Feasibility of Cellular Networks and Radar Coexistence in S Spectrum Band

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ABSTRACT

Spectrum sharing is an efficient solution for spectrum shortage. This paper presents mathematical analysis of a typical coexistence scenario between LTE and radar systems in 2.7-2.9 GHz bands. We present the interference analysis of multiple base stations (BSs) and mobile stations (MSs) on the primary radar. The joint distribution of power-controlled and non power-controlled interference is also presented in this study. Our simulation and analytical results closely approximate each other and the LTE downlink data rate for the shared spectrum is quite comparable to an LTE system operating in a dedicated spectrum band. Thus, the feasibility of spectral coexistence is shown.

General Terms

Spectrum Sharing, Coexistence

Keywords

Spectrum sharing, power control, Poisson, Log-normal distribution.

1. INTRODUCTION

The advent of 5G requires sufficient spectrum to make it a reality. The current form of spectrum allocation is static which has led to the inefficient utilization of this precious resource (ie spectrum). Federal communication commission (FCC) and other world telecommunication regulatory bodies have identified huge chunks of spectrum held by fixed wireless communication services been underutilized. Spectrum sharing is a promising technique that will help in harnessing more spectrums for the 5G systems. Much study has been done on this important subject. Raymond et al in [1] studied spectral coexistence between radar systems and power-controlled cellular systems in the 2.7-2.9 GHz bands. They devised mechanisms to ensure secondary access to this band by cellular systems. M.I Rahman in [2] also studied coexistence between LTE-BS and radar systems considering only the radar main beam gain. An account was not made for the radar antenna sidelobes and its time varying gain due to the rotation. M. Tercero et al in [3] analyzed the impact of aggregate interference on the primary radar mathematical using log-normal approximations without incorporating a guard region for the secondary users (SUs). In [4], a qualitative analysis was done indicating the possibility of sharing radar spectrum with other wireless services. Other researchers in [5] proposed the projection of the signals of the two systems in the null space of each using a blind learning approach. The authors of [5-10] presented other techniques in analyzing spectral coexistence between radar systems and other wireless communication services.

This paper presents mathematical modeling of the probability distribution of power control (PC) and non (PC) interference from cellular BSs or MSs using joint log normal and Poisson distributions. To the best of our knowledge this is the first paper that considers joint interference distribution as the authors of [1] and [3] separately model the two scenarios. The rest of the paper is organized as follows; we present mathematical derivation of the joint probability distribution functions of the PC and non PC BSs and MSs interference on the primary radar system in section 2. In section 3, we present the results showing the feasibility of spectral coexistence in the S spectrum band with cellular systems and section 4 concludes the paper..

2. PROBABILITY DISTRIBUTION OF INTERFERENCE

2.1 Joint Probability Distribution of BSs Interference on the Radar

The authors of [1] analyzed the distribution of the interference from power controlled BSs on the primary radar. This section of the paper models the joint distribution of power controlled BSs and non power controlled BSs interference mathematically. The representation of this model is depicted in Fig.1. BSs in the annular region R2-R1 activates the power control algorithm developed in [1] to prevent harmful interference on the radar system whereas those in R3-R2 operates normally as an LTE system operating in a dedicated spectrum band. The probability distribution functions of the separation distances of PC and no PC BSs from the radar (d_{k1}, d_{k2}) are given by (1) and (2) respectively as

$$f_{d_{k1}}(x_1) = \frac{2x_1}{R_2^2 - R_1^2}, R_1 \leq x_1 \leq R_2, \quad (1)$$

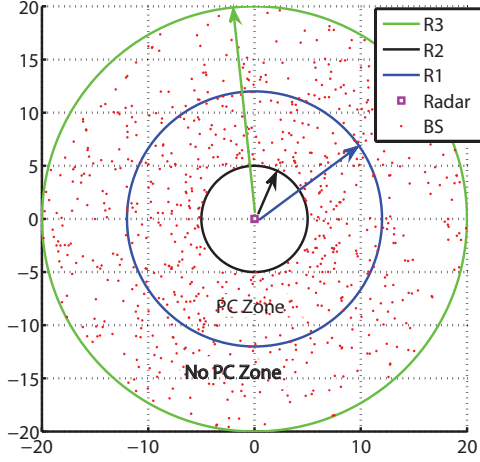$$f_{d_{k2}}(x_2) = \frac{2x_2}{R_3^2 - R_1^2}, R_2 \leq x_2 \leq R_2. \quad (2)$$

Fig. 1. Representation of Multiple BSs Interference scenario.

The PDF of the arbitrary interference γ_{k1} and γ_{k2} from PC and no PC BSs are respectively derived as

$$f_{\gamma_{k1}}(z) = \int_{z/Q_1}^{z/Q_2} \frac{1}{|x_1|} f_{x_{1k1}}(x_1) f_{U_{1k1}}\left(\frac{z}{x_1}\right) dx_1 \qquad (3)$$

and

$$f_{\gamma_{k2}}(z) = \int_{z/Q_2}^{z/Q_3} \frac{1}{|x_2|} f_{x_{2k2}}(x_2) f_{U_{2k2}}\left(\frac{z}{x_2}\right) dx_2, \qquad (4)$$

where γ_{k1} and U_{k1} are defined respectively as $\gamma_{k1} = U_{1k}X_k = P_T GL(d_{k1})X_{k1}, R_1 \leq d_{k1} \leq R_2$. γ_{k2} and U_{k2} are similarly defined with only a change in the subscript from 1 to 2. $G = G_t G_r / 10^{\text{FDR}/10}$ and $L(d_k) = L_o \left(\frac{d_k}{d_o}\right)^{-n}$, $d_o \leq R_1 \leq R_2$. P_T, G_t and G_r are the BS transmit power, maximum antenna gains of the BS and radar respectively, whereas L_o, d_o and n are the path loss, reference distance and path loss exponent respectively as similarly given in [1].
The joint PDF of the PC and no PC interference is derived from (3) and (4) as

$$f_{\gamma_{k1},\gamma_{k1}}(z) = \int_{z/Q_2}^{z/Q_3} \int_{z/Q_2}^{z/Q_3} f_{\gamma_{k1},\gamma_{k2}}(x_1, x_2) dx_1 dx_2. \qquad (5)$$

The solutions of (3-5) are respectively given by (6), (9) and (13) as similarly derived in [11] .

$$S_1 = \Omega_z^{\frac{-2}{n}-1}(A_1 - B_1), \qquad (6)$$

where

$$A_1 = \text{erf}\left(\frac{\ln(z/Q_2) - 2\sigma_{x_1 k_1}^2/n}{\sqrt{2\sigma_{x_1 k_1}^2}}\right), \qquad (7)$$

$$B_1 = \text{erf}\left(\frac{\ln(z/Q_1) - 2\sigma_{x_1 k_1}^2/n}{\sqrt{2\sigma_{x_1 k_1}^2}}\right), \qquad (8)$$

$$\Omega_1 = \frac{1}{R_2^2 - R_1^2}\left(\frac{d_o^{-n}}{GP_t L_o}\right)^{\frac{-2}{n}} \exp(2\sigma_{x_1 k_1}^2/n^2) \quad , \quad Q_1 = GP_t L_o(R_1/d_o)^{-n} \text{ and } Q_2 = GP_t L_o(R_2/d_o)^{-n}.$$

$$S_2 = \Omega_z^{\frac{-2}{n}-1}(A_2 - B_2), \qquad (9)$$

where

$$A_2 = \text{erf}\left(\frac{\ln(z/Q_3) - 2\sigma_{x_k}^2/n}{\sqrt{2\sigma_{x_2 k_2}^2}}\right), \qquad (10)$$

$$B_2 = \text{erf}\left(\frac{\ln(z/Q_2) - 2\sigma_{x_2 k_2}^2/n}{\sqrt{2\sigma_{x_2 k_2}^2}}\right), \qquad (11)$$

$$\Omega_2 = \frac{1}{R_3^2 - R_2^2}\left(\frac{d_o^{-n}}{GP_t L_o}\right)^{\frac{-2}{n}} \exp(2\sigma_{x_2 k_2}^2/n^2) \quad , \quad Q_2 = GP_t L_o(R_2/d_o)^{-n} \text{ and } Q_3 = GP_t L_o(R_3/d_o)^{-n}.$$

$$f_{\gamma_{k_1},\gamma_{k_1}}(z) = \int_{z/Q_2}^{z/Q_3} \int_{z/Q_1}^{z/Q_2} \frac{1}{|x_1|} f_{x_{k_1}}(x_1) f_{U_{k_1}} \left(\frac{z}{x_1}\right) \frac{1}{|x_2|} f_{x_{k_2}}(x_2) f_{U_{k_2}} \left(\frac{z}{x_2}\right) dx_1 dx_2. \qquad (12)$$

Since S_1 is a constant with respect to S_2 the solution of above integral becomes

$$S_3 = \int_{z/Q_2}^{z/Q_3} S_1 \frac{1}{|x_2|} f_{x_{k_2}}(x_2) f_{U_{k_2}} \left(\frac{z}{x_2}\right) dx_2$$

$$S_1 \int_{z/Q_2}^{z/Q_3} \frac{1}{|x_2|} f_{x_{k_2}}(x_2) f_{U_{k_2}} \left(\frac{z}{x_2}\right) dx_2 \qquad (13)$$

$$= S_1 S_2$$

2.2 Mathematical Analysis of MSs Interference on Primary Radar

The arrival of MSs at their respective serving BSs follows a Poisson process with a mean arrival rate denoted by μ and the number of MSs arriving at the BSs is K. Without the loss of generality, the distribution of the interference from the multiple MSs follows the Poisson distribution. The probability distribution of the random MSs is modeled as

$$P_k = \frac{e^{-\mu}\mu^k}{k!} \qquad (14)$$

Denoting ζ_k as the interference from an arbitrary arriving MS. The probability distribution of the interference is similarly given as

$$P_{\zeta_k} = \frac{e^{-\mu\zeta_k}\mu_{\zeta_k}^k}{k!} \qquad (15)$$

The mean and variance of the random variable ζ_k are computed from the first and second moments respectively in (8) and (10).

$$E[\zeta_k] = \sum_{k=0}^{N} KP_k = \sum_{k=0}^{N} \frac{Ke^{-\mu_{\zeta_k}}\mu_{\zeta_k}^k}{k!}$$

$$= \mu_{\zeta_k}e^{-\mu_{\zeta_k}} \sum_{k=1}^{N} \frac{\mu_{\zeta_k}^{k-1}}{(k-1)!}, \text{ but } \sum_{k=1}^{N} \frac{\mu_{\zeta_k}^{k-1}}{(k-1)!} = e^{\mu_{\zeta_k}}$$

$$= \mu_{\zeta_k}e^{-\mu_{\zeta_k}}e^{\mu_{\zeta_k}} = \mu_{\zeta_k},$$

$$(16)$$

where N is the sum of all interfering MSs. The variance of the interference is given as the second moment. We first evaluate

$$E[\zeta_k(\zeta_k - 1)] = \sum_{k=2}^{N} K(K-1)P_k$$

$$\mu_{\zeta_k}^2 e^{-\mu_{\zeta_k}} \sum_{k=2}^{N} \frac{\mu_{\zeta_k}^{(k-2)}}{(k-2)!}$$

$$(17)$$

$$= \mu_{\zeta_k}^2, \text{ because } \sum_{k=2}^{N} \frac{\mu_{\zeta_k}^{(k-2)}}{(k-2)!} = e^{\mu_{\zeta_k}}$$

The variance of the interference then computed as

$$\sigma_{\zeta_k}^2 = Var[\zeta_k^2] - E[\zeta_k]^2,$$

$$but, E[\zeta_k^2] = E[\zeta_k(\zeta_k - 1) + E[\zeta_k] = \mu_{\zeta_k}^2 + \mu_{\zeta_k} \quad (18)$$

$$therefore, = \mu_{\zeta_k}^2 + \mu_{\zeta_k} - \mu_{\zeta_k}^2 = \mu_{\zeta_k}$$

Thus, the mean and variance of the MSs interference on the radar are equal. The aggregate interference (I_{agg}) on the PU from all MSs is given as

$$I_{agg} = \sum_{k=1}^{N} \zeta_k \quad (19)$$

As the mean number of the arriving MSs increases, the distribution of their interference on the radar system gets more symmetrical. In light of the above-mentioned and without any ambiguity, we assumed the interference is log-normally distributed based on the central limit theorem for independent and identically distributed random variables. The Log-normal CDF of the interference is approximated using (25) in [1].

3. NUMERICAL RESULTS

The results of [1] gives the required separation distances coexistence between the primary radar and the secondary LTE system. We assumed a perfectly planned cellular system with dominant interference from the radar system located at a maximum range of 100 km and a rayleigh fading channel between the LTE BS and MS. The interference from the radar system is computed using (2) in [1] and applying the parameters of tables I and II in [1]. Exclusion radii of R1=4 km and R2=6 km are used for urban and R1=10 km and 15 km for suburban (based on the results of [1]). The ITU-R P.1456 model is used to capture the propagation losses between the radar and the LTE system.

The cellular system's cell edge downlink data rate for MSs located in the power control zone is lower than that of the non power control users as depicted in Fig.2. This is expected because of the low transmit power at the BS during the power control period. It is also observed that, the performance in the urban environment is better
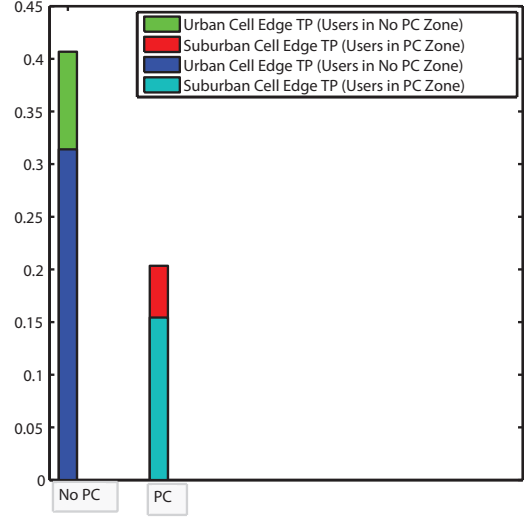

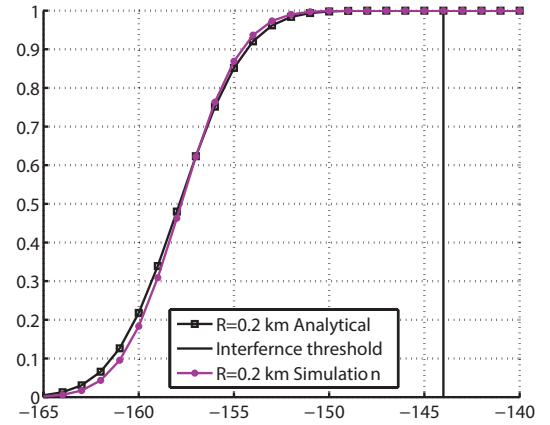Fig. 2.   SU Cell Edge throughput

Fig. 3.   Multiple MSs Interference Scenario

compared to the suburban which is desirable because of the spectrum demand for high broadband applications is in the urban.

An interference-to-noise ratio of -10 dB (correspond to an interference threshold (I_{thr}) of -144 dBW) is used for optimum protection of the radar system from interference emanating from multiple MSs. Our simulation results closely follow the analytical results obtained from (25) in [1] indicating a radar system protection radius requirement of 0.2 for multiple MSs coexistence.

4. CONCLUSION

This paper considers secondary spectrum access to radar bands by cellular systems. We analyze the joint probability distribution of power control and non power control SUs interference on the PU mathematically. Typical parameters of ASR-11 are used to proof the feasibility of our analysis. The performance of the SU systems in terms of cell edge throughput is comparable to an SU operating

in a dedicated spectrum and hence we conclude that coexistence in the S band spectrum is feasible. The authors intend extending this work into millimeter wave spectrum band in the future.

5. ACKNOWLEDGMENT

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