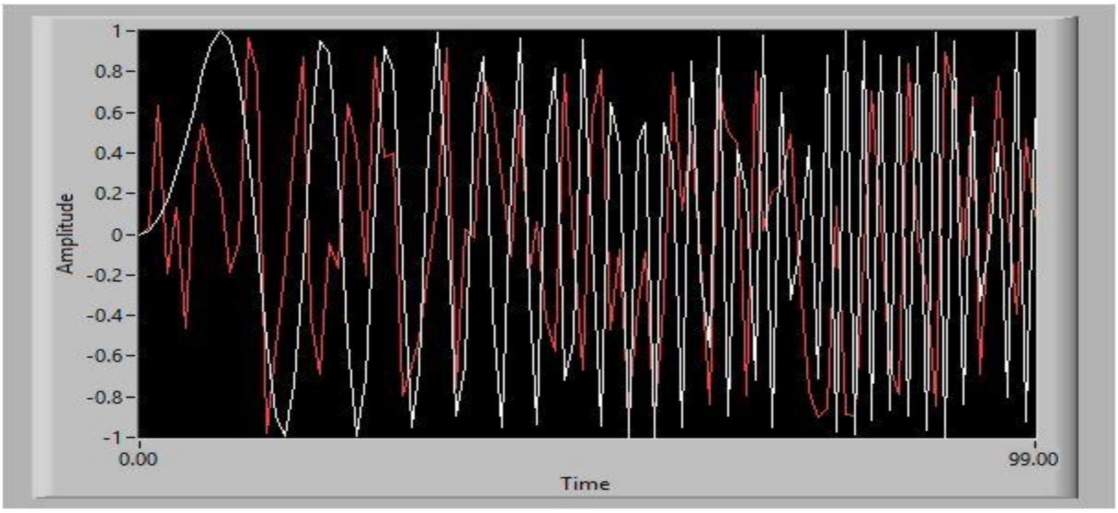

Figure 3.1 Pectral View of Received Signals and noise

The largest singular value from the SVD matrix generated from the uncompressed signal was used to investigate the effect of the value on the simulation. Simulations were carried out involving;

- the Matched filter
- the Full SVD Matrix
- proposed SVD Matrix filter (with reduced SVD Matrix)

Figure 3.2 illustrates a pectoral output of the signal when matched filtering technique was used and figure 3.3 is the pectoral view observed when the matched filter was substituted with the SVD technique.

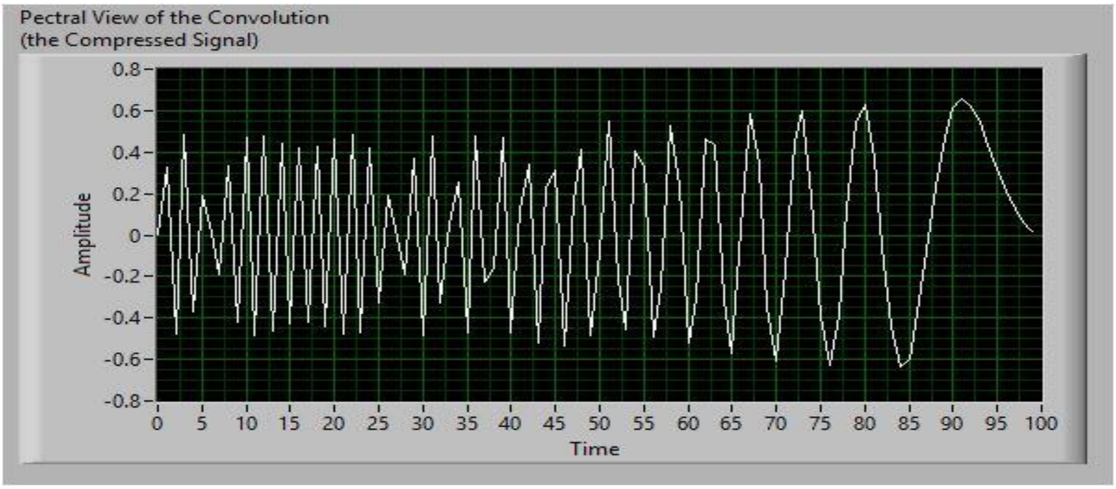

Figure 3.2 Pectral View of Signals Compressed with matched Filter

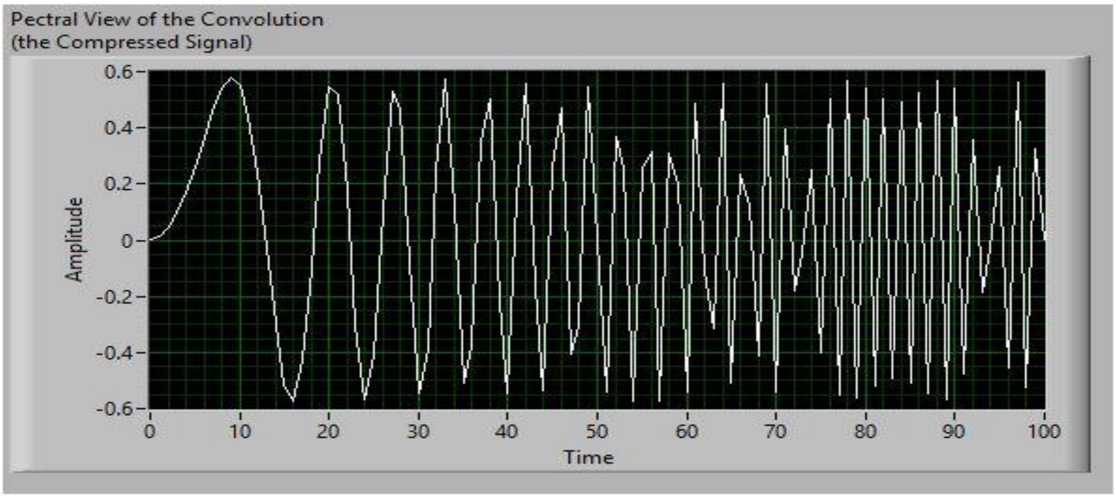

Figure 3.3 Pectral View of Signals Compressed with SVD

The entire design including calculations, for accuracy were done using LabVIEW. The input and output signals to the matched filter and the SVD filters was measured and compared, emphasizing on their SNR, Compression Ratios and Range Resolutions. Using the positivist approach, the research, aimed at improving the SNR, Compression Ratios and Range Resolutions of the received uncompressed radar signal. Comparisons were made to the various outcomes.

4. RESULTS

Table 4.1 compare the simulation results of the matched filter and the full SVD matrix and Table 4.2 compare then matched filter and the reduced SVD matrix. The comparison as indicated in the tables is based on their compression ratio, range resolution and the signal to noise ratio.

Table 4.1: Comparison of compression results for the matched filter and the full SVD matrix filter from their respective simulations

Parameters	Matched filter	Full SVD matrix filter	Difference	% difference wrt matched filter
Compression Ratio	0.513714	1.00493	0.491216	95.62
Range Resolution	1.1974×10^9	6.12102×10^8	5.85298×10^8	48.88
Signal To Noise Ratio	17.5376	17.5726	0.035	0.12

Table 4.2: Comparison of compression results for the matched filter and the reduced SVD matrix filter from their respective simulations

Parameters	Matched Filter	Reduced SVD Matrix Filter	Difference	% Difference wrt Matched Filter
Compression Ratio	0.513714	1.00199	0.488276	95.05

Range Resolution	1.1974 x 10^9	6.139 x 10^8	5.835 x 10^8	48.73
Signal To Noise Ratio	17.5376	17.2197	- 0.3179	-1.81

i. Improving compression ratio of received radar signal using SVD filter.

As illustrated in Table 4 1, the matched filter recorded a compression ratio of 0.513714 as against 1.00493 by the full SVD matrix filter. A difference of 0.491216 representing 95.62% of the value of the matched filter.

Table 4.2 also illustrates a comparison of compression ratio for the matched filter and the reduced SVD matrix filter. A difference of 0.488276 representing 95.05% of the value of the matched filter was recorded.

ii. Improving the range resolution of received radar signal using SVD filter.

From Table 4 1, the range resolution after the compression with the matched filter was observed to be 1.1974 x 109 and that of full SVD matrix filter was 6.12102 x 108. A difference of 5.85298 x 108 representing 48.88% was recorded.

In Table 4 2, the range resolution for the matched filter was recorded as 1.1974 x 109 and reduced SVD matrix filter was 6.139 x 108. A difference of 5.835 x 108 representing 48.73% was noticed in the two filters.

iii. Improving the signal to noise ratio of a received radar signal using SVD filter

The matched filter recorded 17.5376 while the full SVD matrix recorded 17.5726. A difference of 0.035 representing 0.12% was noticed as represented in table 4-1. The research recorded 17.5376 for matched filter and 17.2197 for the reduced SVD matrix with a difference of -0.3179 representing 0.18 less than that of the matched filter.

4.1 Interesting Findings

Based on the information gathered from the simulation;

There was a change of 0.491216 representing 95.62% increase in the compression ratio over the matched filter by the full SVD filter.

The signal to noise ratio of the compression increased with a value 0.035 representing 0.12% between the full SVD matrix filter values and the matched filter.

There was an improvement of 48.88% in the range resolution between the full SVD filter and the matched filter.

The range resolution reduced by 0.15% when the reduced SVD matrix filter was used for the simulation with respect to the full SVD matrix filter.

There was a 0.035 increase in signal to noise ratio over the matched filter by the full SVD matrix filter but reduced when the reduced SVD matrix filter was simulated.

There was a decrease in the compression ratio, range resolution and the signal to noise ratio when the reduced SVD was used as compared to the full SVD.

5. CONCLUSIONS

Conclusion made based on the results of the simulations is that;

i. The 95.62% increase in the compression ratio over the matched filter as used in the simulation was due to the compression capability of the SVD matrix filter.

ii. The improvement in the Range resolution of a received radar signal by 48.88% over a matched filter as was used in the simulation was an improvement in the pulse width of the received signal using the full SVD matrix.

iii. The increase in Signal to Noise Ratio of received radar signal by 0.12% over matched filter as was used in the simulation is due to the noise reduction ability of the SVD matrix filter.

iv. The 0.15% reduction in the range resolution observed between the full matrix filter and the reduced SVD matrix filter was due to the percentage energy retained for the reduced SVD matrix. That is 56.80% retained.

v. The reduction in the compression ratio, range resolution and the signal to noise ratio when the reduced SVD matrix was simulated was due to the percentage of the singular values or the 43.20% energy lost in the simulation.

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