Randić Color Energy of a Graph

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ABSTRACT

Let $G = (V, E)$ be a colored graph with vertex set $V(G)$ and edge set $E(G)$ with chromatic number $\chi(G)$ and d_i is the degree of a vertex v_i. The Randić matrix $R(G) = (r_{ij})$ of a graph G, is defined by $r_{ij} = \frac{1}{\sqrt{d_i d_j}}$, if the vertices v_i and v_j are adjacent and $r_{ij} = 0$, otherwise. The Randić energy [5] $RE(G)$ is the sum of absolute values of the eigenvalues of $R(G)$. The concept of Randić color energy $E_{RC}(G)$ of a colored graph G is defined and obtained the Randić color energy $E_{RC}(G)$ of some graphs with minimum number of colors.

Keywords

Colored graph, Randić matrix, Randić color energy

1. INTRODUCTION

Let $G = (V, E)$ be a graph on vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$ and edge set $E(G)$. Let d_i be the degree of a vertex v_i for $i = 1, 2, \ldots, n$. The adjacency matrix $A(G) = (a_{ij})$ of a graph G is a square matrix of order n, where

$$(a_{ij}) = \begin{cases} 1, & \text{if the vertices } v_i \text{ and } v_j \text{ are adjacent;} \\ 0, & \text{otherwise.} \end{cases}$$

Since $A(G)$ is symmetric, its eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$ are all real numbers, whose sum is equal to zero. The energy $E(G)$ [14] of a graph G is

$$E(G) = \sum_{i=1}^{n} |\lambda_i|.$$

In 1975 by Milan Randić [10], is defined Randić index as $R = R(G) = \sum_{i \sim j} \frac{1}{d_i d_j}$, where $\sum_{i \sim j}$ indicates summation over all pairs of adjacent vertices v_i and v_j. Randić matrix [5] $R(G) = (r_{ij})$ of G is a $n \times n$ symmetric matrix defined by

$$r_{ij} = \begin{cases} 0, & i = j; \\ \frac{1}{\sqrt{d_i d_j}}, & v_i \text{ and } v_j \text{ are adjacent;} \\ 0, & v_i \text{ and } v_j \text{ are not adjacent.} \end{cases}$$

Let $\rho_1, \rho_2, \ldots, \rho_n$ be the eigenvalues of the Randić matrix $R(G)$, these eigenvalues are real numbers, and that their sum is zero, the Randić energy [5] of a graph G is defined as

$$RE = RE(G) = \sum_{i=1}^{n} |\rho_i|.$$

In the last few years, research publications on Randić spectrum, Randić indices and Randić energy can be found in literature [3], [4], [5], [7], [8].

A coloring of graph [1] G is a coloring of its vertices such that no two adjacent vertices receive the same color. The minimum number of colors needed for coloring of a graph G is called chromatic number and denoted by $\chi(G)$. The color adjacency matrix [1] $A_C(G)$ are as follows: If $c(v_i)$ is the color of v_i, then

$$a_{ij} = \begin{cases} 1, & \text{if } v_i \text{ and } v_j \text{ are adjacent with } c(v_i) \neq c(v_j); \\ -1, & \text{if } v_i \text{ and } v_j \text{ are non-adjacent with } c(v_i) = c(v_j); \\ 0, & \text{otherwise.} \end{cases}$$

If $\lambda_1, \lambda_2, \ldots, \lambda_n$ are eigenvalues of $A_c(G)$ are called color eigenvalues. Color energy [1] of a graph is

$$E_c(G) = \sum_{i=1}^{n} |\lambda_i|.$$

For more literature on color energy of a graph, we can see [1], [2], [11], [12], [13].

1.1 Randić color matrix and Randić color energy

Motivated by Color Energy of a Graph [1] and Randić Matrix and Randić Energy [5], have obtained a new matrix called Randić color matrix.

Let G be a simple colored graph with n vertices. The Randić color matrix $A_{RC}(G) = (r_{ij})$ is a square $n \times n$ matrix defined by

$$r_{ij} = \begin{cases} \frac{1}{\sqrt{d_i d_j}}, & \text{if } v_i \text{ and } v_j \text{ are adjacent with } c(v_i) \neq c(v_j); \\ \frac{-1}{\sqrt{d_i d_j}}, & \text{if } v_i \text{ and } v_j \text{ are non-adjacent with } c(v_i) = c(v_j); \\ 0, & \text{otherwise.} \end{cases}$$

The characteristic polynomial of $A_{RC}(G)$ is $|\rho I - A_{RC}(G)|$. Let $\rho_1, \rho_2, \ldots, \rho_n$ be eigenvalues of Randić color matrix $A_{RC}(G)$. Since $A_{RC}(G)$ is real and symmetric matrix, so its eigenvalues are real numbers and that their sum is zero. If the eigenvalues of $A_{RC}(G)$ are $\rho_1, \rho_2, \ldots, \rho_n$ with their multiplicities are $m_1, m_2, \ldots, m_r$ then spectrum of $A_{RC}(G)$ is denoted by

$$Spec_{RC}(G) = \begin{pmatrix} \rho_1 & \rho_2 & \ldots & \rho_{n-1} & \rho_n \\ m_1 & m_2 & \ldots & m_{r-1} & m_r \end{pmatrix}.$$

The Randić color energy $E_{RC}(G)$ of a colored graph G is defined as

$$E_{RC}(G) = \sum_{i=1}^{n} |\rho_i|.$$

2. RANDIĆ COLOR ENERGY OF SOME GRAPHS

THEOREM 1. *If K_n is complete graph with n vertices and $\chi(K_n) = n$, then Randić color energy of K_n is $E_{RC}(K_n) = 2$.*

PROOF. Let K_n be the complete graph with n vertices. Since $\chi(K_n) = n$, we have Randić color matrix

$$|\rho I - A_{RC}(K_n)| = \begin{vmatrix} \rho & \frac{-1}{n-1} & \frac{-1}{n-1} & \cdots & \frac{-1}{n-1} & \frac{-1}{n-1} & \frac{-1}{n-1} \\ \frac{-1}{n-1} & \rho & \frac{-1}{n-1} & \cdots & \frac{-1}{n-1} & \frac{-1}{n-1} & \frac{-1}{n-1} \\ \frac{-1}{n-1} & \frac{-1}{n-1} & \rho & \cdots & \frac{-1}{n-1} & \frac{-1}{n-1} & \frac{-1}{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \frac{-1}{n-1} & \frac{-1}{n-1} & \frac{-1}{n-1} & \cdots & \rho & \frac{-1}{n-1} & \frac{-1}{n-1} \\ \frac{-1}{n-1} & \frac{-1}{n-1} & \frac{-1}{n-1} & \cdots & \frac{-1}{n-1} & \rho & \frac{-1}{n-1} \\ \frac{-1}{n-1} & \frac{-1}{n-1} & \frac{-1}{n-1} & \cdots & \frac{-1}{n-1} & \frac{-1}{n-1} & \rho \end{vmatrix}_{n \times n}.$$

$R'_1 = R_1 + R_2 + \cdots + R_{n-1} + R_n$, then we get $(\rho - 1)$ common in first row

$$|\rho I - A_{RC}(K_n)| = (\rho - 1) \begin{vmatrix} 1 & 1 & 1 & \cdots & 1 & 1 & 1 \\ \frac{-1}{n-1} & \rho & \frac{-1}{n-1} & \cdots & \frac{-1}{n-1} & \frac{-1}{n-1} & \frac{-1}{n-1} \\ \frac{-1}{n-1} & \frac{-1}{n-1} & \rho & \cdots & \frac{-1}{n-1} & \frac{-1}{n-1} & \frac{-1}{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \frac{-1}{n-1} & \frac{-1}{n-1} & \frac{-1}{n-1} & \cdots & \rho & \frac{-1}{n-1} & \frac{-1}{n-1} \\ \frac{-1}{n-1} & \frac{-1}{n-1} & \frac{-1}{n-1} & \cdots & \frac{-1}{n-1} & \rho & \frac{-1}{n-1} \\ \frac{-1}{n-1} & \frac{-1}{n-1} & \frac{-1}{n-1} & \cdots & \frac{-1}{n-1} & \frac{-1}{n-1} & \rho \end{vmatrix}_{n \times n}.$$

$R'_k = R_k + \frac{R_1}{n-1}, k = 2, 3, \ldots, n$, we get, Characteristic polynomial $|\rho I - A_{RC}(K_n)| = (\rho - 1)\left(\rho + \dfrac{1}{n-1}\right)^{n-1}$,

$Spec_{RC}(K_n) = \begin{pmatrix} \dfrac{-1}{n-1} & 1 \\ n-1 & 1 \end{pmatrix}$, Randić Color Energy of K_n is $E_{RC}(K_n) = 2$.

□

THEOREM 2. *If S_n is star graph with n vertices and $\chi(S_n) = 2$, then Randić color energy of S_n is $E_{RC}(S_n) = \sqrt{n^2 - 4n + 8} + (n-2)$.*

PROOF. Let S_n be the star graph with n vertices. Since $\chi(S_n) = 2$, we have Randić color matrix

$$|\rho I - A_{RC}(S_n)| = \begin{vmatrix} \rho & \frac{-1}{\sqrt{n-1}} & \frac{-1}{\sqrt{n-1}} & \cdots & \frac{-1}{\sqrt{n-1}} & \frac{-1}{\sqrt{n-1}} & \frac{-1}{\sqrt{n-1}} \\ \frac{-1}{\sqrt{n-1}} & \rho & 1 & \cdots & 1 & 1 & 1 \\ \frac{-1}{\sqrt{n-1}} & 1 & \rho & \cdots & 1 & 1 & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \frac{-1}{\sqrt{n-1}} & 1 & 1 & \cdots & \rho & 1 & 1 \\ \frac{-1}{\sqrt{n-1}} & 1 & 1 & \cdots & 1 & \rho & 1 \\ \frac{-1}{\sqrt{n-1}} & 1 & 1 & \cdots & 1 & 1 & \rho \end{vmatrix}_{n \times n}.$$

$R'_k = R_k - R_{k-1}, k = 3, 4, 5, \ldots, (n-1), n$, then taking $(\rho - 1)$ common from R_3 to R_n, we get

$$|\rho I - A_{RC}(S_n)| = (\rho - 1)^{n-2} \begin{vmatrix} \rho & \frac{-1}{\sqrt{n-1}} & \frac{-1}{\sqrt{n-1}} & \cdots & \frac{-1}{\sqrt{n-1}} & \frac{-1}{\sqrt{n-1}} & \frac{-1}{\sqrt{n-1}} \\ \frac{-1}{\sqrt{n-1}} & \rho & 1 & \cdots & 1 & 1 & 1 \\ 0 & -1 & 1 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 & 0 \\ 0 & 0 & 0 & \cdots & -1 & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & -1 & 1 \end{vmatrix}_{n \times n}.$$

$C'_k = C_k + C_{k+1} + \cdots + C_n, k = 2, 3, \ldots, (n-1), n$, then

$$|\rho I - A_{RC}(S_n)| = (\rho - 1)^{n-2} \begin{vmatrix} \rho & \frac{-(n-1)}{\sqrt{n-1}} \\ \frac{-1}{\sqrt{n-1}} & \rho + (n-2) \end{vmatrix},$$

Characteristic polynomial

$$|\rho I - A_{RC}(S_n)| = (\rho - 1)^{n-2}\left[\rho^2 + (n-2)\rho - 1\right],$$

$$Spec_{RC}(S_n) = \begin{pmatrix} \frac{-(n-2) - \sqrt{n^2-4n+8}}{2} & \frac{-(n-2) + \sqrt{n^2-4n+8}}{2} & 1 \\ 1 & 1 & n-2 \end{pmatrix},$$

Randić Color Energy of S_n is

$$E_{RC}(S_n) = \sqrt{n^2 - 4n + 8} + (n-2).$$

□

THEOREM 3. *If $K_{n,n}$ is complete bipartite graph with $2n$ vertices and $\chi(K_{n,n}) = 2$, then Randić color energy of $K_{n,n}$ is $E_{RC}(K_{n,n}) = \frac{4n-2}{n}$.*

PROOF. Let $K_{n,n}$ be the complete bipartite graph with $2n$ vertices. Since $\chi(K_{n,n}) = 2$, we have Randić color matrix

$$|\rho I - A_{RC}(K_{n,n})| = \begin{vmatrix} \rho & \frac{1}{n} & \cdots & \frac{1}{n} & \frac{1}{n} & \frac{-1}{n} & \frac{-1}{n} & \cdots & \frac{-1}{n} & \frac{-1}{n} \\ \frac{1}{n} & \rho & \cdots & \frac{1}{n} & \frac{1}{n} & \frac{-1}{n} & \frac{-1}{n} & \cdots & \frac{-1}{n} & \frac{-1}{n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{1}{n} & \frac{1}{n} & \cdots & \rho & \frac{1}{n} & \frac{-1}{n} & \frac{-1}{n} & \cdots & \frac{-1}{n} & \frac{-1}{n} \\ \frac{1}{n} & \frac{1}{n} & \cdots & \frac{1}{n} & \rho & \frac{-1}{n} & \frac{-1}{n} & \cdots & \frac{-1}{n} & \frac{-1}{n} \\ \frac{-1}{n} & \frac{-1}{n} & \cdots & \frac{-1}{n} & \frac{-1}{n} & \rho & \frac{1}{n} & \cdots & \frac{1}{n} & \frac{1}{n} \\ \frac{-1}{n} & \frac{-1}{n} & \cdots & \frac{-1}{n} & \frac{-1}{n} & \frac{1}{n} & \rho & \cdots & \frac{1}{n} & \frac{1}{n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{-1}{n} & \frac{-1}{n} & \cdots & \frac{-1}{n} & \frac{-1}{n} & \frac{1}{n} & \frac{1}{n} & \cdots & \rho & \frac{1}{n} \\ \frac{-1}{n} & \frac{-1}{n} & \cdots & \frac{-1}{n} & \frac{-1}{n} & \frac{1}{n} & \frac{1}{n} & \cdots & \frac{1}{n} & \rho \end{vmatrix}_{2n\times 2n}.$$

$R_1' = (R_1 + R_2 + \cdots + R_n) - (R_{n+1} + R_{n+2} + \cdots + R_{2n})$, $R_k' = R_k + R_{2n}$, $k = 2, 3, \ldots, n$ and $R_k' = R_k - R_{2n}$, $k = n+1, n+2, \ldots, 2n-1$, then we get $\left(\rho + \frac{2n-1}{n}\right)$ common in first row and $\left(\rho - \frac{1}{n}\right)$ common from R_2 to R_{2n-1}

$$= \left(\rho + \frac{2n-1}{n}\right)\left(\rho - \frac{1}{n}\right)^{2n-2} \begin{vmatrix} 1 & 1 & \cdots & 1 & 1 & -1 & -1 & \cdots & -1 & -1 \\ 0 & 1 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 0 & 0 & 1 & 0 & \cdots & 0 & -1 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 1 & \cdots & 0 & -1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 1 & -1 \\ \frac{-1}{n} & \frac{-1}{n} & \cdots & \frac{-1}{n} & \frac{-1}{n} & \frac{1}{n} & \frac{1}{n} & \cdots & \frac{1}{n} & \rho \end{vmatrix}_{2n\times 2n}.$$

$R_{2n}' = R_{2n} + \frac{R_1}{n}$, we get Characteristic polynomial

$$|\rho I - A_{RC}(K_{n,n})| = \left(\rho + \frac{2n-1}{n}\right)\left(\rho - \frac{1}{n}\right)^{2n-1},$$

$$Spec_{RC}(K_{n,n}) = \begin{pmatrix} -\left(\frac{2n-1}{n}\right) & \frac{1}{n} \\ 1 & 2n-1 \end{pmatrix}.$$

Randić Color Energy of $K_{n,n}$ is $E_{RC}(K_{n,n}) = \frac{4n-2}{n}$. □

THEOREM 4. *If $K_{n,n+1}$ is complete bipartite graph with $2n+1$ vertices and $\chi(K_{n,n+1}) = 2$, then Randić color energy of $K_{n,n+1}$ is*

$$E_{RC}(K_{n,n+1}) = \frac{2\left[\sqrt{n^2+2n+2} + n\right]}{n+1}.$$

PROOF. Let $K_{n,n+1}$ be the complete bipartite graph with $2n+1$ vertices. Since $\chi(K_{n,n+1}) = 2$, we have Randić color matrix $A_{RC}(K_{n,n+1})$ is a square matrix of order $2n+1$.

$$|\rho I - A_{RC}(K_{n,n+1})| =$$

$$\begin{vmatrix} \rho & \frac{1}{n+1} & \cdots & \frac{1}{n+1} & \frac{-1}{\sqrt{n(n+1)}} & \cdots & \frac{-1}{\sqrt{n(n+1)}} & \frac{-1}{\sqrt{n(n+1)}} \\ \frac{1}{n+1} & \rho & \cdots & \frac{1}{n+1} & \frac{-1}{\sqrt{n(n+1)}} & \cdots & \frac{-1}{\sqrt{n(n+1)}} & \frac{-1}{\sqrt{n(n+1)}} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{1}{n+1} & \frac{1}{n+1} & \cdots & \frac{1}{n+1} & \frac{-1}{\sqrt{n(n+1)}} & \cdots & \frac{-1}{\sqrt{n(n+1)}} & \frac{-1}{\sqrt{n(n+1)}} \\ \frac{1}{n+1} & \frac{1}{n+1} & \cdots & \rho & \frac{-1}{\sqrt{n(n+1)}} & \cdots & \frac{-1}{\sqrt{n(n+1)}} & \frac{-1}{\sqrt{n(n+1)}} \\ \frac{-1}{\sqrt{n(n+1)}} & \frac{-1}{\sqrt{n(n+1)}} & \cdots & \frac{-1}{\sqrt{n(n+1)}} & \rho & \cdots & \frac{1}{n} & \frac{1}{n} \\ \frac{-1}{\sqrt{n(n+1)}} & \frac{-1}{\sqrt{n(n+1)}} & \cdots & \frac{-1}{\sqrt{n(n+1)}} & \frac{1}{n} & \cdots & \frac{1}{n} & \frac{1}{n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{-1}{\sqrt{n(n+1)}} & \frac{-1}{\sqrt{n(n+1)}} & \cdots & \frac{-1}{\sqrt{n(n+1)}} & \cdots & \rho & \frac{1}{n} & \\ \frac{-1}{\sqrt{n(n+1)}} & \frac{-1}{\sqrt{n(n+1)}} & \cdots & \frac{-1}{\sqrt{n(n+1)}} & \frac{1}{n} & \cdots & \frac{1}{n} & \rho \end{vmatrix}$$

$R_k' = R_k - R_n, k = 1, 2, 3, \ldots, n-1$, $R_m' = R_m - R_{n+1}$, $m = n+2, n+3, \ldots, 2n+1$ then taking $\left(\rho - \frac{1}{n+1}\right)$ common from R_1 to R_{n-1} and $\left(\rho - \frac{1}{n}\right)$ from R_{n+2} to R_{2n+1}

$$\left(\rho - \frac{1}{n+1}\right)^{n-1}\left(\rho - \frac{1}{n}\right)^{n}$$

$$\begin{vmatrix} 1 & 0 & \cdots & -1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & -1 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -1 & 0 & 0 & \cdots & 0 \\ \frac{1}{n+1} & \frac{1}{n+1} & \cdots & \rho & \frac{-1}{\sqrt{n(n+1)}} & \frac{-1}{\sqrt{n(n+1)}} & \cdots & \frac{-1}{\sqrt{n(n+1)}} \\ \frac{-1}{\sqrt{n(n+1)}} & \frac{-1}{\sqrt{n(n+1)}} & \cdots & \frac{-1}{\sqrt{n(n+1)}} & \rho & \frac{1}{n} & \cdots & \frac{1}{n} \\ 0 & 0 & \cdots & 0 & -1 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \\ 0 & 0 & \cdots & 0 & -1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & -1 & 0 & \cdots & 1 \end{vmatrix}$$

$C_{n+1}' = C_{n+1} + C_{n+2} + \cdots + C_{2n+1}$

$$= \left(\rho - \frac{1}{n+1}\right)^{n-1}\left(\rho - \frac{1}{n}\right)^{n}$$

$$\begin{vmatrix} 1 & 0 & \cdots & 0 & -1 & 0 \\ 0 & 1 & \cdots & 0 & -1 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -1 & 0 \\ \frac{1}{n+1} & \frac{1}{n+1} & \cdots & \frac{1}{n+1} & \rho & \frac{-(n+1)}{\sqrt{n(n+1)}} \\ \frac{-1}{\sqrt{n(n+1)}} & \frac{-1}{\sqrt{n(n+1)}} & \cdots & \frac{-1}{\sqrt{n(n+1)}} & \frac{-1}{\sqrt{n(n+1)}} & \rho+1 \end{vmatrix}_{(n+1)\times(n+1)}$$

$C'_n = C_1 + C_2 + \cdots + C_{n-1} + C_n$, then

$$|\rho I - A_{RC}(K_{n,n+1})| = \left(\rho - \frac{1}{n+1}\right)^{n-1}\left(\rho - \frac{1}{n}\right)^{n}\begin{vmatrix} \rho + \frac{n-1}{n+1} & \frac{-(n+1)}{\sqrt{n(n+1)}} \\ \frac{-n}{\sqrt{n(n+1)}} & \rho + 1 \end{vmatrix}$$

Characteristic polynomial

$$|\rho I - A_{RC}(K_{n,n+1})| = \left(\rho - \frac{1}{n+1}\right)^{n-1}\left(\rho - \frac{1}{n}\right)^{n}\left[(n+1)\rho^2 + 2n\rho - 2\right],$$

$$Spec_{RC}(K_{n,n+1}) = \begin{pmatrix} \frac{-n-\sqrt{n^2+2n+2}}{n+1} & \frac{-n+\sqrt{n^2+2n+2}}{n+1} & \frac{1}{n+1} & \frac{1}{n} \\ 1 & 1 & n-1 & n \end{pmatrix},$$

Randić Color Energy of $K_{n,n+1}$ is

$$E_{RC}(K_{n,n+1}) = \frac{2\left[\sqrt{n^2+2n+2}+n\right]}{n+1}.$$

□

THEOREM 5. *If $K_{m,n}$ is complete bipartite graph with $m+n$ vertices and $\chi(K_{m,n}) = 2$, then Randić color energy of $K_{m,n}$ is*

$$E_{RC}(K_{m,n}) = \frac{\sqrt{(m^2+n^2-m-n)^2 + 4mn(m+n-1)} + (m^2+n^2-m-n)}{mn},$$

where $m < n$.

PROOF. Let $K_{m,n}$ be the complete bipartite graph with $m+n$ vertices. Since $\chi(K_{m,n}) = 2$, we have Randić color matrix

$$|\rho I - A_{RC}(K_{m,n})| =$$

$$\begin{vmatrix} \rho & \frac{1}{n} & \cdots & \frac{1}{n} & \frac{1}{n} & \frac{-1}{\sqrt{mn}} & \frac{-1}{\sqrt{mn}} & \cdots & \frac{-1}{\sqrt{mn}} & \frac{-1}{\sqrt{mn}} \\ \frac{1}{n} & \rho & \cdots & \frac{1}{n} & \frac{1}{n} & \frac{-1}{\sqrt{mn}} & \frac{-1}{\sqrt{mn}} & \cdots & \frac{-1}{\sqrt{mn}} & \frac{-1}{\sqrt{mn}} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{1}{n} & \frac{1}{n} & \cdots & \rho & \frac{1}{n} & \frac{-1}{\sqrt{mn}} & \frac{-1}{\sqrt{mn}} & \cdots & \frac{-1}{\sqrt{mn}} & \frac{-1}{\sqrt{mn}} \\ \frac{1}{n} & \frac{1}{n} & \cdots & \frac{1}{n} & \rho & \frac{-1}{\sqrt{mn}} & \frac{-1}{\sqrt{mn}} & \cdots & \frac{-1}{\sqrt{mn}} & \frac{-1}{\sqrt{mn}} \\ \frac{-1}{\sqrt{mn}} & \frac{-1}{\sqrt{mn}} & \cdots & \frac{-1}{\sqrt{mn}} & \frac{-1}{\sqrt{mn}} & \rho & \frac{1}{m} & \cdots & \frac{1}{m} & \frac{1}{m} \\ \frac{-1}{\sqrt{mn}} & \frac{-1}{\sqrt{mn}} & \cdots & \frac{-1}{\sqrt{mn}} & \frac{-1}{\sqrt{mn}} & \frac{1}{m} & \rho & \cdots & \frac{1}{m} & \frac{1}{m} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{-1}{\sqrt{mn}} & \frac{-1}{\sqrt{mn}} & \cdots & \frac{-1}{\sqrt{mn}} & \frac{-1}{\sqrt{mn}} & \frac{1}{m} & \frac{1}{m} & \cdots & \rho & \frac{1}{m} \\ \frac{-1}{\sqrt{mn}} & \frac{-1}{\sqrt{mn}} & \cdots & \frac{-1}{\sqrt{mn}} & \frac{-1}{\sqrt{mn}} & \frac{1}{m} & \frac{1}{m} & \cdots & \frac{1}{m} & \rho \end{vmatrix}$$

$R'_k = R_k - R_m, k = 1,2,3,\ldots,m-1$, $R'_d = R_d - R_{m+1}$, $d = m+2, m+3, \ldots, m+n$, then taking $\left(\rho - \frac{1}{n}\right)$ common from R_1 to R_{m-1} and taking $\left(\rho - \frac{1}{m}\right)$ common from R_{m+2} to R_{m+n}, we get

$$\left(\rho - \frac{1}{n}\right)^{m-1}\left(\rho - \frac{1}{m}\right)^{n-1}$$

$$\begin{vmatrix} 1 & 0 & \cdots & 0 & -1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & -1 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -1 & 0 & 0 & \cdots & 0 & 0 \\ \frac{1}{n} & \frac{1}{n} & \cdots & \frac{1}{n} & \rho & \frac{-1}{\sqrt{mn}} & \frac{-1}{\sqrt{mn}} & \cdots & \frac{-1}{\sqrt{mn}} & \frac{-1}{\sqrt{mn}} \\ \frac{-1}{\sqrt{mn}} & \frac{-1}{\sqrt{mn}} & \cdots & \frac{-1}{\sqrt{mn}} & \frac{-1}{\sqrt{mn}} & \rho & \frac{1}{m} & \cdots & \frac{1}{m} & \frac{1}{m} \\ 0 & 0 & \cdots & 0 & 0 & -1 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & -1 & 0 & \cdots & 1 & 0 \\ 0 & 0 & \cdots & 0 & 0 & -1 & 0 & \cdots & 0 & 1 \end{vmatrix}$$

Characteristic polynomial

$$|\rho I - A_{RC}(K_{m,n})| = \left(\rho - \frac{1}{n}\right)^{m-1}\left(\rho - \frac{1}{m}\right)^{n-1}\left[mn\rho^2 + (n^2+m^2-n-m)\rho - (m+n-1)\right]$$

$$Spec_{RC}(K_{m,n}) = \begin{bmatrix} \frac{-(m^2+n^2-m-n) \pm \sqrt{(m^2+n^2-m-n)^2 + 4mn(m+n-1)}}{2mn} & \frac{1}{n} & \frac{1}{m} \\ 1 & m-1 & n-1 \end{bmatrix}$$

Randić Color Energy of $K_{m,n}$ is

$$E_{RC}(K_{m,n}) = \frac{\sqrt{(m^2+n^2-m-n)^2 + 4mn(m+n-1)} + (m^2+n^2-m-n)}{mn}.$$

□

THEOREM 6. *If S_n^0 is crown graph with $2n$ vertices and $\chi(S_n^0) = 2$, then Randić color energy of S_n^0 is $E_{RC}(S_n^0) = \frac{4n-4}{n}$.*

PROOF. Let S_n^0 be the crown graph with $2n$ vertices. Since $\chi(S_n^0) = 2$, we have Randić color matrix

$$|\rho I - A_{RC}(S_n^0)| = \begin{vmatrix} \rho & \frac{1}{n} & \cdots & \frac{1}{n} & \frac{1}{n} & 0 & \frac{-1}{n} & \cdots & \frac{-1}{n} & \frac{-1}{n} \\ \frac{1}{n} & \rho & \cdots & \frac{1}{n} & \frac{1}{n} & \frac{-1}{n} & 0 & \cdots & \frac{-1}{n} & \frac{-1}{n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{1}{n} & \frac{1}{n} & \cdots & \rho & \frac{1}{n} & \frac{-1}{n} & \frac{-1}{n} & \cdots & 0 & \frac{-1}{n} \\ \frac{1}{n} & \frac{1}{n} & \cdots & \frac{1}{n} & \rho & \frac{-1}{n} & \frac{-1}{n} & \cdots & \frac{-1}{n} & 0 \\ 0 & \frac{-1}{n} & \cdots & \frac{-1}{n} & \frac{-1}{n} & \rho & \frac{1}{n} & \cdots & \frac{1}{n} & \frac{1}{n} \\ \frac{-1}{n} & 0 & \cdots & \frac{-1}{n} & \frac{-1}{n} & \frac{1}{n} & \rho & \cdots & \frac{1}{n} & \frac{1}{n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{-1}{n} & \frac{-1}{n} & \cdots & 0 & \frac{-1}{n} & \frac{1}{n} & \frac{1}{n} & \cdots & \rho & \frac{1}{n} \\ \frac{-1}{n} & \frac{-1}{n} & \cdots & \frac{-1}{n} & 0 & \frac{1}{n} & \frac{1}{n} & \cdots & \frac{1}{n} & \rho \end{vmatrix}_{2n\times 2n}.$$

$C'_k = C_k + C_{n+k}, k = 1,2,3,\ldots,n$ then $R'_{n+k} = R_{n+k} - R_k$, $k = 1,2,3,\ldots,n$ then we get,

$$|\rho I - A_{RC}(S_n^0)| = \rho^n \begin{vmatrix} \rho & \frac{2}{n} & \cdots & \frac{2}{n} & \frac{2}{n} \\ \frac{2}{n} & \rho & \cdots & \frac{2}{n} & \frac{2}{n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{2}{n} & \frac{2}{n} & \cdots & \rho & \frac{2}{n} \\ \frac{2}{n} & \frac{2}{n} & \cdots & \frac{2}{n} & \rho \end{vmatrix}_{n\times n}.$$

Characteristic polynomial

$$|\rho I - A_{RC}(S_n^0)| = \rho^n\left(\rho + \frac{2(n-1)}{n}\right)\left(\rho - \frac{2}{n}\right)^{n-1},$$

$$Spec_{RC}(S_n^0) = \begin{pmatrix} -\left(\frac{2(n-1)}{n}\right) & 0 & \frac{2}{n} \\ 1 & n & n-1 \end{pmatrix}.$$

Randić Color Energy of S_n^0 is $E_{RC}(S_n^0) = \frac{4(n-1)}{n}$. □

3. ACKNOWLEDGEMENT:

The author would like to thank the Prof. R. Rangarajan, DOS in Mathematics, University of Mysore, Mysuru, for many useful suggestions.

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