Solving Large-scale Three-level Linear Fractional Programming Problem with Rough Coefficient in Objective Function

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ABSTRACT
In this paper a Large-Scale three level fractional problem is considered with random rough coefficient in objective function, in order to solve this problem, The intervals technique used to convert rough nature in objective into equivalent crisp , Then Tailor Series transformation is used to convert the Large-Scale three level fractional to an equivalent three level linear programming problem , then a Traditional Method used to constructed solution of the three- level programming problem, then we will use Decomposition Technique to solve Large-ScaleProblem. Finally an auxiliary problem is discussed as well as an example is presented.

Keywords
Three-level problem; Fractional Problem; Rough interval Coefficient; Large-Scale; Tailor Series

1. INTRODUCTION
Hierarchical optimization or multiple level programming (MLP) techniques are formulated in order to solve decentralized planning problems involving several decision makers (DMs) in a hierarchical organization based on the concept of Stackelberg game theory [1].

Multilevel programming involves optimization problems where the constraint region of the first level problem is implicitly determined by the second level problem and the constrained region of the second level problem is determined by the third level problem, and so on.

During the past few decades, many methodological developments have been reported for multi-level programming problem (MLPP). However, these methods are proven to be computationally ineffective and can handle only simple MLLPs. To overcome the shortcomings of the traditional methods, the concept of membership function of fuzzy set theory was incorporated for large and complex hierarchical optimization problems. Lai [1] at first proposed a new solution concept based on tolerance membership functions as well as multiple objective optimizations to develop an effective fuzzy approach for solving MLPP.

Shih et al. [2] extended Lai's concept and proposed a supervised search procedure by employing non-compensatory max-min aggregation operator for solving MLPP.

Tirayaki [3] discussed interactive compensatory fuzzy programming for decentralized MLLPs to obtain a preferred compensatory compromise Pareto-optimal solution.

Pramanik and Roy [4] developed a fuzzy goal programming (FGP) technique for MLPPs for proper distribution of decision powers to the DMs to reach a satisfying decision.

Baky [5] presented two FGP algorithms to solve multi-objective MLPPs to achieve highest degree of each of the membership goals by minimizing over and under deviational variables. Arbaiy and Watada [5] discussed additive FGP model for solving multi-objective MLPPs for obtaining satisfaction solution.

When the objective functions of level DMs of a MLPP are linear fractional in nature, then the MLPP is called multi-level linear fractional programming problem (MLFPP). Lachhwani and Poonia[6] proposed a different FGP approach for MLFPP by defining separate membership functions for numerator and denominator functions of the fractional objective functions at each level.

2. PROBLEMFORMULATION AND SOLUTION CONCEPT
Consider a three-level Large-Scale programming problem of maximization-type with random rough coefficient in the objective function at each level can be written as:

$$\underset{X_1}{\mathrm{Max}}\ Z_1(x) = \frac{\left[\underline{c}_j^L, \underline{c}_j^U\right], \left[\overline{c}_j^L, \overline{c}_j^U\right] x + \alpha_1}{b_1 x + \beta_1} \quad (1)$$

$$\underset{X_2}{\mathrm{Max}}\ Z_2(x) = \frac{\left[\underline{b}_j^L, \underline{b}_j^U\right], \left[\overline{b}_j^L, \overline{b}_j^U\right] x + \alpha_2}{b_2 x + \beta_2} \quad (2)$$

$$\underset{X_3}{\mathrm{Max}}\ Z_3(x) = \frac{\left[\underline{a}_j^L, \underline{a}_j^U\right], \left[\overline{a}_j^L, \overline{a}_j^U\right] x + \alpha_3}{b_3 x + \beta_3} \quad (3)$$

Subject to

$x \in s = \{A_1x_1 + A_2x_2 + \cdots + A_jx_j(\geq, =, \leq)B, x \geq 0\}$. (4)

Here S is the multilevel convex constraints set and Z_1, Z_1 and Z_3 are the objective functions of the first level decision maker (FLDM), second level decision maker (SDLM) and third level decision maker (TLDM), $[\underline{c}_j^L, \underline{c}_j^U], [\overline{c}_j^L, \overline{c}_j^U], [\underline{b}_j^L, \underline{b}_j^U], [\overline{b}_j^L, \overline{b}_j^U], [\underline{a}_j^L, \underline{a}_j^U], [\overline{a}_j^L, \overline{a}_j^U]$ are rough intervals coefficient of the objective function for the three levels. Also let (j=1,2,…,n), $x = (x_1, x_2, \ldots, x_n)$ denote the vector of all decision variables.

To tackle problem (1)-(4) and to deal with rough nature in (TLPP) the problem is transformed using the Intervals method to transform the rough coefficient in objective function into crisp number presented in the following section .

3. THE EQUIVALENT CRISP MODEL FOR THREE-LEVEL LINEAR PROBLEM WITH ROUGH IN OBJECTIVE FUNCTION

Conversion of (TLPP) with rough coefficient in objective number into upper and lower approximation is usually a hard work for many cases, but transformation process needs to know the following definitions:

Definition 3.1:

Rough Interval (RI) can be considered as a qualitative value from vague concept defined on a variable xin R.

Definition 3.2:

The qualitative value A is called a rough interval when one can assign two closed intervals $A*$ and $A*$ on R to it where $A* \subseteq A*$.

Definition 3.3:

$A*$and$A*$ are called the lower approximation interval (LAI) and the upper approximation interval (UAI) of A, respectively. Further, A is denoted by $A = (A* \text{ and } A*)$.

Definition 3.4:

if the Problem (3.1) has qi $(i = 1, \ldots, m)$qi rough coefficients in right and/or left hand sides and q0 rough coefficients in the objective function, then there exist 2Q corresponding Linear problem with interval coefficient (LPIC) problems and 4Q corresponding LP problems where $Q = q0 \times q1 \times \ldots \times qm$.

Definition 3.5:

Consider all of the corresponding LPIC problems and LP problems of Problem (3.1).

The interval $[Z_*^l, Z_*^u]([Z^{*l}, Z^{*u}])$ is called the surely optimal range of problem (3.1), if the optimal range of each LPIC problem is a superset of $[Z_*^l, Z_*^u]([Z^{*l}, Z^{*u}])$.

Let $[Z_*^l, Z_*^u]([Z^{*l}, Z^{*u}])$ be surely optimal range of problem (4.1).then the rough interval $([Z_*^l, Z_*^u]([Z^{*l}, Z^{*u}])$ is called the rough optimal range of problem (3.1).

The optimal solution of each corresponding LP problem of the problem (4.1) which its optimal value belongs to $[Z_*^l, Z_*^u]([Z^{*l}, Z^{*u}])$ is called a completely satisfactory solution of the problem (3.1).

Now, the equivalent problem of the first level with rough coefficient in objective function by using Intervals method can obtained by getting the surely optimal range of LPIC problem (4.1) by solving two classical LPs as Follow:

$\underline{z}^L := \max \sum_{j=1}^{n} \underline{c}_j^L y_1 + \alpha_1 z$ (5) subject to $b_1y_1 + \beta_1 z = 1,$ $A_1y_1 + A_2y_2 + \cdots + A_jy_j - Bz(\geq, =, \leq)0,$ $x_1 \geq 0, \ldots, x_j \geq 0, z \geq 0.$	$\underline{z}^U := \max \sum_{j=1}^{n} \underline{c}_j^U y_1 + \alpha_1 z$ (6) subject to $b_1y_1 + \beta_1 z = 1,$ $A_1y_1 + A_2y_2 + \cdots + A_jy_j - Bz(\geq, =, \leq)0,$ $x_1 \geq 0, \ldots, x_j \geq 0, z \geq 0.$

While the possibly optimal range of LPIC Problem (4.1) can be obtained by solving two classical LPs as follows:

$\overline{z}^L := \max \sum_{j=1}^{n} \overline{c}_j^L y_1 + \alpha_1 z$ (7) subject to $b_1y_1 + \beta_1 z = 1,$ $A_1y_1 + A_2y_2 + \cdots + A_jy_j - Bz(\geq, =, \leq)0,$ $x_1 \geq 0, \ldots, x_j \geq 0, z \geq 0.$	$\overline{z}^U := \max \sum_{j=1}^{n} \overline{c}_j^U y_1 + \alpha_1 z$ (8) subject to $b_1y_1 + \beta_1 z = 1,$ $A_1y_1 + A_2y_2 + \cdots + A_jy_j - Bz(\geq, =, \leq)0,$ $x_1 \geq 0, \ldots, x_j \geq 0, z \geq 0.$

After using intervals method to convert problem (4.1) for the first level from rough nature to crisp, that resulted in four multi-level programming problems. These steps will be repeated for second and third level, so the problem of (TLFPP) converted into twelve linear problems with four problems at each level.

Then each level has his\her own optimal solution. To deal with the conflict between the solutions, the fuzzy approach is used.

4. DECOMPOSITION ALGORITHM FOR SOLVING A THREE LEVEL LARGE SCALE LINEAR PROGRAMMING PROBLEM

The three level large scale linear programming problem is solved by adopting the leader-follower Stackelberg strategy combine with Dantzig and Wolf decomposition method [7,8,9].

One first gets the optimal solution that is acceptable to FLDM using the decomposition method to break the large scale problem into n-sub problems that can be solved directly.

The decomposition principle is based on representing the TLLSLPP in terms of the extreme points of the sets $d_j x_j \leq b_j, x_j \geq 0, j = 1,2,..,m$. To do so, the solution space described by each $d_j x_j \leq b_j, x_j \geq 0$, $j = 1,2,..,m$ must be bounded and closed, for more details see.

Then by inserting the FLDM decision variable to the SLDM for him/her to seek the optimal solution using Dantzig and Wolf decomposition method [10] then the decomposition method break the large scale problem into n-sub problems that can be solved directly.

Finally the TLDM do the same action till he obtains the optimal solution of his problem which is the optimal solution to TLLSLPP.

4.1 The First-Level Decision-Maker (FLDM) Problem

The first-level decision-maker problem of the TLLSLPP is as follows:

[First Level]

$$Max\ F_1(x) = Max \sum_{j=1}^{m} c_{1j} x_j , \quad (9)$$

Subject to

$$x \in G.$$

To obtain the optimal solution of the SLDM problem; the SLDM solves his master problem by the decomposition method [10] as the FLDM.

4.2 The Second-Level Decision-Maker (SLDM) Problem

Secondly, according to the mechanism of theTLLSLPP, the FLDM variables x_1^F, x_2^F must be given to the SLDM; hence, the SLDM problem of the (TLLSLPPFN) can be written as follows:

[Second Level]

$$Max\ F_2(x) = Max \sum_{j=1}^{m} c_{2j} x_j , \quad (10)$$

Subject to

$$x \in G_2.$$

Where

$$G_2 = (x_1^F, x_2^F, ..., x_m).$$

To obtain the optimal solution of the SLDM problem; the SLDM solves his master problem by the decomposition method [10] as the FLDM.

4.3 The Third-Level Decision-Maker (TLDM) Problem

Finally, according to the mechanism of the TLLSLPP, the SLDM variables $x_1^F, x_2^F, x_3^S, x_4^S$ must be given to the TLDM; hence, the TLDM problem can be written as follow:

[Third Level]

$$Max\ F_3(x) = Max \sum_{j=1}^{m} c_{3j} x_j , \quad (11)$$

Subject to

$$x \in G_3.$$

Where

$$G_3 = (x_1^F, x_2^F, x_3^S, x_4^S, ..., x_m).$$

To obtain the optimal solution of the TLDM problem; the TLDM solves his master problem by the decomposition method [10] as the FLDM and SLDM.

Now the optimal solution $(x_1^F, x_2^F, x_3^S, x_4^S, x_5^T, x_6^T, ..., x_m^T)$ of the TLDM is the optimal solution of the TLLSLPP.

5. NUMERICAL EXAMPLE

First level:

P1 = max (X1X2) = $\frac{11x_1+3x_2+4x_5}{x_1+4x_2+6}$

P2 = max (X1X2) = $\frac{15x_1+6x_2+9x_5+4x_6}{x_1+4x_2+6}$

P3 = max (X1X2) = $\frac{12x_1+2x_2+5x_5+2x_6}{x_1+4x_2+6}$

P3 = max (X1X2) = $\frac{14x_1+5x_2+7x_5+3x_6}{x_1+4x_2+6}$

Subject To:

X1 + X2 + X3 + X4 + X5 + X6≤ 10

X1 + X2≤ 3

X3 + 2X4≤4

X5 +$\frac{1}{3}$X6≤3

X1, X2, X3, X4, X5, X6 ≥0

Solving by applying tailor series to convert production objectives function to linear Function

P1=MAX 0.85X1-0.32X2+0.36X5+0.74

P2=MAX 1.08X1-0.57X2+0.81X5+0.36X6+1.41

P3=MAX 1.08X1-0.39X2+0.45X5+0.27X6+.68

P=MAX 1.03X1-0.50X2+0.63X5+0.27X6+1.2

Solving by applying Decomposition Algorithm

X6	X5	X4	X3	X2	X1	**P1**
0	3	0	0	0	3	4.37
X6	X5	X4	X3	X2	X1	**P2**
5.97	1.02	0	0	0	3	7.633
X6	X5	X4	X3	X2	X1	**P3**
5.99	1.02	0	0	0	3	5.99
X6	X5	X4	X3	X2	X1	**P4**
5.97	1.02	0	0	0	3	6.55

Second Level

Upper

$$P1 = \frac{3x_3 + 4x_4 + x_5 + x_6}{6x_3 + 4x_4 + 2}$$

$$P2 = \frac{6x_3 + 4x_4 + 4x_5 + x_6}{6x_3 + 4x_4 + 2}$$

Lower

$$P3 = \frac{4x_3 + 5x_4 + 2x_5 + x_6}{6x_3 + 4x_4 + 2}$$

$$P4 = \frac{5x_3 + 7x_4 + 3x_5 + x_6}{6x_3 + 4x_4 + 2}$$

Subject To:

X1 + X2 + X3 + X4 + X5 + X6≤ 10

X1 + X2≤ 3

X3 + 2X4≤4

X5 +$\frac{1}{3}$X6≤3

X1, X2, X3, X4, X5, X6 ≥0

Solving by applying tailor series to convert production objectives function to linear Function

P1=MAX -.125X3+0.083X4+0.083X5+0.083X6+.626

P2= MAX -.125X3-0.083X4+0.166X5+0.083X6+1.209

P3= MAX -.166X3+0.083X4+0.166X5+0.083X6+0.834

P4= MAX -.25X3+0.138X4+0.25X5+0.083X6+1.109

Solving by applying Decomposition Algorithm Given X1,X2 from first level

X6	X5	X4	X3	X2	X1	P1
2.98	2.014	2	0	0	0	1.207
X6	X5	X4	X3	X2	X1	P2
5.970	1.029	0	0	0	0	1.875
X6	X5	X4	X3	X2	X1	P3
2.98	2.014	2	0	0	0	1.58
X6	X5	X4	X3	X2	X1	P4
2.985	2.014	2	0	0	0	2.136

Solving by applying DecompositionAlgorithm in isolation from first level

X6	X5	X4	X3	X2	X1	P1
7.462	.537	2	0	0	0	1.456
X6	X5	X4	X3	X2	X1	P2
9.090	0	0	0	0	0	1.963
X6	X5	X4	X3	X2	X1	P3
7.462	.537	2	0	0	0	1.708

Third Level

Upper

$$P1 = \frac{x_1 + x_5 + 4x_6}{x_5 + x_6 + 6}$$

$$P2 = \frac{x_1 + 4x_5 + 9x_6}{x_5 + x_6 + 6}$$

Lower

$$P3 = \frac{x_1 + 2x_5 + 5x_6}{x_5 + x_6 + 6}$$

$$P4 = \frac{x_1 + 3x_5 + 7x_6}{x_5 + x_6 + 6}$$

Subject to

$x_1 + x_2 + x_3 + x_4 + x_3 + x_4 \geq 10,$

$x_1 + x_2 \leq 3,$

$x_3 + 2x_4 \leq 4,$

$x_5 + 1/3x_6 \leq 3,$

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, , x_5 \geq 0, x_6 \geq 0,$

Solving by applying tailor series to convert production objectives function to linear Function

P1= MAX 0.125X1+0.218X5+0.593X6-0.186

P2= MAX 0.125X1+0.718X5+1.343X6-0.436

P3= MAX 0.125X1+0.375X5+0.75X6-1.125

P4= MAX 0.125X1+0.546X5+1.046X6-0.342

Solving by applying Decomposition Algorithm Given X1,X2 from first level and X3,X4 from Level 2

X6	X5	X4	X3	X2	X1	P1
5	0	0	0	0	0	3.154
X6	X5	X4	X3	X2	X1	P2
7	0	0	0	0	0	9.30
X6	X5	X4	X3	X2	X1	P3
5	0	0	0	0	0	3
X6	X5	X4	X3	X2	X1	P4
5	0	0	0	0	0	5.263

Solving by applying Decomposition Algorithm in isolation from first level and second level

X6	X5	X4	X3	X2	X1	P1
9.090	0	0	0	0	0.90 90	5.318
X6	X5	X4	X3	X2	X1	P2
9.090	0	0	0	0	0.90 90	11.886
X6	X5	X4	X3	X2	X1	P3
9.090	0	0	0	0	0.90 90	5.806
X6	X5	X4	X3	X2	X1	P4
9.090	0	0	0	0	0.90 90	9.280

6. CONCLUSION

This paper presents a solving method for a three level fractional problem with a rough coefficient in the objective function, we first began by converting the rough nature of the problem into its equivalent crisp problem by using interval method then converting Fractional using Tailor series finally solving the three level problem using the compromised approach.

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