Robust Adaptive Beamforming using Woodward-Lawson Array Design Method

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ABSTRACT
In practice, the knowledge of the desired steering vector can be imprecise due to estimation errors in the direction of arrival (DOA) of the desired signal or imperfect array calibration. In these situations, the performances of the conventional adaptive beamformers are known to degrade substantially. In this paper, an effective method for designing a robust adaptive beamforming is presented. This method is based on Woodward-Lawson array design method, where the main beamformer in the upper channel is designed to form a main lobe with high gain in the direction of desired signal while the blocking structure in the lower channel is designed to form a wide and deep null toward and around the direction of desired signal. By generating this wide null, the proposed method provides robustness against arbitrary mismatches in the desired signal steering vector. Simulation results in ideal situations (where the desired signal steering vector is known exactly) and in more realistic situations with signal steering vector errors are presented to illustrate the performance of the proposed method.

Keywords
Adaptive Beamforming, Woodward-Lawson array design, Direction-of-arrival mismatch, desired signal cancellation

1. INTRODUCTION
In modern communication systems, there is an increasing interest toward adaptive beamforming, capable of processing multiple signals, adapting to possible scenario variations, and mitigating the effects of the background noise and interfering signals. However, in practical situations the adaptive beamforming techniques are known to be sensitive to model mismatches. In such cases, robust approaches (since their performance is robust against mismatches) to adaptive beamforming are required. The linearly constrained minimum variance (LCMV) beamformer [1] or more generally the generalized sidelobe canceller (GSC) [2] adaptively selects the weight vector to minimize the array output power subject to the linear constrained that the desired signal does not suffer from any distortion. The GSC has a better performance than the data independent beamformers such as delay-and-sum beamformer, provided that the desired steering vector is accurately known. In practice, the knowledge of the desired steering vector can be imprecise, which often occurs due to estimation errors in the direction of arrival (DOA) of the desired signal or imperfect array calibration. In these situations, the performance of the LCMV or GSC beamformers are known to degrade substantially [3,4]. Similar degradation occurs when the number of snapshots used for covariance matrix is insufficient [5]. To mitigate this, robust Capon beamformers have been designed by assuming that the array steering vector belongs to a spherical uncertainty set [6-8]. These beamformers are dependent on the choice of a user parameter related to the size of the uncertainty set. However, overestimating the size of the uncertainty set will degrade spatial selectivity more than necessary, whereas underestimating it will lead to signal cancellation. Alternatively, instead of creating an adaptive null in the direction of the interfering signal, it is sometimes more convenient to design an array with decreased sidelobe levels. In [9], a simple technique that involves the addition of two elements for canceling predefined sidelobe, that is, the sidelobe into which the interference signal is coming, in the sum and difference patterns of monopulse antenna was presented. This technique is successfully extended to obtain an antenna radiation pattern with extremely low sidelobes in [10].

In this paper, we propose an effective method to design the main beamformer structure (upper channel) and the blocking structure (lower channel) in a GSC structure using Woodward-Lawson method. The task of the main beamformer is to favor the protected region, while the blocking beamformer's task is the opposite, to stop the desired signal from reaching the adaptive filter. The maximum allowable deviation in the desired-direction can be specified by the designer.

The paper is organized as follows. The steering vector error in the conventional adaptive beamforming is mathematically formulated in Section II. A set of selected results is reported in Section III to point out the potentialities of the proposed approach. Eventually, some conclusions are drawn (Section IV).

2. BACKGROUND AND PROBLEM FORMULATION
The output of a narrow band beamformer composed by N antennas is given by

$$y(k) = \mathbf{w}^{\mathbf{H}}\mathbf{x}(k) \tag{1}$$

where k is the time index, $\mathbf{x}(k) = [x_0(k) \quad x_1(k) \ldots x_{N-1}(k)]^T$ is the complex vector of received signal, $\mathbf{w} = [w_0 \quad w_1 \ldots w_{N-1}]^T$ is the beamformer weight vector, T and H denote transpose and conjugate transpose, respectively. The received signal at time instant k is given by

$$\begin{aligned}\mathbf{x}(k) &= \mathbf{s}(\mathrm{k}) + \mathbf{i}(\mathrm{k}) + \mathbf{n}(\mathrm{k}) \\ &= s(k)\mathbf{a}(\Phi_s) + \sum_{j=1}^{\mathrm{I}} i_j(k)\mathbf{a}(\Phi_j) + n(k)\end{aligned} \tag{2}$$

where I is the number of interference signals. Here, $s(k)$ and $i_j(k)$ are the signal and interference symbol samples. The signal and interference directions of arrival are Φ_s and Φ_j, $j = 1,\ldots,I$, respectively, with corresponding steering vectors $\mathbf{a}(\Phi_s)$ and $\mathbf{a}(\Phi_j)$. Let $\mathbf{R}_{xx}$ denote the $N \times N$ theoretical covariance matrix of the received signal. Assume that $\mathbf{R}_{xx}$ is a positive definite matrix with the following form

$$\mathbf{R}_{xx} = \sigma_s^2 \mathbf{a}(\Phi_s)\mathbf{a}(\Phi_s)^H + \sum_{j=1}^{I} \sigma_j^2 \mathbf{a}(\Phi_j)\mathbf{a}(\Phi_j)^H + \mathbf{Q} \quad (3)$$

where σ_s^2 and σ_j^2, $j = 1,\ldots,I$ are the powers of the uncorrelated impinging signals $s(k)$ and $i_j(k)$ respectively, and $\mathbf{Q}$ is the noise covariance matrix. A linearly constrained minimum variance (LCMV) beamformer performs the minimization of the output signal's variance with respect to some given constraints. The LCMV can be formulated as

$$\min_{\mathbf{w}} \mathbf{w}^{\mathbf{H}} \mathbf{R}_{xx} \mathbf{w} \quad \textit{subject to} \quad \mathbf{w}^{\mathbf{H}} \mathbf{a}(\Phi_{\mathbf{s}}) = 1 \quad (4)$$

The constrained optimization of the LCMV problem in (4) can be conveniently solved using a GSC as shown in Fig.1. Since $\mathbf{w}_{\mathbf{q}}$ is designed to satisfy the specified constraints, the desired signal will pass through the beamformer having a desired response independent of $\mathbf{w}_{\mathbf{a}}$. In the lower channel, the blocking matrix $\mathbf{B}$ is required to block the desired signal so that only interfering signals and noise exist. When adapting $\mathbf{w}_{\mathbf{a}}$, the scheme will tend to cancel the interference and noise component from $d(k)$, while minimizing the variance of the output signal $e(k)$.

The blocking matrix $\mathbf{B}$ in the GSC is sensitive to the steering vector error and easily leaks the desired signal (see the blocking matrix response in Fig. 1). Thus, desired signal cancellation is a serious problem [11, 12]. In order to overcome this problem a new construction of the quiescent vector and the blocking matrix is described in the following section.

3. THE PROPOSED APPROACH

The structure of the proposed beamformer is shown in Fig.2. This structure consists of main beamformer, blocking beamformer, and transversal adaptive filter.

Main Beamformer: Consider an array of N isotropic radiating elements along the x-axis and separated by equal intervals of size $d = \lambda/2$. If N is odd, say $N = 2M + 1$, we can write the array factor as [13]:

$$AF_{main}(\Psi) = \sum_{m=-M}^{M} w_m e^{jm\Psi} = w_0 + \sum_{m=1}^{M} \left[w_m e^{jm\Psi} + w_{-m} e^{-jm\Psi} \right] \quad (N = 2M + 1) \quad (5a)$$

where $\Psi = kd \cos\Phi$, k is the wave number, Φ is the angular position of the field point and w_m represents the amplitude weights. On the other hand, if N is even, say $N = 2M$, the array factor will be [13]:

$$AF_{main}(\Psi) = \sum_{m=1}^{M} \left[w_m e^{j(m-1/2)\Psi} + w_{-m} e^{-j(m-1/2)\Psi} \right] \quad (N = 2M) \quad (5b)$$

In order to compute the N array weights w_m we may use the Woodward-Lawson design method. This method is based on performing an inverse N-point discrete Fourier transform (DFT). It assumes that N samples of the desired array factor $AF_{main}(\Psi)$ are available, that is, $AF_{main}(\Psi_i), i = 0,1,\ldots,N-1$, where Ψ_i are the N DFT frequencies:

$$\Psi_i = \frac{2\pi i}{N}, i = 0,1,\ldots,N-1 \quad (6)$$

The frequency samples $AF_{main}(\Psi_i)$ are related to the array weights via the forward N-point DFT's obtained by evaluating (5) at the N DFT frequencies:

$$AF_{main}(\Psi_i) = w_0 + \sum_{m=1}^{M} \left[w_m e^{jm\Psi_i} + w_{-m} e^{-jm\Psi_i} \right], (N = 2M + 1) \quad (7a)$$

$$AF_{main}(\Psi_i) = \sum_{m=1}^{M} \left[w_m e^{j(m-1/2)\Psi_i} + w_{-m} e^{-j(m-1/2)\Psi_i} \right], (N = 2M) \quad (7b)$$

where Ψ_i are given by (6). The corresponding inverse N-point DFT's are as follows

For odd $N = 2M + 1$

$$w_m = \frac{1}{N} \sum_{i=0}^{N-1} \left[AF_{main}(\Psi_i) e^{-jm\Psi_i} \right], m = 0, \pm1, \pm2, \ldots, \pm M \quad (8a)$$

And for even $N = 2M$

$$w_{\pm m} = \frac{1}{N} \sum_{i=0}^{N-1} \left[AF_{main}(\Psi_i) e^{\mp j(m-1/2)\Psi_i} \right], m = 1,2,\ldots,M \quad (8b)$$

Fig.3 shows the design of the main beamformer response compared to the quiescent beamformer response in the GSC. In this figure, we assume a uniform linear array with $N = 20$ elements and half-wavelength element spacing. The DOA of the desired signal is assumed at 0^o. The maximum tolerance of the main beamformer to errors in the direction of the desired signal is 10^o, that is, $\Phi_2 - \Phi_1 = 10^o$. Thus, the main beam is centered at DOA of the desired signal and has width equal to 10^o. As Φ ranges over $[\Phi_1, \Phi_2]$, the wave number $\Psi = kd \cos\Phi$ will range over $kd \cos\Phi_2 \le \Psi \le kd \cos\Phi_1$. For all DFT frequencies Ψ_i that lie in this interval, we set $AF_{main}(\Psi_i) = 1$, otherwise, we set $AF_{main}(\Psi_i) = 0$. From Fig.3, it can be seen that the main beamformer which designed with a rectangular window introduces unwanted ripples in the desired direction. This is known as Gibbs phenomenon [14]. It can be minimized using an appropriate window, but at the expense of wider transition regions as depicted in Fig.3.

Blocking Beamformer: The task of the blocking beamformer is the opposite, to stop the desired signal from reaching the adaptive filter. To calculate the N array weights in the blocking beamformer we may use the inverse (reciprocal) of the Woodward-Lawson design method. That is, for all DFT frequencies Ψ_i that lie in the range $[\Phi_1, \Phi_2]$, we set

$AF_{blocking}(\Psi_i) = 0$, otherwise, we set $AF_{blocking}(\Psi_i) = 1$. In this case, the band stop condition can be written as:

$$kd \cos \Phi_2 \leq \Psi_i \leq kd \cos \Phi_1 \quad (9)$$

Using (6) and solving for the DFT index i, we find

$$\frac{Nd}{\lambda} \cos \Phi_2 \leq i \leq \frac{Nd}{\lambda} \cos \Phi_1 \quad (10)$$

this range determines the DFT indices i for which $AF_{blocking}(\Psi_i) = 0$. Fig.4 shows the response of the blocking beamformer designed with a rectangular and Hamming windows compared to the blocking matrix response in the GSC. Observe that the blocking beamformer designed with a hamming window introduces a wide null, but not deep as that of GSC, towards and around the desired direction in the lower channel of the proposed beamformer. The maximum tolerance of the blocking beamformer to errors in the direction of desired signal is 10^o. Thus, the lower channel in the proposed beamformer does not contain any desired signal component and it is reference for interference-plus-noise only.

Another example is shown in Figs.5 and 6. In these figures, the maximum tolerance to errors in the look direction is designed to be at 20^o. It can be seen from Fig. 6 that the blocking beamformer response has a more deeply and broad null at the look direction. In this way, the desired signal and interference was separated completely and avoid the desired signal cancellation. Notice that the major advantage using this method is the direct approach. The desired resolution and maximum tolerance directly determine the main and blocking beamformers of the proposed beamformer.

The outputs of the main beamformer (upper channel) and blocking beamformer (lower channel) now feed the two inputs of the standard transversal (Tapped Delay Line) adaptive filter.

TDL Adaptive Filter: Fig.7(a) shows the structure of the adaptive filter in the conventional GSC that has P channels (where $P < N$), and Fig.7(b) shows the structure of the proposed beamformer that has one channel followed by a tapped delay line (TDL). The use of a TDL in adaptive beamforming topic has been successfully applied to reduce the number of auxiliary channels to only one channel, while suppressing multiple interfering signals [15, 16]. In Fig.7(b), $d(k)$ is the received signal of the upper channel, $\mathbf{u} = [u(k), u(k-1), \ldots, u(k-P+1)]^T$ are delayed signals of the lower channel, $e(k)$ is the output signal, and $\mathbf{w}_a = [w_{a1}, w_{a2}, \ldots, w_{aP}]^T$ indicates the adaptive complex weights. The output signal of the proposed beamformer is given by

$$e(k) = d(k) - \sum_{n=0}^{P-1} w_{an}^* u(k-n) = d(k) - \mathbf{w}_\mathbf{a}^\mathbf{H} \mathbf{u} \quad (11)$$

Minimizing the mean square error of the output signal and by taking the instantaneous estimates of correlation matrix of the received signal, the LMS solution is given by [17]

$$\mathbf{w}_\mathbf{a}(k+1) = \mathbf{w}_\mathbf{a}(k) + \mu e^*(k) \mathbf{u}(k) \quad (12)$$

where μ is the step size parameter.

4. SIMULATION RESULT

Our goal for studying adaptive beamforming problem is to get the maximum signal-to-interference-plus-noise ratio (SINR) in the presence of strong interferences in radar applications as well as uncertainty in the target echo steering vector. To evaluate the performance of the proposed approach, some computer simulations have been carried out in an ideal scenario without desired steering vector errors and more realistic situations with steering vector errors. In all scenarios, the responses of the main and blocking beamformers are designed with a rectangular window. We assume the tolerance range is $\Phi_2 - \Phi_1 = 20^o$. Thus, the maximum allowable uncertainty in the desired direction is $\pm 10^o$. The array antenna to be considered is a 20-element array with half wavelength spacing. Two interfering signals with powers of $\sigma_i^2 = 30dB, \;\; i = 1,2$ are assumed to impinging on the array from the DOAs -24^o and 35^o, respectively. The presumed DOA of the desired signal is $\Phi_s = 0^o$ and has a power of $\sigma_s^2 = 10dB$. The noise, $n(k)$ is spatially and temporally white and it has a complex Gaussian zero mean distribution with variance $\sigma_n^2 = 0.1$. The weight vector in the adaptive filter for both GSC and proposed method has adaptively computed (varying with time) using LMS algorithm.

4.1 Scenario 1: Exactly Known Signal Steering Vector

In this scenario, we assume that both the presumed and actual DOA of the desired signal is impinging from $\Phi_s = 0^o$. Fig. 8(a) shows the beam patterns of the proposed beamformer compared to the GSC beamformer for the fixed snapshot size of 1000. It can be seen that, when the signal steering vector is exactly known, the both beam patterns have nulls at the DOAs of the interferences and maintain a distortionless response for the desired direction. As mentioned earlier, the proposed beamformer designed with a rectangular window introduces unwanted ripples in the desired direction. However, the GSC response presents very narrow main lobe compared to that of the proposed method. In this way, the beampattern of the proposed method can account for possible signal steering vector errors, while with the GSC it can result in deep degradations in case of any mismatch error. Fig.8(b) depicts the output SINR versus the number of available snapshots for both beamforming methods. Note that the proposed method achieves a better performance than the GSC especially when the number of snapshots is small.

4.2 Scenario 2: Signal Steering Vector Errors

In this scenario, some uncertainty in the desired steering vector is considered. First, we assume that both the presumed and actual desired DOAs are $\Phi_s = 0^o$ and $\Phi_a = 3^o$, respectively. The priory uncertainty in the desired DOA was $\pm 3^o$. Fig.9 shows the performance of the tested beamformers for the fixed snapshot size of 1000. Second, we assume that both the presumed and actual desired DOAs are $\Phi_s = 0^o$ and $\Phi_a = 8^o$, respectively. In this case, the priory uncertainty in the desired DOA was $\pm 8^o$. A plot of the beampatterns as a function of DOA is shown in Fig.10 (the snapshot size for this case is also 1000). Note that the beampattern of the GSC has unity gain in the assumed DOA. Also from figures 9 and 10, it can be seen that the GSC fails in its operation, allocating a null in the actual DOA since the desired signal is considered as an interfering signal. On the other hand, the desired signal is preserved by the proposed method for all DOAs in the range of $\left[-10^o, 10^o\right]$. Also notice that the GSC beampattern has high sidelobe levels compared to the proposed method. This can result in further degradations in case of unexpected interferences. The output SINR verses the mismatch in the signal DOA is illustrated in Fig.11 for different number of snapshots. We can highlight that the proposed method performs much better than the GSC beamformer for all DOAs mismatch and independently on the number of available snapshots. It provides excellent robustness against uncertainty in the signal DOA. On the other hand, the GSC is highly depends on the number of available snapshots.

5. CONCLUSIONS

In this paper we have described a procedure based on Woodward-Lawson method for designing adaptive beamforming that is robust against signal steering vector errors. This method modifies the conventional GSC beamformer with the goals: 1) increasing the beamformer robustness against arbitrary unknown mismatches in the desired signal steering vector by providing variable tolerance which can be specified by the user, 2) maintaining the features of simplicity and practicality. The satisfactory performance of the proposed method in terms of both interference rejection and robustness was demonstrated through computer simulations in different situations. Its operation was shown to outperform the GSC beamformer especially when the number of available snapshots is insufficient. Moreover, the presented method can be extended by investigating other array design methods such as Dolph-Chebyshev arrays.

6. REFERENCES

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7. APPENDIX

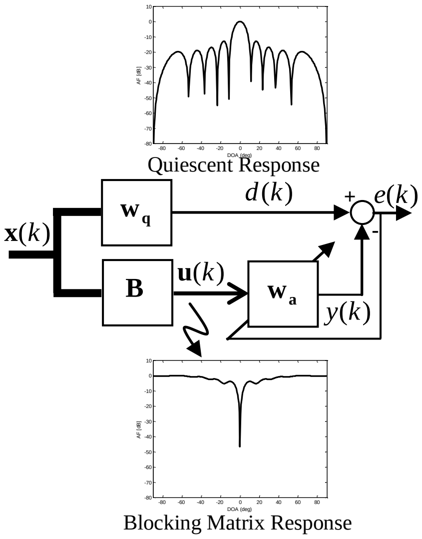


Fig.1 Structure of the Traditional GSC.

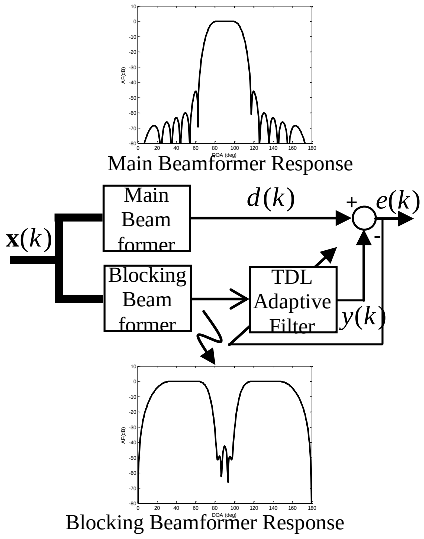


Fig. 2 Structure of the Proposed Method.

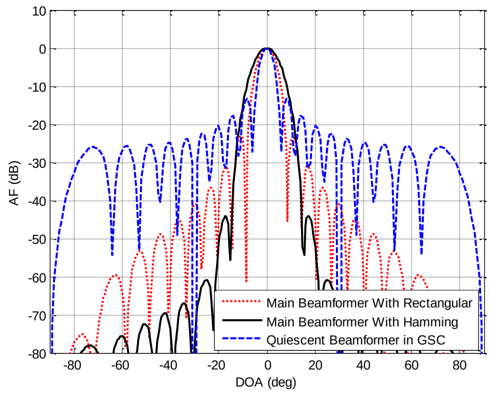


Fig.3 Main Beamformer Designed With Rectangular and Hamming Windows (Maximum Tolerance= 10^o **).**

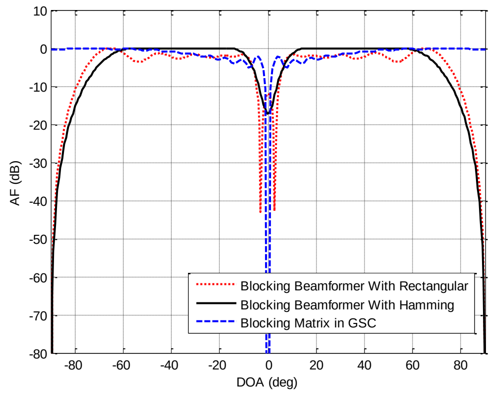


Fig.4 Blocking Beamformer Designed With Rectangular and Hamming Windows (Maximum Tolerance= 10^o **).**

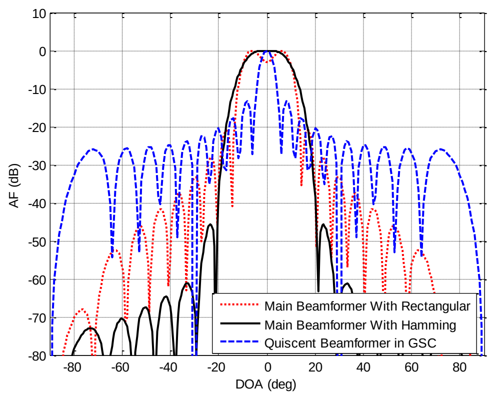


Fig.5 Main Beamformer Designed With Rectangular and Hamming Windows (Maximum Tolerance= 20^o **).**

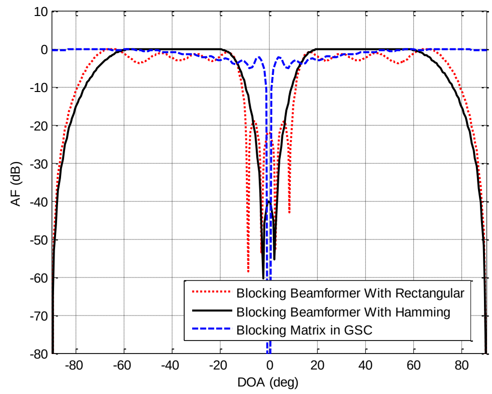
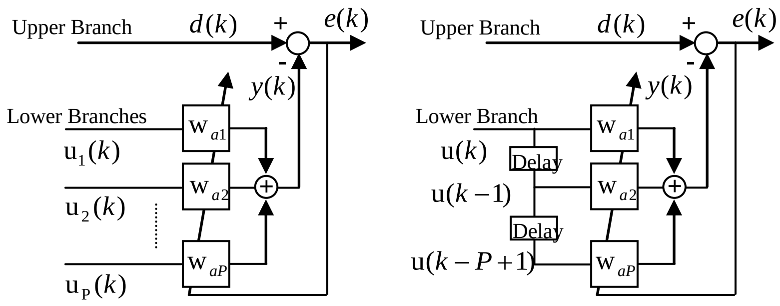
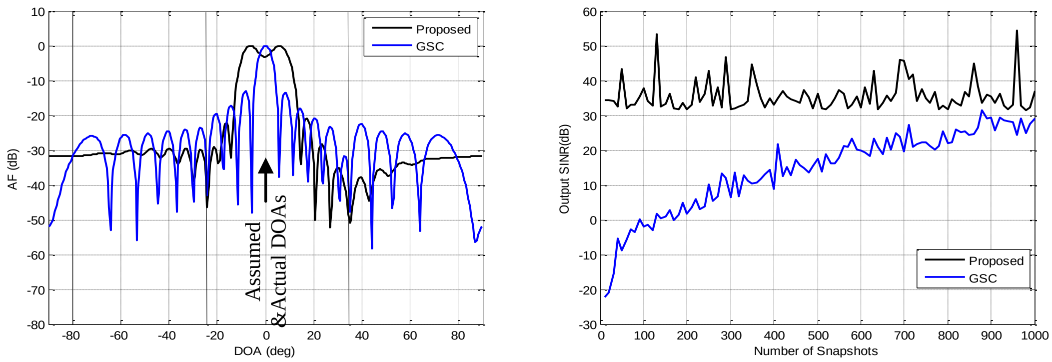


Fig.6 Blocking Beamformer Designed With Rectangular and Hamming Windows (Maximum Tolerance= 20^o **).**



(a) (b)

Fig.7 Structure of the Adaptive Filter in The (a) GSC, (b) Proposed Method.



(a) **(b)**

Fig.8 Performance Comparison of the GSC and Proposed Beamformers (No-mismatch Scenario). (a) Beampatterns For Fixed Snapshots= 1000, (b) Output SINR versus the Number of Snapshots.

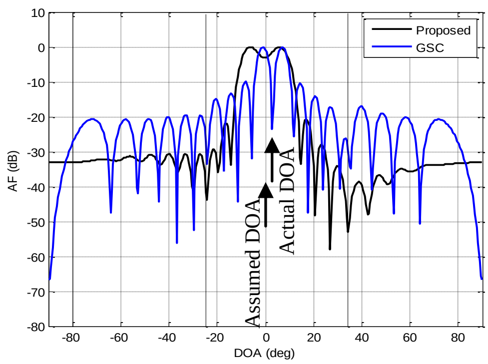


Fig.9 Comparison of the GSC and Proposed Beampatterns (The priory uncertainty in the DOA was $\pm 3^o$).

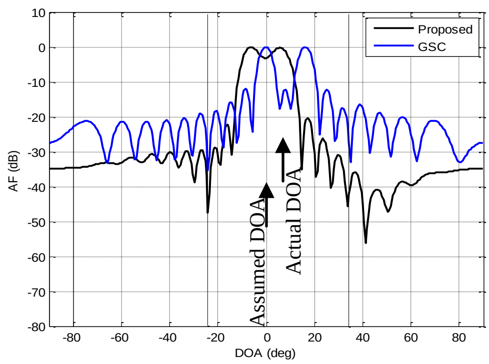
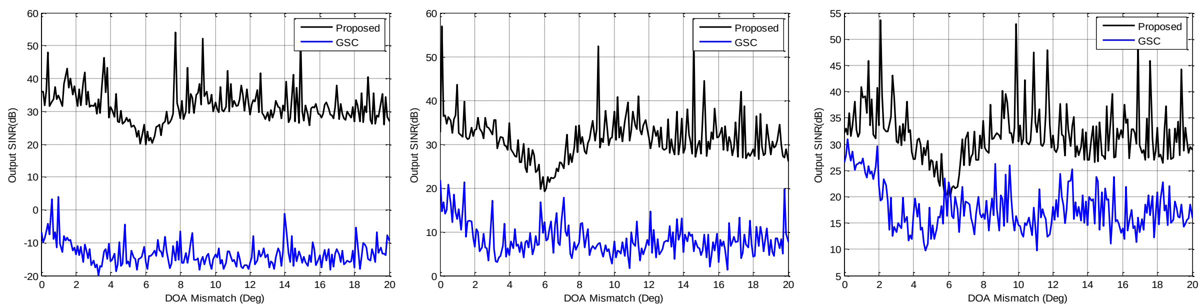


Fig.10 Comparison of the GSC and Proposed Beampatterns (The priory uncertainty in the DOA was $\pm 8^o$).



(a) **(b)** **(c)**

Fig.11 Output SINR versus DOA Mismatch for Different Number of Available Snapshots. (a) Snapshot =50, (b) Snapshot =500, and (c) Snapshot =1000.