Robust Water Distribution System using Minimal Spanning Tree Algorithm

Ahmad A. Maidamisa
Department of Statistics & Operations Research
Modibbo Adama University of Technology
P.M.B. 2076, Yola; 640001, Adamawa State, Nigeria

Noel Eckson
Department of Mathematics & Statistics
Federal Polytechnic Mubi
P.M.B. 35, Mubi, Adamawa State, Nigeria

ABSTRACT

People living in Numan town draw water for use from River Benue and other sources. An integrated water distribution system is currently needed in place that can supply adequate water to the entire community. The government of Adamawa State had been planning to bring pipe-borne water to the town but unfortunately not much had happened in regard of achieving this noble objective. The objective of this paper is to design an efficient water distribution system that will connect 15 nodes (pumping stations) in the town with a network of pipe lines to ensure that the total branch length is minimized. This will guarantee an optimal distribution of water from source of origin to demand destination at a minimum cost. The problem is viewed as a typical case of network optimization problem. A greedy algorithm is employed to accomplish this objective. Firstly, the problem is converted into a Minimal Spanning Tree (MST) which involves using the edges of the network to reach all the vertices of the network in such a fashion that the total length of all the edges is minimized. The paper shows a combined length of fourteen point five (14.5 km) kilometer of pipe as against thirty five point five (35.5 km) kilometer in earlier proposed TCIA [10]. The capacity to deliver water is tremendously enhanced, since all the fifteen pumping stations are connected. This will allow maintenance to be carried out easily and routinely, since the possibility of the system to dispense leakages is greatly minimized. The total distance is shorten by twenty one kilometer (21km). The paper is the first to design a water distribution system using minimal spanning tree algorithm in Numan town.

General Terms

Greedy algorithm, minimal spanning tree

Keywords

Planning, piped water, optimization tool

1. INTRODUCTION

Water is essential for human existence. The ability to get access to water has always been a problem for humanity since time immemorial. This problem becomes enormous when a town or city does not have inadequate or no water supply. Numan town, like most towns in Adamawa State, is without water. It is for this reason that we attempt to develop a robust water distribution network system which will be sufficiently efficient to alleviate the problem of water scarcity in the town.
Ojo [7] Investigated the inaccessibility of water to reach household for domestic use, by studying the difference in household requirements with a standard benchmark, taking into account the amount of water that is obtained per household per day. Zebedee [13] Drew a blueprint that will pump water to sixteen ward (nodes) in a city using minimal spanning tree protocol at a minimal cost. Water is received by household through a network of pipe that span across sixteen nodes. Results show a combined length of 29.65km pipe is required to connect the entire town. Mundia [6] Used minimal spanning tree algorithm to design a shortest route model for drawing water from Kapingazi River and pumping the water to a central reservoir. A water collection point will receive the water from the reservoir and then pump it to targeted homestead at a minimal cost. The model will provide water to 90% of people in homesteads in Kangaru village in Kenya, since the flow of water is permanently guaranteed from the River. Ahmed [1] Demonstrated a simulated model for optimizing water to distribute to canal. The model was applied to real cases of wet and dry regions of Sudan, and sensitivity to changes in outlet inflow rate was made for the two cases. The model shows capabilities to optimize water scheduling and allocation process can be used to serve irrigation schemes in Sudan.
The rest of the paper is organized as follows: section 1 presents the Introduction. The Minimal spanning tree methodology is described in section 2. Data is presented in section 3. Section 4 presents the results. Finally, the conclusion is presented in section 5.

2. DEFINITIONS

2.1 Tree Graph

A graph T is called a tree if T is connected and T has no cycles, Seymour and Taha [8, 9].

2.2 Spanning Tree

A subgraph T of a connected graph G is called a spanning tree of G if T is a tree and T includes all the vertices of G, Winston and Seymour [12, 8].

2.3 Minimum Spanning Tree

Suppose G is a connected weighted graph. That is, each edge of G is assigned a nonnegative number called the weight of the edge. Then any spanning tree T of G is assigned a total weight obtained by adding the weights of the edges in T. A minimal spanning t of G is a spanning tree whose total weight is as small as possi Hillier and Seymour [4, 8].

3. METHODOLOGY

A greedy algorithm for solving an optimization problem is one t iteratively construct a solution such that at each step the curr partial solution is augmented by an attempt to realize maxim immediate gain, Jensen [5]. Two additional characteristics of s algorithms are firstly, that the selection at each step is not alte in subsequent steps and secondly, that each step is a polynom function of problem size. A survey of greedy algorithm for problem can be found in, Cheriton and Torkestani [2, 11]. (of the best known greedy algorithm in operations research was signed to solve the minimal spanning tree problem on a graph v m vertices and n edges, Hillier and Ford [4, 3].

3.1 Greedy Algorithm

(1) (Initialization) Let m be the number of vertices in the graph, and let S_1 and S_2 be two disjoint sets. Arbitrarily select any vertex i and place it in S_i. Place the remaining $m-1$ vertices in S_2, Jensen and Taha [5, 9].

(2) (Selection) From all the edges with one end in S_1 and the other end in S_2, select the edge with the smallest length. Call this edge (i, j) , where $i \in S_1$ and $j \in S_2$, Jensen [5].

(3) (Construction) Add edge (i, j) to the spanning tree. Remove vertex j from set S_2 and place it in set S_1. If set $S_2 = \emptyset$, stop with the MST; otherwise, repeat step 2, Jensen [5].

4. DATA

Figure 1 shows 15 pumping stations in Numan town that are connected to 15 settlements. Vertex is considered as pumping station and edge is the length from one vertex to another. An arc is connected from station 1 to station 2. Stations are labeled from A to O and numbered from 1 to 15 serially. A pumping stations for instance, A, which stands for treatment plant can pump water to station B, then distribute to the entire Government Reservation Area (GRA) settlement. Water is pumped from A to C to D and to I. The same kind of circle is applied to all the station in the network diagram. Distance is written on the arcs. For example, the distance from A to B is 2.5km, these are measured on a motorcycle using preset meter. A combined total of 32 arcs shows possible connection route on the diagram.
Table 1, shows the coding of pumping stations. Water is received and dispensed at each station. A reservoir is proposed to collect the water and distribute to the entire settlement, through the network of pipes.

4.1 The Study Area

Consider the undirected network shown in Figure 1. The graph consists of 15 vertices and 32 edges, where each edge has an associated length. The problem is to select a set of edges such that there is a path between two vertices. The sum of the edge length is to be minimized. When the edge lengths are all nonnegative, as assumed here, the optimal selection of edges forms a spanning tree. This characteristics gives rise to the name minimal spanning tree, or MST, Jensen [5] . The problem can be solved with a greedy algorithms attributable to R. Prim, Hillier et. al. [4, 9, 3] .

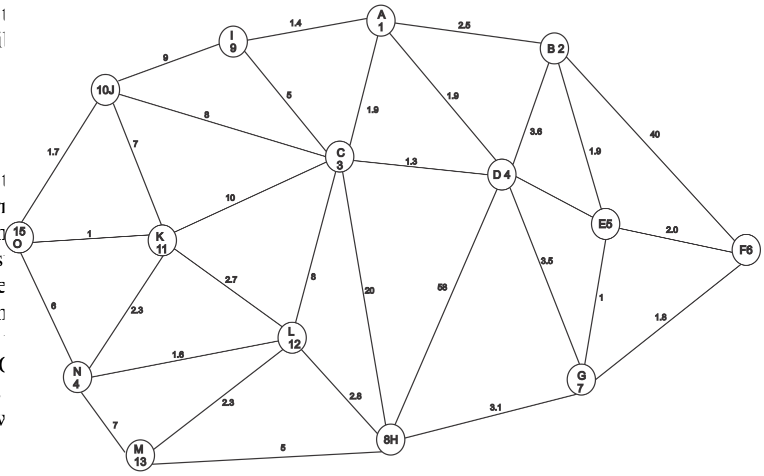

Fig. 1. Sketch map showing 15 vertices and 32 edges

Table 1. Codes for pumping station

S/N	Code	Pumping Station
1	a	Treatment Plant
2	b	Government Reserve Area (GRA)
3	c	Distribution office
4	d	Sabon Pegi
5	e	Nasarawo Demsa
6	f	Dakata
7	g	Anguwa Abuja
8	h	Green Village
9	i	Makera
10	j	Ngbayofe
11	k	Wodi Pare
12	l	Mbatato
13	m	Gwaida Mallam
14	n	Wayam Barkindo
15	o	VSS/GTC

5. RESULTS

Table 2, shows the optimum solution obtained from computer program. TORA is a computer software employed to find the minimum spanning tree. Then I is connected to C, and so on. Ties are resolved arbitrarily, the connection will continue until all the stations in the network are reached without a cycle. The shortest path the water would flow through to the community are (A,I), (C,I), (C,D), (C,J), (C,L), (D,E), (B,E), (E,G), (F,G), (J,K), (K,O), (O,N), (N,M) finally (M,H) that is the pipelines for the minimum connection which minimized the number and distance of the pipelines to 14.5km as against 35.5km, TCIA [10]. Results obtained show a combined length of 14.5km.
Applying the greedy algorithms to the graph in Figure 1, we obtained the sequence of iterations given in Table 2. The columns of the table show the set S_1 with the vertex labels listed in the order in which they are added to the set, the edge selected at step 2, the length of that edge, and the cumulative length of the tree. Node A was selected as starting point and served as a central reservoir. The solution is given by the minimal spanning tree shown in Table 2 and Figure 2. The resulting minimum pipe length to provide the desired water service are: 1.4 + 0.5 + 0.8 + 0.8 + 0.7 + 1.0 + 0.6 + 0.7 +

Table 2. Minimal Spanning Tree for Figure 2

S1	Selected	Length	Total
[A]	(I,A)	1.4	—
[A,I]	(C,I)	0.5	1.4
[A,I,C]	(L,C)	0.8	1.9
[A,I,C,L]	(J,C)	0.8	2.7
[A,I,C,L,J]	(K,J)	0.7	3.5
[A,I,C,L,J,K]	(O,K)	1.0	4.2
[A,I,C,L,J,K,O]	(N,O)	0.6	5.2
[A,I,C,L,J,K,O,N]	(M,N)	0.7	5.8
[A,I,C,L,J,K,O,N,M]	(H,M)	0.5	6.5
[A,I,C,L,J,K,O,N,M,H]	(D,C)	1.3	7.0
[A,I,C,L,J,K,O,N,M,H,D]	(E,D)	1.5	8.3
[A,I,C,L,J,K,O,N,M,H,D,E]	(G,E)	1.0	9.8
[A,I,C,L,J,K,O,N,M,H,D,E,G]	(F,G)	1.8	10.8
[A,I,C,L,J,K,O,N,M,H,D,E,G,F]	(B,E)	1.9	12.6
[A,I,C,L,J,K,O,N,M,H,D,E,G,F,B]	—	—	14.5

0.5 + 1.3 + 1.5 + 1.0 + 1.8 + 1.9 = 14.5 km. This total is obtained irrespective of the starting node and regardless of the selection in the case of ties. The shortest path are:
I=C=D=J=L=E=B=G=F=K=O=N=M=H

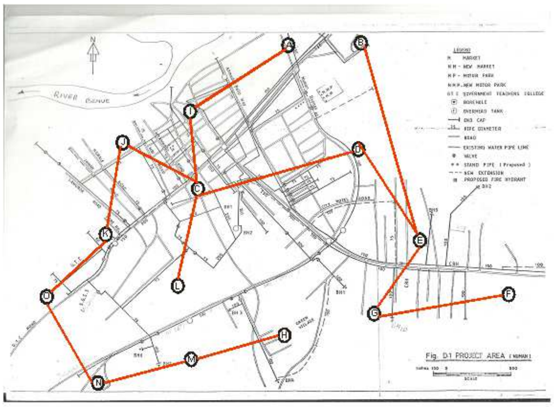

Fig. 2. Map of the study area (Source: TCIA)

Figure 2, shows the sketch map of Numan town that was adopted from, TCIA [10]. The former Gongola state Government hired this consultant and assigned the responsibility to execute water project in five towns in Adamawa state (former Gongola state). Figure 2, shows all the 15 stations that are connected with a single arc, which gives an optimum solution. Fourteen arcs connect the 15 stations, with a combined total length of 14.5 km. These connection form the spanning tree.

6. CONCLUSION

The distribution system proposed is intended to be strong efficient and durable that can serve the needs of Numan community. The result from this study shows a combined total length of fourteen point five kilometer of pipe is required to connect the main line in fifteen settlements. This demonstrate a remarkable improvement of saving of approximately twenty one kilometer of pipe as against earlier proposal made by the government consultants.

The shortest path the water would flow through to the community are (A,I), (C,I), (C,D), (C,J), (C,L), (D,E), (B,E), (E,G), (F,G), (J,K), (K,O), (O,N), (N,M) finally (M,H) that is the pipelines for the minimum connection which minimized the number and distance of the pipelines to 14.5km as against 35.5km, TCIA [10].

In view of the findings in the paper it is infer that the shortest path problem especially Prims algorithm, Ford [3] can be used to reduce the network systems. Numan township pipe water distribution networks have water supply problems mainly due to inadequate pipe network. The minimum spanning tree which is good for both small and large network system was used for the computation of the data. The total distance covered by the pipe network system was the minimal that is 14.5km as against 35.5km obtained from consultants following the method that was applied. The extension of the piped water distribution network was also included at new site. The purpose of environmental impact assessment is to ensure that decision makers consider the environmental impacts when deciding whether or not to proceed with the project. The environmental assessment is left for further studies.

7. ACKNOWLEDGMENT

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