Separation Axioms in Soft Tritopological Spaces with Respect to Ordinary Points

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ABSTRACT
In the present paper the definitions of separation axioms in $soft$ −tritopological spaces are introduced dependent to the $soft-\delta^*-open$ set and their basic properties are investigated with respect to ordinary points. That is, the $soft-\delta^*-\mathrm{T}_i$; ($i=0,1,2,3,4$) spaces and notions of $soft-\delta^*$ −normal and $soft-\delta^*$ −regular spaces are discussed in detail, also we introduce some theorems shows how one of the $soft$ −spaces implies the others with the help of an examples it is established that the converse does not hold.

General Terms
2010 Mathematics Subject Classification: 54E55, 54D10.

Keywords
$soft$ −tritopological space, $soft-\delta^*-open$ set, $soft-\delta^*-\mathrm{T}_i$; ($i=0,1,2,3,4$) space, $soft-\delta^*$ −regular space, $soft-\delta^*$ −normal space.

1. INTRODUCTION
The $soft$ −theory was first initiated by Molodtsov [1] in 1999 as a mathematical tool for uncertain objects. In [1,2], the researcher successfully applied the theory in several directions, like operations research, probability, theory of measurement, and others. To develop $soft$ −set theory, the operations of the $soft$ −sets are defined in [3].

A topology on a $soft$ −sets, called ''$soft$ −topology'', and its related properties which presented in 2011 by M Shabir & M. Naz, [4] , Cagman et. al. [5], and they introduced the foundations of the theory of $soft$ −topological spaces. A bitopology on a $soft$ −sets, called '$soft$ −bitopology', and its related properties which presented by Ittanagi [6] in 2014, and he introduced the foundations of the theory of $soft$ −bitopological spaces.

Furthermore, the theory of '$soft$ −tritopology' was first initiated by Asmhan [7] in 2017, and she was presented the foundations and its related properties. And in 2019 she was first initiated the theory of $fuzzy-soft$ −tritopology [8]. In 2019, the Italian researcher Giorgio [9] introduced the theory of '$soft-N$ −topology', and study it in details.

Separation axioms in $soft$ −topology are among the most interesting notions. Various generalizations of separation axioms have been studied for generalized $soft$ −topological spaces, such as $soft$ −bitopological. It is very interesting to see that when classical $soft$ −notions are replaced by new generalized $soft$ −notions. D. N. Georgiou et. al [10] , A. Kandil et. al. [11], El-Sheikh et. al. [12] and Shabir et. al [4], studied and defined some separation axioms in $soft$ −topological spaces using (ordinary) points. A. M. Khattak et. al. [13] introduced $soft$ −b-separation axioms in $soft$ −bitopological spaces and discussed it with respect to ordinary and $soft$ −points. And Ittanagi [6] introduced some types of $soft$ −separation axioms in $soft$ −bitopological spaces with respect to ordinary points. Also, The concept of pair-wise $soft$ −separation axioms for bi- $soft$ −topological spaces studied by M. Naz et. al. in [14].

In this paper, we define and explore several properties of $\mathrm{soft}-\delta^*-\mathrm{T}_i$; ($i=0,1,2,3,4$) space, $\mathrm{soft}-\delta^*$ −regular space, $\mathrm{soft}-\delta^*$ −normal space. All these axioms are defined using ordinary points. We also discuss some properties and obtained some results. We hope that these results will be useful for the future works on $soft$ −tritopology to carry out general framework for some practical applications, such as medical, uncertainties problems and biomathematics.

Now, our motivation in the present paper is to state and continue the foundations of the $soft$ −tritopological spaces theory. For more exactly, to define and study the $soft-\delta^*$ −separation axioms (based on $soft-\delta^*-open$ set) w.r.t. ordinary points and study the main properties for these $soft$ −tritopological spaces.

2. PRELIMINARIES
In the following, some foundations concepts about $soft$ − sets, $soft$ −topological spaces and $soft$ − tritopological spaces are given.

Definition 2.1[1] Let the set $\mathbb{X}$ be an initial universe and $\mathbb{E}$ be a set of parameters. Let $\mathcal{P}(\mathbb{X})$ denotes the power set of $\mathbb{X}$ and $\mathbb{A}$ be a non-empty subset of $\mathbb{E}$. A pair $(\mathbb{F},\mathbb{A})$ is said to be a $soft$ −set over $\mathbb{X}$, where $\mathbb{F}$ is a mapping given by $\mathbb{F}:\mathbb{A}\to\mathcal{P}(\mathbb{X})$.In other words, a $soft$ −set over $\mathbb{X}$ is a parametrized family of subsets of the universe $\mathbb{X}$. For $e\in\mathbb{A}$, $\mathbb{F}(e)$ may be considered as the set of e−approximate elements of the $soft$ −set $(\mathbb{F},\mathbb{A})$. Clear that, a $soft$ −set is not a set.

Definition 2.2 [3] The complement (relative complement) of a $soft$ −set $(\mathbb{F},\mathbb{E})$ is denoted by $(\mathbb{F},\mathbb{E})^c$ and is defined by $(\mathbb{F},\mathbb{E})^c=(\mathcal{F}^c,\mathbb{E})$ where $\mathcal{F}^c:\mathbb{E}\to\mathcal{P}(\mathbb{X})$ is a mapping given by $\mathcal{F}^c(e)=\mathbb{X}-\mathbb{F}(e)$ for all $e\in\mathbb{E}$.

Definition 2.3 [3] Let $\mathbb{Y}$ be a non-empty subset of $\mathbb{X}$, then $\mathbb{Y}$ denotes the $soft$ −set $(\mathbb{Y},\mathbb{E})$ over $\mathbb{X}$ for which $\mathbb{Y}(e)=\mathbb{Y}$, for all $e\in\mathbb{E}$. In particular, $(\mathbb{X},\mathbb{E})$ will be denoted by $\mathcal{X}$.

Definition 2.4 [3] Let $x \in \mathrm{X}$. Then $(x, \mathbb{E})$ denotes the $soft$ −set over X for which $x(e) = \{x\}$, for all $e \in \mathbb{E}$.

Definition 2.5 [3] Let $(\mathrm{F}, \mathbb{E})$ be a $soft$ −set over X and $x \in \mathrm{X}$. We say that $x \in (\mathrm{F}, \mathbb{E})$ read as x belongs to the $soft$ −set $(\mathrm{F}, \mathbb{E})$ whenever $x \in \mathrm{F}(e)$ for all $e \in \mathbb{E}$. Note that for any $x \in \mathrm{X}$, $x \notin (\mathrm{F}, \mathbb{E})$, if $x \notin \mathrm{F}(e)$ for some $e \in \mathbb{E}$.

Definition 2.6 [3] The Union of two $soft$ −sets $(\mathrm{F}, \mathbb{E})$ and $(\mathrm{G}, \mathbb{E})$ over the common universe X is the $soft$ −set $(\mathrm{H}, \mathbb{E})$ where $\mathrm{H}(e) = \mathrm{F}(e) \cup \mathrm{G}(e)$ for all $e \in \mathbb{E}$. We write $(\mathrm{F}, \mathbb{E}) \cup (\mathrm{G}, \mathbb{E}) = (\mathrm{H}, \mathbb{E})$.

Definition 2.7 [3] The $soft$ −intersection of two $soft$ −sets $(\mathrm{F}, \mathbb{E})$ and $(\mathrm{G}, \mathbb{E})$ over the common universe X is the $soft$ −set $(\mathrm{H}, \mathbb{E})$ where $\mathrm{H}(e) = \mathrm{F}(e) \cap \mathrm{G}(e)$ for all $e \in \mathbb{E}$. We write $(\mathrm{F}, \mathbb{E}) \cap (\mathrm{G}, \mathbb{E}) = (\mathrm{H}, \mathbb{E})$.

Definition 2.8 [4] Let τ be the collection of $soft$ −sets over X, then τ is said to be a $soft$ −topology on X, if

(1) the $soft$ −sets Φ , $\mathcal{X}$ belongs to τ.

(2) the union of any number of $soft$ −sets in τ belongs to τ.

(3) the intersection of any two $soft$ −sets in τ belongs to τ .

Then, the triple $(\mathcal{X}, \tau, \mathbb{E})$ is called a $soft$ −topological space over X.

Definition 2.9 [4] Let $(\mathcal{X}, \tau, \mathbb{E})$ be a $soft$ −topological space over X, then the members of τ are said to be $soft - open$ sets in X.

Definition 2.10 [4] Let $(\mathcal{X}, \tau, \mathbb{E})$ be a $soft$ −topological space over X. A $soft - open$ set $(\mathrm{F}, \mathbb{E})$ over X is said to be a $soft$ −closed set in X, if its complement $(\mathrm{F}, \mathbb{E})'$ belongs to τ.

Definition 2.11 [7] Let $(\mathcal{X}, \tau_1, \mathbb{E})$, $(\mathcal{X}, \tau_2, \mathbb{E})$ and $(\mathcal{X}, \tau_3, \mathbb{E})$ be the three $soft$ −topological spaces on $\mathcal{X}$. Then $(\mathcal{X}, \tau_1, \tau_2, \tau_3, \mathbb{E})$ is called a $soft$ −tritopological space. The three $soft$ −topological spaces $(\mathcal{X}, \tau_1, \mathbb{E})$, $(\mathcal{X}, \tau_2, \mathbb{E})$ and $(\mathcal{X}, \tau_3, \mathbb{E})$ are independently satisfy the axioms of $soft$ −topological space.

Definition 2.12 [7] Let $(\mathcal{X}, \tau_1, \tau_2, \tau_3, \mathbb{E})$ be a $soft$ −tritopological space, then a $soft$ −set $(\mathrm{F}, \mathbb{E})$ in the universe $\mathcal{X}$ is called a $soft - \delta^* - open$ set iff:

$(\mathrm{F}, \mathbb{E}) \subseteq \mathrm{S}.\tau_1 - \mathrm{int}(\mathrm{S}.\tau_2 - \mathrm{cl}(\mathrm{S}.\tau_3 - \mathrm{int}(\mathrm{F}, \mathbb{E})))$. The complement of $soft - \delta^* - open$ set is called a $soft - \delta^* - closed$ set. The family of all $soft - \delta^* - open$ sets is denoted by $\mathrm{S}.\delta^*.\mathrm{O}(\mathcal{X})$. And the family of all $soft - \delta^* - closed$ sets is denoted by $\mathrm{S}.\delta^*.\mathrm{C}(\mathcal{X})$.

Definition 2.13 [15] A $soft$ −set $(\mathrm{F}, \mathbb{E})$ in a $soft$ −topological space $(\mathcal{X}, \tau, \mathbb{E})$, then:

(i) The $soft - \delta^*$ −closure of $(F, \mathbb{E})$, denoted by $S.\delta^* - cl(F, \mathbb{E})$ is defined by:

$S.\delta^* - cl(\mathrm{F}, \mathbb{E}) = \bigcap\{(\mathrm{G}, \mathbb{E}) : (\mathrm{F}, \mathbb{E}) \subseteq (\mathrm{G}, \mathbb{E}), \text{and} (\mathrm{G}, \mathbb{E})$ is $soft - \delta^* - closed\}$

(ii) The $soft - \delta^*$ −interior of $(\mathrm{F}, \mathbb{E})$, denoted by $S.\delta^* - int(\mathrm{F}, \mathbb{E})$ is defined by:

$S.\delta^* - int(\mathrm{F}, \mathbb{E}) = \bigcup\{(\mathrm{H}, \mathbb{E}) : (\mathrm{H}, \mathbb{E}) \subseteq (\mathrm{F}, \mathbb{E}), \text{and} (\mathrm{H}, \mathbb{E})$ is $soft - \delta^* - open\}$

Definition 2.14 [16] Let $(\mathcal{X}, \tau_1, \tau_2, \tau_3, \mathbb{E})$ be a $soft$ −tritopological space over X , $\mathrm{Y} \neq \emptyset$ and $\mathrm{Y} \subseteq \mathrm{X}$. Then the $soft$ −relative space ($soft$ −subspace) for $(\mathcal{X}, \tau_1, \tau_2, \tau_3, \mathbb{E})$ on Y (with respect to a $soft - \delta^* - open$ set) is the collection $S.\delta^*.\mathrm{O}(\mathcal{X})_{\mathrm{Y}}$, given by:

$S.\delta^*.\mathrm{O}(\mathcal{X})_{\mathrm{Y}} = \{(F_{\mathrm{Y}}, \mathbb{E}) = \mathcal{Y} \cap (F, \mathbb{E}) \mid (F, \mathbb{E}) \in S.\delta^*.\mathrm{O}(\mathcal{X})\}$, where $\mathcal{Y}$ is the $soft$ −set $(\mathrm{Y}, \mathbb{E})$ and $\mathrm{F}_{\mathrm{Y}}(e) = \mathrm{Y} \cap \mathrm{F}(e)$, for all $e \in \mathbb{E}$. The members of $S.\delta^*.\mathrm{O}(\mathcal{X})_{\mathrm{Y}}$ are said to be $soft - \delta^*_{\mathrm{Y}} - open$ sets in Y.

Note 2.15 Throughout this paper, $\mathcal{X}$ [for simply] denote the $soft$ −tritopological space over X, [and simply $\mathbb{Y}$] denote to the $soft$ −subspace of a $soft$ −tritopological space $\mathcal{X}$.

3. $Soft - \delta^* - \mathrm{T_i}$; ($\mathrm{i} = 0, 1, 2$) spaces

In this section the definition and some foundations of separation axioms in $soft$ −tritopological spaces ($soft - \delta^* - \mathrm{T}_0$, $soft - \delta^* - \mathrm{T}_1$, $soft - \delta^* - \mathrm{T}_2$) w.r.t. ordinary points are given.

Definition 3.1. Let $(\mathcal{X}, \tau_1, \tau_2, \tau_3, \mathbb{E})$ be a $soft$ −tritopological space over X , and $x_1, x_2 \in \mathrm{X}$ such that $x_1 \neq x_2$. If there exist $soft - \delta^* - open$ sets $(F_1, \mathbb{E})$ and $(F_2, \mathbb{E})$ such that " $x_1 \in (F_1, \mathbb{E})$, $x_2 \notin (F_1, \mathbb{E})$ " or " $x_1 \notin (F_2, \mathbb{E})$, $x_2 \in (F_2, \mathbb{E})$ ", then $(\mathcal{X}, \tau_1, \tau_2, \tau_3, \mathbb{E})$ is said to be a $soft - \delta^* - \mathrm{T}_0 -$ space.

Example 3.2. Let $\mathrm{X} = \{x_1, x_2\}$ be the universe set and $\mathbb{E} = \{e_1, e_2\}$ be the set of parameters, Then $(\mathrm{X}, \mathbb{E}) = \{(e_1, \{x_1, x_2\}), (e_2, \{x_1, x_2\})\}$. By [5], its cardinality is defined by $|\mathcal{P}(\mathcal{X})| = 2^{\sum_{e \in \mathbb{E}} |\mathcal{F}(e)|}$, where $|\mathcal{F}(e)|$ is the cardinality of $\mathcal{F}(e)$. (i.e. $|\mathcal{P}(\mathcal{X})| = 2^4 = 16$ $soft$ −set).

Let $(\mathcal{X}, \tau_1, \mathbb{E})$, $(\mathcal{X}, \tau_2, \mathbb{E})$ and $(\mathcal{X}, \tau_3, \mathbb{E})$ be the three $soft$ −topological spaces on X, where their topologies defined as follows:

$\tau_1 = \{\Phi, \mathcal{X}, (\mathcal{M}_1, \mathbb{E}), (\mathcal{M}_2, \mathbb{E})\}$,

$\tau_2 = \{\Phi, \mathcal{X}, (\mathrm{G}_1, \mathbb{E}), (\mathrm{G}_2, \mathbb{E})\}$ and

$\tau_3 = \{\Phi, \mathcal{X}, (\mathcal{H}_1, \mathbb{E})\}$. Where $(\mathcal{M}_1, \mathbb{E}), (\mathcal{M}_2, \mathbb{E}), (\mathrm{G}_1, \mathbb{E}), (\mathrm{G}_2, \mathbb{E}), (\mathcal{H}_1, \mathbb{E})$ are $soft$ −open sets defined as :

$\mathcal{M}_1(e_1) = \emptyset$, $\mathcal{M}_1(e_2) = \emptyset$

$\mathcal{M}_2(e_1) = \mathrm{X}$, $\mathcal{M}_2(e_2) = \mathrm{X}$

$\mathrm{G}_1(e_1) = \mathrm{X}$, $\mathrm{G}_1(e_2) = \{x_1\}$

$\mathrm{G}_2(e_1) = \emptyset$, $\mathrm{G}_2(e_2) = \{x_1\}$

$\mathcal{H}(e_1) = \mathrm{X}$, $\mathcal{H}(e_2) = \{x_1\}$

The complement of the $soft$ −open sets of τ_2 Are ;

${\mathrm{G}_1}^{\mathrm{c}}(e_1) = \emptyset$, ${\mathrm{G}_1}^{\mathrm{c}}(e_2) = \{x_2\}$

$G_2{}^c(e_1) = X$, $G_2{}^c(e_2) = \{x_2\}$

Hence the family of all $soft - \delta^* - open$ sets of the $soft$ –tritopological space $(\mathcal{X}, \tau_1, \tau_2, \tau_3, \mathbb{E})$ is,

$S.\delta^*.O(\mathcal{X}) = \{\mathcal{X}, \Phi, (\mathcal{F}, \mathbb{E})\}$.

$\mathcal{F}(e_1) = X$, $\mathcal{F}(e_2) = \{x_1\}$

Let x_1 , $x_2 \in X$ such that $x_1 \neq x_2$

$\exists (\mathcal{F}, \mathbb{E}) \in S.\delta^*.O(\mathcal{X})$ such that

$x_1 \in (\mathcal{F}, \mathbb{E})$, $x_2 \notin (\mathcal{F}, \mathbb{E})$

Thus $(\mathcal{X}, \tau_1, \tau_2, \tau_3, \mathbb{E})$ is a $soft - \delta^* - T_0 -$ space.

Proposition 3.3. Let $(\mathcal{X}, \tau_1, \tau_2, \tau_3, \mathbb{E})$ be a $soft -$ tritopological space over X and Y be a non-empty subset of X. If $\mathcal{X}$ is a $soft - \delta^* - T_0$ –space then the $soft$ –subspace $\mathbb{Y}$ is a $soft - \delta^* - T_0 -$ space.

Proof. Let $y_1 \neq y_2 \in Y$. There exists a two $soft - \delta^* -$ open sets $(\mathcal{A}, \mathbb{E}), (\mathcal{B}, \mathbb{E})$ in $\mathcal{X}$ (in $S.\delta^*.O(\mathcal{X})$), such that $y_1 \in (\mathcal{A}, \mathbb{E})$, $y_2 \notin (\mathcal{A}, \mathbb{E})$ or $y_1 \notin (\mathcal{B}, \mathbb{E})$, $y_2 \in (\mathcal{B}, \mathbb{E})$. Now, $y_1 \in Y$ implies that $y_1 \in (Y, \mathbb{E})$. So $y_1 \in (Y, \mathbb{E})$ and $y_1 \in (\mathcal{A}, \mathbb{E})$. Hence $y_1 \in (Y, \mathbb{E}) \cap (\mathcal{A}, \mathbb{E}) = (\mathcal{A}_Y, \mathbb{E})$ where $(\mathcal{A}, \mathbb{E}) \in S.\delta^*.O(\mathcal{X})$. Consider $y_2 \notin (\mathcal{A}, \mathbb{E})$, this means that $y_2 \notin \mathcal{A}(e)$ for some $e \in \mathbb{E}$. Then $y_2 \notin Y \cap \mathcal{A}(e) = Y(e) \cap \mathcal{A}(e)$. Therefore $y_2 \notin (Y, \mathbb{E}) \cap (\mathcal{A}, \mathbb{E}) = (\mathcal{A}_Y, \mathbb{E})$. Similarly it can be proved that If $y_2 \in (\mathcal{B}, \mathbb{E})$ and $y_1 \notin (\mathcal{B}, \mathbb{E})$ then $y_2 \in (\mathcal{B}_Y, \mathbb{E})$, $y_1 \notin (\mathcal{B}_Y, \mathbb{E})$.

Thus the $soft$ –subspace $\mathbb{Y}$ is a soft $- \delta^* - T_0$ –space.

Definition 3.4. Let $(\mathcal{X}, \tau_1, \tau_2, \tau_3, \mathbb{E})$ be a $soft$ –tritopological space over X, and $x_1, x_2 \in X$ such that $x_1 \neq x_2$. If there exist $soft - \delta^* - open$ sets $(F_1, \mathbb{E})$ and $(F_2, \mathbb{E})$ such that " $x_1 \in (F_1, \mathbb{E})$, $x_2 \notin (F_1, \mathbb{E})$ " and " $x_1 \notin (F_2, \mathbb{E})$, $x_2 \in (F_2, \mathbb{E})$ ", then $(\mathcal{X}, \tau_1, \tau_2, \tau_3, \mathbb{E})$ is said to be a $soft - \delta^* - T_1 -$ space.

Example 3.5. Let $X = \{x_1, x_2\}$ be the universe set and $\mathbb{E} = \{e_1, e_2\}$ be the set of parameters, Then $(X, \mathbb{E}) = \{(e_1, \{x_1, x_2\}), (e_2, \{x_1, x_2\})\}$. By [5], its cardinality is defined by $|\mathcal{P}(\mathcal{X})| = 2^{\sum_{e \in \mathbb{E}} |\mathcal{F}(e)|}$, where $|\mathcal{F}(e)|$ is the cardinality of $\mathcal{F}(e)$. (i.e. $|\mathcal{P}(\mathcal{X})| = 2^4 = 16$ $soft$ –set).

Let $(\mathcal{X}, \tau_1, \mathbb{E})$, $(\mathcal{X}, \tau_2, \mathbb{E})$ and $(\mathcal{X}, \tau_3, \mathbb{E})$ be the three $soft$ –topological spaces on X, where their $soft$ –topologies defined as follows:

$\tau_1 = \{\Phi, \mathcal{X}, (\mathcal{M}_1, \mathbb{E}), (\mathcal{M}_2, \mathbb{E})\}$,

$\tau_2 = \{\Phi, \mathcal{X}, (G_1, \mathbb{E}), (G_2, \mathbb{E}), (G_3, \mathbb{E}), (G_4, \mathbb{E})\}$ and

$\tau_3 = \{\Phi, \mathcal{X}, (\mathcal{H}_1, \mathbb{E}), (\mathcal{H}_2, \mathbb{E}), (\mathcal{H}_3, \mathbb{E}), (\mathcal{H}_4, \mathbb{E})\}$. Where $(\mathcal{M}_1, \mathbb{E}), (\mathcal{M}_2, \mathbb{E}), (G_1, \mathbb{E}), (G_2, \mathbb{E}), (G_3, \mathbb{E}), (G_4, \mathbb{E})$, $(\mathcal{H}_1, \mathbb{E})$, $(\mathcal{H}_2, \mathbb{E}), (\mathcal{H}_3, \mathbb{E})$ and $(\mathcal{H}_4, \mathbb{E})$ are $soft - open$ sets defined as:

$\mathcal{M}_1(e_1) = \{x_1\}$, $\mathcal{M}_1(e_2) = X$
$\mathcal{M}_2(e_1) = \{x_1\}$, $\mathcal{M}_2(e_2) = \{x_2\}$

$G_1(e_1) = X$, $G_1(e_2) = \emptyset$

$G_2(e_1) = \{x_2\}$, $G_2(e_2) = \{x_1\}$

$G_3(e_1) = \{x_2\}$, $G_3(e_2) = \emptyset$

$G_4(e_1) = X$, $G_4(e_2) = \{x_1\}$

$\mathcal{H}_1(e_1) = X$, $\mathcal{H}_1(e_2) = \{x_2\}$

$\mathcal{H}_2(e_1) = \{x_1\}$, $\mathcal{H}_2(e_2) = X$

$\mathcal{H}_3(e_1) = \{x_2\}$, $\mathcal{H}_3(e_2) = \emptyset$

$\mathcal{H}_4(e_1) = \{x_1\}$, $\mathcal{H}_4(e_2) = \{x_2\}$

The complement of the $soft - open$ sets of $soft$ –topology τ_2 Are ;

$G_1{}^c(e_1) = \emptyset$, $G_1{}^c(e_2) = X$

$G_2{}^c(e_1) = \{x_1\}$, $G_2{}^c(e_2) = \{x_2\}$

$G_3{}^c(e_1) = \{x_1\}$, $G_3{}^c(e_2) = X$

$G_4{}^c(e_1) = \emptyset$, $G_4{}^c(e_2) = \{x_2\}$

Hence the family of all $soft - \delta^* - open$ sets of the $soft$ –tritopological space $(\mathcal{X}, \tau_1, \tau_2, \tau_3, \mathbb{E})$ is, $S.\delta^*.O(\mathcal{X}) = \{\mathcal{X}, \Phi, (\mathcal{F}_1, \mathbb{E}), (\mathcal{F}_2, \mathbb{E}), (\mathcal{F}_3, \mathbb{E})\}$.

$\mathcal{F}_1(e_1) = \{x_1\}$, $\mathcal{F}_1(e_2) = X$

$\mathcal{F}_2(e_1) = X$, $\mathcal{F}_2(e_2) = \{x_2\}$

$\mathcal{F}_3(e_1) = \{x_1\}$, $\mathcal{F}_3(e_2) = \{x_2\}$

Let x_1 , $x_2 \in X$, such that $x_1 \neq x_2$

$\exists (\mathcal{F}_1, \mathbb{E}), (\mathcal{F}_2, \mathbb{E}) \in S.\delta^*.O(\mathcal{X})$ such that

$x_1 \in (\mathcal{F}_1, \mathbb{E})$, $x_2 \notin (\mathcal{F}_1, \mathbb{E})$ and

$x_2 \in (\mathcal{F}_2, \mathbb{E})$, $x_1 \notin (\mathcal{F}_2, \mathbb{E})$

Thus $(\mathcal{X}, \tau_1, \tau_2, \tau_3, \mathbb{E})$ is a $soft - \delta^* - T_1$ –space.

Remark 3.6. If $(\mathcal{X}, \tau_1, \tau_2, \tau_3, \mathbb{E})$ is a soft $- \delta^* - T_1$ –space then (X , $\delta^*.\tau_e$) may not be a T_1 –space for every parameter $e \in \mathbb{E}$. Where $\delta^*.\tau_e = \{F(e) | (F, \mathbb{E}) \in S.\delta^*.O(\mathcal{X})\}$ [15].

Example 3.7. Clearly in example 3.5 above, Neither

(X , $\delta^*.\tau_{e_1}$) nor (X , $\delta^*.\tau_{e_2}$) is a T_1 –space. Where $\delta^*.\tau_{e_1} = \{ X, \emptyset, \{x_1\} \}$ and $\delta^*.\tau_{e_2} = \{ X, \emptyset, \{x_2\} \}$

Proposition 3.8. Every $soft - \delta^* - T_1$ –space is a $soft - \delta^* - T_0$ –space.

Proof. Let $(\mathcal{X}, \tau_1, \tau_2, \tau_3, \mathbb{E})$ be a $soft$ –tritopological space over X and $x_1, x_2 \in X$ such that $x_1 \neq x_2$. If $(\mathcal{X}, \tau_1, \tau_2, \tau_3, \mathbb{E})$ is a $soft - \delta^* - T_1$ –space then there exist $soft - \delta^* - open$ sets $(\mathcal{A}, \mathbb{E})$ and $(\mathcal{B}, \mathbb{E})$ such that $x_1 \in (\mathcal{A}, \mathbb{E})$, $x_2 \notin (\mathcal{A}, \mathbb{E})$ and $x_1 \notin (\mathcal{B}, \mathbb{E})$, $x_2 \in (\mathcal{B}, \mathbb{E})$. Obviously, then we have $x_1 \in (\mathcal{A}, \mathbb{E}), x_2 \notin (\mathcal{A}, \mathbb{E})$ or $x_1 \notin (\mathcal{B}, \mathbb{E})$, $x_2 \in (\mathcal{B}, \mathbb{E})$

Thus $(\mathcal{X}, \tau_1, \tau_2, \tau_3, \mathbb{E})$ is a $soft - \delta^* - T_0$ –space.

Proposition 3.9. Let $(\mathcal{X}, \tau_1, \tau_2, \tau_3, \mathbb{E})$ be a $soft -$ tritopological space over X and Y be a non-empty subset of X. If $\mathcal{X}$ is a $soft - \delta^* - T_1$ –space then the $soft -$ subspace $\mathbb{Y}$ is a $soft - \delta^* - T_1$ –space.

Proof. Let $y_1 \neq y_2 \in Y$. There exists a two $soft - \delta^* - open$

sets $(\mathcal{A},\mathbb{E}),(\mathcal{B},\mathbb{E})$ in $\mathcal{X}$ (in $S.\delta^*.O(\mathcal{X})$), such that $y_1 \in (\mathcal{A},\mathbb{E})$, $y_2 \notin (\mathcal{A},\mathbb{E})$ and $y_1 \notin (\mathcal{B},\mathbb{E})$, $y_2 \in (\mathcal{B},\mathbb{E})$. Now $y_1 \in Y$ implies that $y_1 \in (Y,\mathbb{E})$. So $y_1 \in (Y,\mathbb{E})$ and $y_1 \in (\mathcal{A},\mathbb{E})$.

Hence $y_1 \in (Y,\mathbb{E}) \cap (\mathcal{A},\mathbb{E})= (\mathcal{A}_Y,\mathbb{E})$ where $(\mathcal{A},\mathbb{E}) \in S.\delta^*.O(\mathcal{X})$. Consider $y_2 \notin (\mathcal{A},\mathbb{E})$, this means that $y_2 \notin \mathcal{A}(e)$ for some $e \in \mathbb{E}$. Then $y_2 \notin Y \cap \mathcal{A}(e) = Y(e) \cap \mathcal{A}(e)$. Therefore $y_2 \notin (Y,\mathbb{E}) \cap (\mathcal{A},\mathbb{E}) = (\mathcal{A}_Y,\mathbb{E})$. Similarly it can be proved that If $y_2 \in (\mathcal{B},\mathbb{E})$ and $y_1 \notin (\mathcal{B},\mathbb{E})$ then $y_2 \in (\mathcal{B}_Y,\mathbb{E})$ and $y_1 \notin (\mathcal{B}_Y,\mathbb{E})$. Thus $\mathbb{Y}$ is a $soft - \delta^* - T_1$ –space.

Definition 3.10. Let $(\mathcal{X},\tau_1,\tau_2,\tau_3,\mathbb{E})$ be a $soft-$tritopological space over X , and $x_1,x_2 \in X$ such that $x_1 \neq x_2$. If there exist $soft-\delta^*-open$ sets $(F_1,\mathbb{E})$ and $(F_2,\mathbb{E})$ such that " $x_1 \in (F_1,\mathbb{E})$, $x_2 \in (F_2,\mathbb{E})$ and $(F_1,\mathbb{E}) \cap (F_2,\mathbb{E}) = \Phi$ ", then $(\mathcal{X},\tau_1,\tau_2,\tau_3,\mathbb{E})$ is said to be a $soft-\delta^*-T_2-$ space.

Example 3.11. Let $X = \{x_1,x_2\}$ be the universe set and $\mathbb{E} = \{e_1,e_2\}$ be the set of parameters, Then $(X,\mathbb{E}) = \{(e_1,\{x_1,x_2\}),(e_2,\{x_1,x_2\})\}$. By [5], its cardinality is defined by $|\mathcal{P}(\mathcal{X})| = 2^{\sum_{e\in\mathbb{E}}|\mathcal{F}(e)|}$, where $|\mathcal{F}(e)|$is the cardinality of $\mathcal{F}(e)$. (i.e. $|\mathcal{P}(\mathcal{X})| = 2^4 = 16$ $soft-$set).

Let $(\mathcal{X},\tau_1,\mathbb{E})$, $(\mathcal{X},\tau_2,\mathbb{E})$ and $(\mathcal{X},\tau_3,\mathbb{E})$ be the three $soft-$topological spaces on $\mathcal{X}$, and their $soft-$topologies defined as follows:

$\tau_1 = \{\Phi,\mathcal{X},(\mathcal{M}_1,\mathbb{E}),(\mathcal{M}_2,\mathbb{E}),(\mathcal{M}_3,\mathbb{E}),(\mathcal{M}_4,\mathbb{E})\}$,

$\tau_2 = \{\Phi,\mathcal{X},(G_1,\mathbb{E}),\dots,(G_7,\mathbb{E})\}$ and

$\tau_3 = \{\Phi,\mathcal{X},(\mathcal{H}_1,\mathbb{E}),\dots,(\mathcal{H}_7,\mathbb{E})\}$. Where the $soft-open$ sets over X of three $soft-$topologies, defined as follows:

$\mathcal{M}_1(e_1) = \{x_1\}$, $\mathcal{M}_1(e_2) = \{x_1\}$

$\mathcal{M}_2(e_1) = \{x_2\}$, $\mathcal{M}_2(e_2) = \{x_2\}$

$\mathcal{M}_3(e_1) = X$, $\mathcal{M}_3(e_2) = \{x_1\}$

$\mathcal{M}_4(e_1) = \{x_2\}$, $\mathcal{M}_4(e_2) = \emptyset$

$G_1(e_1) = X$, $G_1(e_2) = \emptyset$

$G_2(e_1) = \{x_2\}$, $G_2(e_2) = \{x_2\}$

$G_3(e_1) = \{x_1\}$, $G_3(e_2) = \{x_1\}$

$G_4(e_1) = \{x_2\}$, $G_4(e_2) = \emptyset$

$G_5(e_1) = \{x_1\}$, $G_5(e_2) = \emptyset$

$G_6(e_1) = X$, $G_6(e_2) = \{x_1\}$

$G_7(e_1) = X$, $G_7(e_2) = \{x_2\}$

$\mathcal{H}_1(e_1) = \{x_1\}$, $\mathcal{H}_1(e_2) = \{x_1\}$

$\mathcal{H}_2(e_1) = \{x_2\}$, $\mathcal{H}_2(e_2) = \{x_2\}$

$\mathcal{H}_3(e_1) = X$, $\mathcal{H}_3(e_2) = \emptyset$

$\mathcal{H}_4(e_1) = \{x_2\}$, $\mathcal{H}_4(e_2) = \emptyset$

$\mathcal{H}_5(e_1) = \{x_1\}$, $\mathcal{H}_5(e_2) = \emptyset$

$\mathcal{H}_6(e_1) = X$, $\mathcal{H}_6(e_2) = \{x_1\}$

$\mathcal{H}_7(e_1) = X$, $\mathcal{H}_7(e_2) = \{x_2\}$

The complement of the $soft-open$ sets in τ_2 Are ;

$G_1{}^c(e_1) = \emptyset$, $G_1{}^c(e_2) = X$

$G_2{}^c(e_1) = \{x_1\}$, $G_2{}^c(e_2) = \{x_1\}$

$G_3{}^c(e_1) = \{x_2\}$, $G_3{}^c(e_2) = \{x_2\}$

$G_4{}^c(e_1) = \{x_1\}$, $G_4{}^c(e_2) = X$

$G_5{}^c(e_1) = \{x_2\}$, $G_5{}^c(e_2) = X$

$G_6{}^c(e_1) = \emptyset$, $G_6{}^c(e_2) = \{x_2\}$

$G_7{}^c(e_1) = \emptyset$, $G_7{}^c(e_2) = \{x_1\}$

Hence the family of all $soft-\delta^*-open$ sets of the $soft-$tritopological space $(\mathcal{X},\tau_1,\tau_2,\tau_3,\mathbb{E})$ is, $S.\delta^*.O(\mathcal{X}) = \{\mathcal{X},\Phi,(\mathcal{F}_i,\mathbb{E})\}$(with $i = 1,2,\dots,7$) defined as

$\mathcal{F}_1(e_1) = \{x_1\}$, $\mathcal{F}_1(e_2) = \{x_1\}$

$\mathcal{F}_2(e_1) = \{x_1\}$, $\mathcal{F}_2(e_2) = \emptyset$

$\mathcal{F}_3(e_1) = \{x_2\}$, $\mathcal{F}_3(e_2) = \{x_2\}$

$\mathcal{F}_4(e_1) = \{x_2\}$, $\mathcal{F}_4(e_2) = \emptyset$

$\mathcal{F}_5(e_1) = X$, $\mathcal{F}_5(e_2) = \{x_1\}$

$\mathcal{F}_6(e_1) = X$, $\mathcal{F}_6(e_2) = \{x_2\}$

$\mathcal{F}_7(e_1) = X$, $\mathcal{F}_7(e_2) = \emptyset$

If we take x_1 , $x_2 \in X$ and $x_1 \neq x_2$

$\exists$ $(\mathcal{F}_1,\mathbb{E}),(\mathcal{F}_3,\mathbb{E}) \in S.\delta^*.O(\mathcal{X})$ such that

$x_1 \in (\mathcal{F}_1,\mathbb{E})$ and $x_2 \in (\mathcal{F}_3,\mathbb{E})$,

$\mathcal{F}_1(e_1) \cap \mathcal{F}_3(e_1) = \emptyset$

$\mathcal{F}_1(e_2) \cap \mathcal{F}_3(e_2) = \emptyset$

Then $(\mathcal{F}_1,\mathbb{E}) \cap (\mathcal{F}_3,\mathbb{E}) = \Phi$

Thus $(\mathcal{X},\tau_1,\tau_2,\tau_3,\mathbb{E})$ is a $soft-\delta^*-T_2-$space.

Proposition 3.12. Let the $soft-$tritopological space $(\mathcal{X},\tau_1,\tau_2,\tau_3,\mathbb{E})$ is a $soft-\delta^*-T_2-$space over X , then

(X , $\delta^*.\tau_e$) is a T_2-space for every parameter $e \in \mathbb{E}$.

Proof. Suppose that $(\mathcal{X},\tau_1,\tau_2,\tau_3,\mathbb{E})$ is a $soft-\delta^*-T_2-$space over X. For any $e \in \mathbb{E}$, and $\delta^*.\tau_e = \{F(e)|(F,\mathbb{E}) \in S.\delta^*.O(\mathcal{X})\}$ [15]. Let $x_1,x_2 \in X$, such that $x_1 \neq x_2$, there is a $soft-\delta^*-open$ sets $(F_1,\mathbb{E})$ and $(F_2,\mathbb{E})$ such that $x_1 \in (F_1,\mathbb{E})$, $x_2 \in (F_2,\mathbb{E})$ and $(F_1,\mathbb{E}) \cap (F_2,\mathbb{E}) = \Phi$. This implies that $x_1 \in F_1(e)$, $x_2 \in F_2(e)$ and $F_1(e) \cap F_2(e) = \emptyset$

Thus (X , $\delta^*.\tau_e$) is a T_2-space, for every parameter $e \in \mathbb{E}$ **.**

Proposition 3.13. Every $soft-\delta^*-T_2-$space is a $soft-\delta^*-T_1-$space.

Proof. Let $(\mathcal{X},\tau_1,\tau_2,\tau_3,\mathbb{E})$ be a $soft-$tritopological space over X and $x_1,x_2 \in X$such that $x_1 \neq x_2$. If $(\mathcal{X},\tau_1,\tau_2,\tau_3,\mathbb{E})$ is a $soft-\delta^*-T_2$ −space then there exist two $soft-\delta^*-open$ sets $(\mathcal{A},\mathbb{E})$ and $(\mathcal{B},\mathbb{E})$ such that $x_1 \in (\mathcal{A},\mathbb{E})$, $x_2 \in (\mathcal{B},\mathbb{E})$ and $(\mathcal{A},\mathbb{E}) \cap (\mathcal{B},\mathbb{E}) = \Phi$. Obviously, since $(\mathcal{A},\mathbb{E}) \cap (\mathcal{B},\mathbb{E}) = \Phi$, then we have $x_1 \in (\mathcal{A},\mathbb{E}), x_2 \notin (\mathcal{A},\mathbb{E})$ and $x_1 \notin (\mathcal{B},\mathbb{E})$, $x_2 \in (\mathcal{B},\mathbb{E})$. Thus $(\mathcal{X},\tau_1,\tau_2,\tau_3,\mathbb{E})$ is a $soft-\delta^*-T_1$ −space.

Proposition 3.14. Let $(\mathcal{X},\tau_1,\tau_2,\tau_3,\mathbb{E})$ be a $soft-$tritopological space over X and Y be a non-empty subset of X. If $\mathcal{X}$ is a $soft-\delta^*-T_2$ −space then the $soft$ −subspace $\mathbb{Y}$ is a $soft-\delta^*-T_2$ −space.

Proof. Let $y_1 \neq y_2 \in Y$. There exists a two $soft-\delta^*-open$ sets $(\mathcal{A},\mathbb{E}),(\mathcal{B},\mathbb{E})$ in $\mathcal{X}$ (in $S.\delta^*.O(\mathcal{X})$), such that $y_1 \in (\mathcal{A},\mathbb{E})$, $y_2 \in (\mathcal{B},\mathbb{E})$ and $(\mathcal{A},\mathbb{E}) \cap (\mathcal{B},\mathbb{E}) = \Phi$. So for each $e \in \mathbb{E}$, $y_1 \in \mathcal{A}(e)$, $y_2 \in \mathcal{B}(e)$ and $\mathcal{A}(e) \cap \mathcal{B}(e) = \emptyset$, this implies that $y_1 \in Y \cap \mathcal{A}(e)$, $y_2 \in Y \cap \mathcal{B}(e)$ and $\mathcal{A}(e) \cap \mathcal{B}(e) = \emptyset$. Hence $y_1 \in (\mathcal{A}_Y,\mathbb{E})$, $y_2 \in (\mathcal{B}_Y,\mathbb{E})$ and $(\mathcal{A}_Y,\mathbb{E}) \cap (\mathcal{B}_Y,\mathbb{E}) = \Phi$,

where $(\mathcal{A}_Y,\mathbb{E}),(\mathcal{B}_Y,\mathbb{E}) \in S.\delta^*.O(\mathcal{X})_Y$.

Thus $\mathbb{Y}$ is a $soft-\delta^*-T_2$ −space.

4. $soft-\delta^*$ −regular, $soft-\delta^*$ −normal and $soft-\delta^*-T_i$; (i = 3, 4) spaces

In this section, we define $soft-\delta^*-T_3$ and $soft-\delta^*-T_4$ spaces using ordinary points and characterize $soft-\delta^*-$regular and $soft-\delta^*$ −normal spaces.

Definition 4.1. Let $(\mathcal{X},\tau_1,\tau_2,\tau_3,\mathbb{E})$ be a $soft$ −tritopological space over X , $(H,\mathbb{E})$ be a $soft-\delta^*-closed$ set in $\mathcal{X}$, and $x \notin (H,\mathbb{E})$, if there exist a $soft-\delta^*-open$ sets $(F_1,\mathbb{E})$ and $(F_2,\mathbb{E})$ such that " $x \in (F_1,\mathbb{E})$, $(H,\mathbb{E}) \subseteq (F_2,\mathbb{E})$ and $(F_1,\mathbb{E}) \cap (F_2,\mathbb{E}) = \Phi$ ". Then $(\mathcal{X},\tau_1,\tau_2,\tau_3,\mathbb{E})$ is said to be $soft-\delta^*$ −regular space.

Definition 4.2. A $soft$ −tritopological space $(\mathcal{X},\tau_1,\tau_2,\tau_3,\mathbb{E})$ over X is said to be a $soft-\delta^*-T_3$ −space iff it is a $soft-\delta^*$ −regular and $soft-\delta^*-T_1$ −space.

Example 4.3. Let $X = \{x_1,x_2\}$ be the universe set, and $\mathbb{E} = \{e_1,e_2\}$ be the set of parameters, Then $(X,\mathbb{E}) = \{(e_1,\{x_1,x_2\}),(e_2,\{x_1,x_2\})\}$. By [5], its cardinality is defined by $|\mathcal{P}(\mathcal{X})| = 2^{\sum_{e\in\mathbb{E}}|\mathcal{F}(e)|}$, where $|\mathcal{F}(e)|$ is the cardinality of $\mathcal{F}(e)$. (i.e. $|\mathcal{P}(\mathcal{X})| = 2^4 = 16$ $soft$ −sets).

Let $(\mathcal{X},\tau_1,\mathbb{E})$, $(\mathcal{X},\tau_2,\mathbb{E})$ and $(\mathcal{X},\tau_3,\mathbb{E})$ be the three $soft$ −topological spaces on $\mathcal{X}$, where their $soft$ −topologies defined as follows:

$\tau_1 = \{\Phi,\mathcal{X},(\mathcal{M}_1,\mathbb{E}),(\mathcal{M}_2,\mathbb{E}),(\mathcal{M}_3,\mathbb{E})\}$,

$\tau_2 = \{\Phi,\mathcal{X},(G_1,\mathbb{E}),(G_2,\mathbb{E}),(G_3,\mathbb{E}),(G_4,\mathbb{E})\}$ and $\tau_3 = \{\Phi,\mathcal{X},(\mathcal{H}_1,\mathbb{E}),\dots,(\mathcal{H}_{10},\mathbb{E})\}$. Where the $soft-open$ sets for the three $soft$ −topologies , defined as follows:

$\mathcal{M}_1(e_1) = \{x_2\}$, $\mathcal{M}_1(e_2) = X$
$\mathcal{M}_2(e_1) = \{x_1\}$, $\mathcal{M}_2(e_2) = \{x_2\}$
$\mathcal{M}_3(e_1) = \emptyset$, $\mathcal{M}_3(e_2) = \{x_2\}$
$G_1(e_1) = X$, $G_1(e_2) = \emptyset$
$G_2(e_1) = \{x_2\}$, $G_2(e_2) = \{x_1\}$
$G_3(e_1) = \{x_2\}$, $G_3(e_2) = \emptyset$
$G_4(e_1) = X$, $G_1(e_2) = \{x_1\}$
$\mathcal{H}_1(e_1) = \{x_1\}$, $\mathcal{H}_1(e_2) = \{x_2\}$
$\mathcal{H}_2(e_1) = X$, $\mathcal{H}_2(e_2) = \{x_1\}$
$\mathcal{H}_3(e_1) = \{x_1\}$, $\mathcal{H}_3(e_2) = \emptyset$
$\mathcal{H}_4(e_1) = \{x_2\}$, $\mathcal{H}_4(e_2) = X$
$\mathcal{H}_5(e_1) = \{x_2\}$, $\mathcal{H}_5(e_2) = \{x_1\}$
$\mathcal{H}_6(e_1) = \emptyset$, $\mathcal{H}_6(e_2) = \{x_2\}$
$\mathcal{H}_7(e_1) = \emptyset$, $\mathcal{H}_7(e_2) = X$
$\mathcal{H}_8(e_1) = \emptyset$, $\mathcal{H}_8(e_2) = \{x_1\}$
$\mathcal{H}_9(e_1) = \{x_1\}$, $\mathcal{H}_9(e_2) = X$
$\mathcal{H}_{10}(e_1) = \{x_1\}$, $\mathcal{H}_{10}(e_2) = \{x_1\}$

The complement of the $soft-open$ sets of $soft$ −topology τ_2 Are ;

${G_1}^c(e_1) = \emptyset$, ${G_1}^c(e_2) = X$
${G_2}^c(e_1) = \{x_1\}$, ${G_2}^c(e_2) = \{x_2\}$
${G_3}^c(e_1) = \{x_1\}$, ${G_3}^c(e_2) = X$
${G_4}^c(e_1) = \emptyset$, ${G_4}^c(e_2) = \{x_2\}$

Hence the family of all $soft-\delta^*-open$ sets of the $soft$ −tritopological space $(\mathcal{X},\tau_1,\tau_2,\tau_3,\mathbb{E})$ is, $S.\delta^*.O(\mathcal{X}) = \{\mathcal{X},\Phi,(\mathcal{F}_i,\mathbb{E})\}$ (with i = 1,2, … ,6) as;

$\mathcal{F}_1(e_1) = \{x_1\}$, $\mathcal{F}_1(e_2) = \{x_2\}$
$\mathcal{F}_2(e_1) = \{x_1\}$, $\mathcal{F}_2(e_2) = \emptyset$
$\mathcal{F}_3(e_1) = \{x_2\}$, $\mathcal{F}_3(e_2) = \{x_1\}$
$\mathcal{F}_4(e_1) = \{x_2\}$, $\mathcal{F}_4(e_2) = X$
$\mathcal{F}_5(e_1) = \emptyset$, $\mathcal{F}_5(e_2) = \{x_2\}$
$\mathcal{F}_6(e_1) = X$, $\mathcal{F}_6(e_2) = \{x_1\}$

Hence the family of all $soft-\delta^*-closed$ sets of the $soft$ −tritopological space $(\mathcal{X},\tau_1,\tau_2,\tau_3,\mathbb{E})$ is, $S.\delta^*.C(\mathcal{X}) = \{\mathcal{X},\Phi,(\mathcal{F}_i^c,\mathbb{E})\}$ (with i = 1,2, … ,6) as;

$\mathcal{F}_1^c(e_1) = \{x_2\}$, $\mathcal{F}_1^c(e_2) = \{x_1\}$
$\mathcal{F}_2^c(e_1) = \{x_2\}$, $\mathcal{F}_2^c(e_2) = X$
$\mathcal{F}_3^c(e_1) = \{x_1\}$, $\mathcal{F}_3^c(e_2) = \{x_2\}$
$\mathcal{F}_4^c(e_1) = \{x_1\}$, $\mathcal{F}_4^c(e_2) = \emptyset$
$\mathcal{F}_5^c(e_1) = X$, $\mathcal{F}_5^c(e_2) = \{x_1\}$
$\mathcal{F}_6^c(e_1) = \emptyset$, $\mathcal{F}_6^c(e_2) = \{x_2\}$

Let $x_1, x_2 \in X$ such that $x_1 \neq x_2$

$\exists(\mathcal{F}_6^c, \mathbb{E}) \in S.\delta^*.C(\mathcal{X})$

$x_1 \notin (\mathcal{F}_6^c, \mathbb{E})$

$\exists(\mathcal{F}_6, \mathbb{E}), (\mathcal{F}_5, \mathbb{E}) \in S.\delta^*.O(\mathcal{X})$ such that

$x_1 \in (\mathcal{F}_6, \mathbb{E}), \quad (\mathcal{F}_6^c, \mathbb{E}) \subseteq (\mathcal{F}_5, \mathbb{E})$

$(\mathcal{F}_6, \mathbb{E}) \cap (\mathcal{F}_5, \mathbb{E}) = \Phi$, and so on …

Thus $(\mathcal{X}, \tau_1, \tau_2, \tau_3, \mathbb{E})$ is a $soft - \delta^*$ −regular space.

Let $x_1, x_2 \in X$ such that $x_1 \neq x_2$

$\exists(\mathcal{F}_4, \mathbb{E}), (\mathcal{F}_6, \mathbb{E}) \in S.\delta^*.O(\mathcal{X})$ such that

$x_1 \in (\mathcal{F}_6, \mathbb{E})$, $x_2 \notin (\mathcal{F}_6, \mathbb{E})$ and

$x_2 \in (\mathcal{F}_4, \mathbb{E})$, $x_1 \notin (\mathcal{F}_4, \mathbb{E})$

Thus $(\mathcal{X}, \tau_1, \tau_2, \tau_3, \mathbb{E})$ is a $soft - \delta^* - T_1$ −space .

Therefore $(\mathcal{X}, \tau_1, \tau_2, \tau_3, \mathbb{E})$ is a $soft - \delta^* - T_3$ −space.

Remark 4.4. **(1)** A $soft - \delta^* - T_3$ −space may not be a $soft - \delta^* - T_2$ −space.

(2) If $(\mathcal{X}, \tau_1, \tau_2, \tau_3, \mathbb{E})$ is a $soft - \delta^* - T_3$ −space then $(X, \delta^*.\tau_e)$ may not be a T_3 −space for every parameter $e \in \mathbb{E}$.

Example 4.5. Let $X = \{x_1, x_2, x_3\}$ be the universe set and $\mathbb{E} = \{e_1, e_2\}$ be the set of parameters, Then (the cardinality $|\mathcal{P}(\mathcal{X})| = 2^6 = 64$ $soft$ −set).

And $(\mathcal{X}, \tau_1, \mathbb{E})$, $(\mathcal{X}, \tau_2, \mathbb{E})$ and $(\mathcal{X}, \tau_3, \mathbb{E})$ be three $soft$ −topological spaces on $\mathcal{X}$, and their $soft$ −topologies defined as follows:

$\tau_1 = \{\Phi, \mathcal{X}, (\mathcal{M}_1, \mathbb{E}), (\mathcal{M}_2, \mathbb{E}), (\mathcal{M}_3, \mathbb{E})\}$,

$\tau_2 = \{\Phi, \mathcal{X}, (G_1, \mathbb{E}), (G_2, \mathbb{E}), (G_3, \mathbb{E})\}$ and

$\tau_3 = \{\Phi, \mathcal{X}, (\mathcal{H}_1, \mathbb{E}), (\mathcal{H}_2, \mathbb{E}), \dots, (\mathcal{H}_{30}, \mathbb{E})\}$. Where $(\mathcal{M}_1, \mathbb{E}), (\mathcal{M}_2, \mathbb{E}), (\mathcal{M}_3, \mathbb{E}), (G_1, \mathbb{E}), (G_2, \mathbb{E}), (G_3, \mathbb{E}), (\mathcal{H}_1, \mathbb{E}), (\mathcal{H}_2, \mathbb{E}), \dots, (\mathcal{H}_{30}, \mathbb{E})$ are $soft - open$ sets in $\mathcal{X}$, defined as;

$\mathcal{M}_1(e_1) = X$, $\mathcal{M}_1(e_2) = \{x_1\}$

$\mathcal{M}_2(e_1) = X$, $\mathcal{M}_2(e_2) = \{x_2, x_3\}$

$\mathcal{M}_3(e_1) = X$, $\mathcal{M}_3(e_2) = \emptyset$

$G_1(e_1) = \emptyset$, $G_1(e_2) = X$

$G_2(e_1) = \emptyset$, $G_2(e_2) = \{x_2, x_3\}$

$G_3(e_1) = \emptyset$, $G_3(e_2) = \{x_1\}$

$\mathcal{H}_1(e_1) = X$, $\mathcal{H}_1(e_2) = \emptyset$

$\mathcal{H}_2(e_1) = \{x_1\}$, $\mathcal{H}_2(e_2) = \emptyset$

$\mathcal{H}_3(e_1) = \{x_2\}$, $\mathcal{H}_3(e_2) = \emptyset$

$\mathcal{H}_4(e_1) = \{x_3\}$, $\mathcal{H}_4(e_2) = \emptyset$

$\mathcal{H}_5(e_1) = \{x_1, x_2\}$, $\mathcal{H}_5(e_2) = \emptyset$

$\mathcal{H}_6(e_1) = \{x_1, x_3\}$, $\mathcal{H}_6(e_2) = \emptyset$

$\mathcal{H}_7(e_1) = \{x_2, x_3\}$, $\mathcal{H}_7(e_2) = \emptyset$

$\mathcal{H}_8(e_1) = X$, $\mathcal{H}_8(e_2) = \{x_1\}$

$\mathcal{H}_9(e_1) = \{x_1\}$, $\mathcal{H}_9(e_2) = \{x_1\}$

$\mathcal{H}_{10}(e_1) = \{x_2\}$, $\mathcal{H}_{10}(e_2) = \{x_1\}$

$\mathcal{H}_{11}(e_1) = \{x_3\}$, $\mathcal{H}_{11}(e_2) = \{x_1\}$

$\mathcal{H}_{12}(e_1) = \{x_1, x_2\}$, $\mathcal{H}_{12}(e_2) = \{x_1\}$

$\mathcal{H}_{13}(e_1) = \{x_1, x_3\}$, $\mathcal{H}_{13}(e_2) = \{x_1\}$

$\mathcal{H}_{14}(e_1) = \{x_2, x_3\}$, $\mathcal{H}_{14}(e_2) = \{x_1\}$

$\mathcal{H}_{15}(e_1) = \emptyset$, $\mathcal{H}_{15}(e_2) = \{x_1\}$

$\mathcal{H}_{16}(e_1) = X$, $\mathcal{H}_{16}(e_2) = \{x_2, x_3\}$

$\mathcal{H}_{17}(e_1) = \{x_1\}$, $\mathcal{H}_{17}(e_2) = \{x_2, x_3\}$

$\mathcal{H}_{18}(e_1) = \{x_2\}$, $\mathcal{H}_{18}(e_2) = \{x_2, x_3\}$

$\mathcal{H}_{19}(e_1) = \{x_3\}$, $\mathcal{H}_{19}(e_2) = \{x_2, x_3\}$

$\mathcal{H}_{20}(e_1) = \{x_1, x_2\}$, $\mathcal{H}_{20}(e_2) = \{x_2, x_3\}$

$\mathcal{H}_{21}(e_1) = \{x_1, x_3\}$, $\mathcal{H}_{21}(e_2) = \{x_2, x_3\}$

$\mathcal{H}_{22}(e_1) = \{x_2, x_3\}$, $\mathcal{H}_{22}(e_2) = \{x_2, x_3\}$

$\mathcal{H}_{23}(e_1) = \emptyset$, $\mathcal{H}_{23}(e_2) = \{x_2, x_3\}$

$\mathcal{H}_{24}(e_1) = \{x_1\}$, $\mathcal{H}_{24}(e_2) = X$

$\mathcal{H}_{25}(e_1) = \{x_2\}$, $\mathcal{H}_{25}(e_2) = X$

$\mathcal{H}_{26}(e_1) = \{x_3\}$, $\mathcal{H}_{26}(e_2) = X$

$\mathcal{H}_{27}(e_1) = \{x_1, x_2\}$, $\mathcal{H}_{27}(e_2) = X$

$\mathcal{H}_{28}(e_1) = \{x_1, x_3\}$, $\mathcal{H}_{28}(e_2) = X$

$\mathcal{H}_{29}(e_1) = \{x_2, x_3\}$, $\mathcal{H}_{29}(e_2) = X$

$\mathcal{H}_{30}(e_1) = \emptyset$, $\mathcal{H}_{30}(e_2) = X$

Hence the family of all $soft - \delta^* - open$ sets of the $soft$ −tritopological space $(\mathcal{X}, \tau_1, \tau_2, \tau_3, \mathbb{E})$ is, $S.\delta^*.O(\mathcal{X}) = \{\mathcal{X}, \Phi, (\mathcal{F}_i, \mathbb{E})\}$ (with $i = 1, 2, \dots, 30$) defined as;

$\mathcal{F}_1(e_1) = X$, $\mathcal{F}_1(e_2) = \emptyset$

$\mathcal{F}_2(e_1) = \{x_1\}$, $\mathcal{F}_2(e_2) = \emptyset$

$\mathcal{F}_3(e_1) = \{x_2\}$, $\mathcal{F}_3(e_2) = \emptyset$

$\mathcal{F}_4(e_1) = \{x_3\}$, $\mathcal{F}_4(e_2) = \emptyset$

$\mathcal{F}_5(e_1) = \{x_1, x_2\}$, $\mathcal{F}_5(e_2) = \emptyset$

$\mathcal{F}_6(e_1) = \{x_1, x_3\}$, $\mathcal{F}_6(e_2) = \emptyset$

$\mathcal{F}_7(e_1) = \{x_2, x_3\}$, $\mathcal{F}_7(e_2) = \emptyset$

$\mathcal{F}_8(e_1) = X$, $\mathcal{F}_8(e_2) = \{x_1\}$

$\mathcal{F}_9(e_1) = \{x_1\}$, $\mathcal{F}_9(e_2) = \{x_1\}$

$\mathcal{F}_{10}(e_1) = \{x_2\}$, $\mathcal{F}_{10}(e_2) = \{x_1\}$

$\mathcal{F}_{11}(e_1) = \{x_3\}$, $\mathcal{F}_{11}(e_2) = \{x_1\}$

$\mathcal{F}_{12}(e_1) = \{x_1, x_2\}$, $\mathcal{F}_{12}(e_2) = \{x_1\}$

$\mathcal{F}_{13}(e_1) = \{x_1, x_3\}$, $\mathcal{F}_{13}(e_2) = \{x_1\}$

$\mathcal{F}_{14}(e_1) = \{x_2, x_3\}$, $\mathcal{F}_{14}(e_2) = \{x_1\}$

$\mathcal{F}_{15}(e_1) = \emptyset$, $\mathcal{F}_{15}(e_2) = \{x_1\}$

$\mathcal{F}_{16}(e_1) = X$, $\mathcal{F}_{16}(e_2) = \{x_2, x_3\}$

$\mathcal{F}_{17}(e_1) = \{x_1\}$, $\mathcal{F}_{17}(e_2) = \{x_2, x_3\}$

$\mathcal{F}_{18}(e_1) = \{x_2\}$, $\mathcal{F}_{18}(e_2) = \{x_2, x_3\}$

$\mathcal{F}_{19}(e_1) = \{x_3\}$, $\mathcal{F}_{19}(e_2) = \{x_2, x_3\}$

$\mathcal{F}_{20}(e_1) = \{x_1, x_2\}$, $\mathcal{F}_{20}(e_2) = \{x_2, x_3\}$

$\mathcal{F}_{21}(e_1) = \{x_1, x_3\}$, $\mathcal{F}_{21}(e_2) = \{x_2, x_3\}$

$\mathcal{F}_{22}(e_1) = \{x_2, x_3\}$, $\mathcal{F}_{22}(e_2) = \{x_2, x_3\}$

$\mathcal{F}_{23}(e_1) = \emptyset$, $\mathcal{F}_{23}(e_2) = \{x_2, x_3\}$

$\mathcal{F}_{24}(e_1) = \{x_1\}$, $\mathcal{F}_{24}(e_2) = X$

$\mathcal{F}_{25}(e_1) = \{x_2\}$, $\mathcal{F}_{25}(e_2) = X$

$\mathcal{F}_{26}(e_1) = \{x_3\}$, $\mathcal{F}_{26}(e_2) = X$

$\mathcal{F}_{27}(e_1) = \{x_1, x_2\}$, $\mathcal{F}_{27}(e_2) = X$

$\mathcal{F}_{28}(e_1) = \{x_1, x_3\}$, $\mathcal{F}_{28}(e_2) = X$

$\mathcal{F}_{29}(e_1) = \{x_2, x_3\}$, $\mathcal{F}_{29}(e_2) = X$

$\mathcal{F}_{30}(e_1) = \emptyset$, $\mathcal{F}_{30}(e_2) = X$

Hence the family of all $soft-\delta^*-closed$ sets of the $soft$ −tritopological space $(\mathcal{X}, \tau_1, \tau_2, \tau_3, \mathbb{E})$ is, $S.\delta^*.C(\mathcal{X}) = \{\mathcal{X}, \Phi, (\mathcal{F}_i{}^c, \mathbb{E})\}$ (with $i = 1, 2, \ldots, 30$).

$\mathcal{F}_1{}^c(e_1) = \emptyset$, $\mathcal{F}_1{}^c(e_2) = X$

$\mathcal{F}_2{}^c(e_1) = \{x_2, x_3\}$, $\mathcal{F}_2{}^c(e_2) = X$

$\mathcal{F}_3{}^c(e_1) = \{x_1, x_3\}$, $\mathcal{F}_3{}^c(e_2) = X$

$\mathcal{F}_4{}^c(e_1) = \{x_1, x_2\}$, $\mathcal{F}_4{}^c(e_2) = X$

$\mathcal{F}_5{}^c(e_1) = \{x_3\}$, $\mathcal{F}_5{}^c(e_2) = X$

$F_6{}^c(e_1) = \{x_2\}$, $F_6{}^c(e_2) = X$

$\mathcal{F}_7{}^c(e_1) = \{x_1\}$, $\mathcal{F}_7{}^c(e_2) = X$

$\mathcal{F}_8{}^c(e_1) = \emptyset$, $\mathcal{F}_8^c(e_2) = \{x_2, x_3\}$

$\mathcal{F}_9{}^c(e_1) = \{x_2, x_3\}$, $\mathcal{F}_9{}^c(e_2) = \{x_2, x_3\}$

$\mathcal{F}_{10}^c(e_1) = \{x_1, x_3\}$, $\mathcal{F}_{10}^c(e_2) = \{x_2, x_3\}$

$\mathcal{F}_{11}^c(e_1) = \{x_1, x_2\}$, $\mathcal{F}_{11}^c(e_2) = \{x_2, x_3\}$

$\mathcal{F}_{12}^c(e_1) = \{x_3\}$, $\mathcal{F}_{12}^c(e_2) = \{x_2, x_3\}$

$\mathcal{F}_{13}^c(e_1) = \{x_2\}$, $\mathcal{F}_{13}^c(e_2) = \{x_2, x_3\}$

$\mathcal{F}_{14}^c(e_1) = \{x_1\}$, $\mathcal{F}_{14}^c(e_2) = \{x_2, x_3\}$

$\mathcal{F}_{15}^c(e_1) = X$, $\mathcal{F}_{15}^c(e_2) = \{x_2, x_3\}$

$\mathcal{F}_{16}^c(e_1) = \emptyset$, $\mathcal{F}_{16}^c(e_2) = \{x_1\}$

$\mathcal{F}_{17}{}^c(e_1) = \{x_2, x_3\}$, $\mathcal{F}_{17}^c(e_2) = \{x_1\}$

$\mathcal{F}_{18}^c(e_1) = \{x_1, x_3\}$, $\mathcal{F}_{18}^c(e_2) = \{x_1\}$

$\mathcal{F}_{19}^c(e_1) = \{x_1, x_2\}$, $\mathcal{F}_{19}^c(e_2) = \{x_1\}$

$\mathcal{F}_{20}^c(e_1) = \{x_3\}$, $\mathcal{F}_{20}^c(e_2) = \{x_1\}$

$\mathcal{F}_{21}^c(e_1) = \{x_2\}$, $\mathcal{F}_{21}^c(e_2) = \{x_1\}$

$\mathcal{F}_{22}^c(e_1) = \{x_1\}$, $\mathcal{F}_{22}^c(e_2) = \{x_1\}$

$\mathcal{F}_{23}^c(e_1) = X$, $\mathcal{F}_{23}^c(e_2) = \{x_1\}$

$\mathcal{F}_{24}^c(e_1) = \{x_2, x_3\}$, $\mathcal{F}_{24}^c(e_2) = \emptyset$

$\mathcal{F}_{25}^c(e_1) = \{x_1, x_3\}$, $\mathcal{F}_{25}^c(e_2) = \emptyset$

$\mathcal{F}_{26}^c(e_1) = \{x_1, x_2\}$, $\mathcal{F}_{26}^c(e_2) = \emptyset$

$\mathcal{F}_{27}^c(e_1) = \{x_3\}$, $\mathcal{F}_{27}^c(e_2) = \emptyset$

$\mathcal{F}_{28}^c(e_1) = \{x_2\}$, $\mathcal{F}_{28}^c(e_2) = \emptyset$

$\mathcal{F}_{29}^c(e_1) = \{x_1\}$, $\mathcal{F}_{29}^c(e_2) = \emptyset$

$\mathcal{F}_{30}^c(e_1) = X$, $\mathcal{F}_{30}^c(e_2) = \emptyset$

We note that $(\mathcal{X}, \tau_1, \tau_2, \tau_3, \mathbb{E})$ is a $soft-\delta^*-T_3$ −space but it is not a $soft-\delta^*-T_2$ −space because $x_2, x_3 \in X$ but there do not exist $soft-\delta^*-open$ sets (F, E) and (G, E) such that $x_2 \in (F, E)$, $x_3 \in (G, E)$ and $(F, E) \cap (G, E) = \Phi$. Thus every $soft-\delta^*-T_3$ −space is not necessarily a $soft-\delta^*-T_2$ −space. Now we have,

$\delta^*.\tau_{e_1} = \{\emptyset, X, \{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_2\}, \{x_1, x_3\}, \{x_2, x_3\}\}$ and $\delta^*.\tau_{e_2} = \{\emptyset, X, \{x_1\}, \{x_2, x_3\}\}$

Here $(X, \delta^*.\tau_{e_2})$ is not a T_3 −space. This shows that if $(\mathcal{X}, \tau_1, \tau_2, \tau_3, \mathbb{E})$ is a $soft-\delta^*-T_3$ −space then $(X, \delta^*.\tau_e)$ may not be a T_3 −space for every parameter $e \in \mathbb{E}$,

Proposition 4.6. Let $(\mathcal{X}, \tau_1, \tau_2, \tau_3, \mathbb{E})$ be a $soft$ −tritopological space over X and Y be a non-empty subset of X. If $\mathcal{X}$ is a $soft-\delta^*-T_3$ −space then the $soft$ −subspace $\mathbb{Y}$ is a $soft-\delta^*-T_3$ −space.

Proof. By proposition 3.9. the $soft$ −subspace $\mathbb{Y}$ is a $soft-\delta^*-T_1$ −space. Let $y \in Y$ and (F, E) be a $soft-\delta^*-closed$ set in Y such that $y \notin (F, E)$.

Then $y \notin ((F, E) \cap (G, E))$. where $(F, E) = ((Y, E) \cap (G, E))$. for some $soft-\delta^*-closed$ set in $\mathcal{X}$, by theorem 3.13. in [16]. But $y \in (Y, E)$, so $y \notin (G, E)$. As $\mathcal{X}$ is a $soft-\delta^*-T_3$ −space, so there exist $soft-\delta^*-open$ sets (G_1, E) and (G_2, E) in X such that $y \in (G_1, E)$, $(G, E) \subseteq (G_2, E)$ and $(G_1, E) \cap (G_2, E) = \Phi$. Now if we take;

$(F_1, E) = ((Y, E) \cap (G_1, E))$ and $(F_2, E) = ((Y, E) \cap (G_2, E))$, then (F_1, E) and $(F_2, E) \in S.\delta^*.O(\mathcal{X})_Y$ such that $y \in (F_1, E)$, $(F, E) \subseteq (Y, E) \cap (G_2, E) = (F_2, E)$ and $(F_1, E) \cap (F_2, E) \subseteq (G_1, E) \cap (G_2, E) = \Phi$, i.e. $(F_1, E) \cap (F_2, E) = \Phi$.

Thus the $soft$ −subspace $\mathbb{Y}$ is a $soft-\delta^*-T_3$ −space.

Definition 4.7. Let $(\mathcal{X}, \tau_1, \tau_2, \tau_3, \mathbb{E})$ be a $soft$ −tritopological space over X, $(H, \mathbb{E})$ and $(G, \mathbb{E})$ $soft-\delta^*-closed$ sets in X such that $(H,\mathbb{E}) \cap (G,\mathbb{E}) = \Phi$, If there exist $soft - \delta^* - open$ sets $(F_1,\mathbb{E})$ and $(F_2,\mathbb{E})$ such that

"$(H,\mathbb{E}) \subseteq (F_1,\mathbb{E}),(G,\mathbb{E}) \subseteq (F_2,\mathbb{E})and(F_1,\mathbb{E}) \cap (F_2,\mathbb{E}) = \Phi$". Then $(\mathcal{X},\tau_1,\tau_2,\tau_3,\mathbb{E})$ is called a $soft - \delta^*$ −normal space.

Definition 4.8. A $soft$ −tritopological space $(\mathcal{X},\tau_1,\tau_2,\tau_3,\mathbb{E})$ over X is said to be a $soft - \delta^* - T_4$ −space iff it is a $soft - \delta^*$ −normal and $soft - \delta^* - T_1$ −space.

Example 4.9. Let $X = \{x_1,x_2\}$ be the universe set and $\mathbb{E} = \{e_1,e_2\}$ be the set of parameters, Then $(\mathcal{X},\mathbb{E}) = \{(e_1,\{x_1,x_2\}),(e_2,\{x_1,x_2\})\}$. By [5], its cardinality is defined by $|\mathcal{P}(\mathcal{X})| = 2^{\sum_{e\in\mathbb{E}}|\mathcal{F}(e)|}$, where $|\mathcal{F}(e)|$ is the cardinality of $\mathcal{F}(e)$. (i.e. $|\mathcal{P}(\mathcal{X})| = 2^4 = 16$ $soft$ −set).

Let $(\mathcal{X},\tau_1,\mathbb{E})$, $(\mathcal{X},\tau_2,\mathbb{E})$ and $(\mathcal{X},\tau_3,\mathbb{E})$ be the three $soft$ −topological spaces on $\mathcal{X}$, where their $soft$ −topologies defined as follows:

$\tau_1 = \{\Phi,\mathcal{X},(\mathcal{M}_1,\mathbb{E}),(\mathcal{M}_2,\mathbb{E}),(\mathcal{M}_3,\mathbb{E}),(\mathcal{M}_4,\mathbb{E})\}$
$\tau_2 = \{\Phi,\mathcal{X},(G_1,\mathbb{E}),(G_2,\mathbb{E}),(G_3,\mathbb{E}),(G_4,\mathbb{E})\}$. And $\tau_3 = \{\Phi,\mathcal{X},(\mathcal{H}_1,\mathbb{E}),\dots,(\mathcal{H}_{14},\mathbb{E})$,Where the $soft - open$ sets over $\mathcal{X}$ in three $soft$ −topologies, defined as follows:

$\mathcal{M}_1(e_1) = \{x_1\}$, $\mathcal{M}_1(e_2) = X$
$\mathcal{M}_2(e_1) = \{x_1\}$, $\mathcal{M}_2(e_2) = \{x_2\}$
$\mathcal{M}_3(e_1) = \emptyset$, $\mathcal{M}_3(e_2) = X$
$\mathcal{M}_4(e_1) = \emptyset$, $\mathcal{M}_4(e_2) = \{x_2\}$
$G_1(e_1) = X$, $G_1(e_2) = \emptyset$
$G_2(e_1) = \{x_2\}$, $G_2(e_2) = \{x_1\}$
$G_3(e_1) = \{x_2\}$, $G_3(e_2) = \emptyset$
$G_4(e_1) = X$, $G_4(e_2) = \{x_1\}$
$\mathcal{H}_1(e_1) = X$, $\mathcal{H}_1(e_2) = \{x_2\}$
$\mathcal{H}_2(e_1) = \{x_1\}$, $\mathcal{H}_2(e_2) = X$
$\mathcal{H}_3(e_1) = \{x_2\}$, $\mathcal{H}_3(e_2) = \emptyset$
$\mathcal{H}_4(e_1) = \{x_2\}$, $\mathcal{H}_4(e_2) = X$
$\mathcal{H}_5(e_1) = \{x_1\}$, $\mathcal{H}_5(e_2) = \{x_1\}$
$\mathcal{H}_6(e_1) = \emptyset$, $\mathcal{H}_6(e_2) = \{x_1\}$
$\mathcal{H}_7(e_1) = \{x_1\}$, $\mathcal{H}_7(e_2) = \emptyset$
$\mathcal{H}_8(e_1) = X$, $\mathcal{H}_8(e_2) = \{x_1\}$
$\mathcal{H}_9(e_1) = \{x_2\}$, $\mathcal{H}_9(e_2) = \{x_1\}$
$\mathcal{H}_{10}(e_1) = \{x_1\}$, $\mathcal{H}_{10}(e_2) = \{x_2\}$
$\mathcal{H}_{11}(e_1) = \{x_2\}$, $\mathcal{H}_{11}(e_2) = \{x_2\}$
$\mathcal{H}_{12}(e_1) = \emptyset$, $\mathcal{H}_{12}(e_2) = \{x_2\}$
$\mathcal{H}_{13}(e_1) = X$, $\mathcal{H}_{13}(e_2) = \emptyset$
$\mathcal{H}_{14}(e_1) = \emptyset$, $\mathcal{H}_{14}(e_2) = X$

The complement of the $soft - open$ sets of τ_2 Are ;

${G_1}^c(e_1) = \emptyset$, ${G_1}^c(e_2) = X$
${G_2}^c(e_1) = \{x_1\}$, ${G_2}^c(e_2) = \{x_2\}$
${G_3}^c(e_1) = \{x_1\}$, ${G_3}^c(e_2) = X$
${G_4}^c(e_1) = \emptyset$, ${G_4}^c(e_2) = \{x_2\}$

Hence the family of all $soft - \delta^* - open$ sets of the $soft$ −tritopological space $(\mathcal{X},\tau_1,\tau_2,\tau_3,\mathbb{E})$ is, $S.\delta^*.O(\mathcal{X}) = \{\mathcal{X},\Phi,(\mathcal{F}_i,\mathbb{E})\}$ (with i = 1,2,…,14) as;

$\mathcal{F}_1(e_1) = \{x_1\}$, $\mathcal{F}_1(e_2) = \{x_1\}$
$\mathcal{F}_2(e_1) = \{x_1\}$, $\mathcal{F}_2(e_2) = \{x_2\}$
$\mathcal{F}_3(e_1) = \{x_1\}$, $\mathcal{F}_3(e_2) = X$
$\mathcal{F}_4(e_1) = \{x_1\}$, $\mathcal{F}_4(e_2) = \emptyset$
$\mathcal{F}_5(e_1) = \{x_2\}$, $\mathcal{F}_5(e_2) = \{x_2\}$
$\mathcal{F}_6(e_1) = \{x_2\}$, $\mathcal{F}_6(e_2) = \{x_1\}$
$\mathcal{F}_7(e_1) = \{x_2\}$, $\mathcal{F}_7(e_2) = X$
$\mathcal{F}_8(e_1) = \{x_2\}$, $\mathcal{F}_8(e_2) = \emptyset$
$\mathcal{F}_9(e_1) = \emptyset$, $\mathcal{F}_9(e_2) = \{x_1\}$
$\mathcal{F}_{10}(e_1) = \emptyset$, $\mathcal{F}_{10}(e_2) = \{x_2\}$
$\mathcal{F}_{11}(e_1) = \emptyset$, $\mathcal{F}_{11}(e_2) = X$
$\mathcal{F}_{12}(e_1) = X$, $\mathcal{F}_{12}(e_2) = \emptyset$
$\mathcal{F}_{13}(e_1) = X$, $\mathcal{F}_{13}(e_2) = \{x_1\}$
$\mathcal{F}_{14}(e_1) = X$, $\mathcal{F}_{14}(e_2) = \{x_2\}$

Hence the family of all $soft - \delta^* - closed$ sets of the $soft$ −tritopological space $(\mathcal{X},\tau_1,\tau_2,\tau_3,\mathbb{E})$ is, $S.\delta^*.C(\mathcal{X}) = \{\mathcal{X},\Phi,(\mathcal{F}_i^c,\mathbb{E})\}$, (with i = 1,2,…,14), defined as;

${\mathcal{F}_1}^c(e_1) = \{x_2\}$, ${\mathcal{F}_1}^c(e_2) = \{x_2\}$
${\mathcal{F}_2}^c(e_1) = \{x_2\}$, ${\mathcal{F}_2}^c(e_2) = \{x_1\}$
${\mathcal{F}_3}^c(e_1) = \{x_2\}$, ${\mathcal{F}_3}^c(e_2) = \emptyset$
${\mathcal{F}_4}^c(e_1) = \{x_2\}$, ${\mathcal{F}_4}^c(e_2) = X$
${\mathcal{F}_5}^c(e_1) = \{x_1\}$, ${\mathcal{F}_5}^c(e_2) = \{x_2\}$
${\mathcal{F}_6}^c(e_1) = \{x_1\}$, ${\mathcal{F}_6}^c(e_2) = \{x_1\}$
${\mathcal{F}_7}^c(e_1) = \{x_1\}$, ${\mathcal{F}_7}^c(e_2) = \emptyset$
${\mathcal{F}_8}^c(e_1) = \{x_1\}$, ${\mathcal{F}_8}^c(e_2) = X$
${\mathcal{F}_9}^c(e_1) = X$, ${\mathcal{F}_9}^c(e_2) = \{x_2\}$
${\mathcal{F}_{10}}^c(e_1) = X$, ${\mathcal{F}_{10}}^c(e_2) = \{x_1\}$
${\mathcal{F}_{11}}^c(e_1) = X$, ${\mathcal{F}_{11}}^c(e_2) = \emptyset$
${\mathcal{F}_{12}}^c(e_1) = \emptyset$, ${\mathcal{F}_{12}}^c(e_2) = X$
${\mathcal{F}_{13}}^c(e_1) = \emptyset$, ${\mathcal{F}_{13}}^c(e_2) = \{x_2\}$
${\mathcal{F}_{14}}^c(e_1) = \emptyset$, ${\mathcal{F}_{14}}^c(e_2) = \{x_1\}$

$\exists(\mathcal{F}_1{}^c,\mathbb{E}),(\mathcal{F}_{14}{}^c,\mathbb{E})\in S.\delta^*.C(\mathcal{X})$ such that

$(\mathcal{F}_1{}^c,\mathbb{E})\cap(\mathcal{F}_{14}{}^c,\mathbb{E})=\Phi$

$\exists(\mathcal{F}_5,\mathbb{E}),(\mathcal{F}_9,\mathbb{E})\in S.\delta^*.O(\mathcal{X})$ such that

$(\mathcal{F}_1{}^c,\mathbb{E})\subseteq(\mathcal{F}_5,\mathbb{E}),(\mathcal{F}_{14}{}^c,\mathbb{E})\subseteq(\mathcal{F}_9,\mathbb{E})$

And $(\mathcal{F}_5,\mathbb{E})\cap(\mathcal{F}_9,\mathbb{E})=\Phi$, and so on …

Thus $(\mathcal{X},\tau_1,\tau_2,\tau_3,\mathbb{E})$ is a $soft-\delta^*$ –normal space.

Let $x_1,x_2\in X$ such that $x_1\neq x_2$

$\exists(\mathcal{F}_1,\mathbb{E}),(\mathcal{F}_5,\mathbb{E})\in S.\delta^*.O(\mathcal{X})$ such that

$x_1\in(\mathcal{F}_1,\mathbb{E})$, $x_2\notin(\mathcal{F}_1,\mathbb{E})$ and

$x_2\in(\mathcal{F}_5,\mathbb{E})$, $x_1\notin(\mathcal{F}_5,\mathbb{E})$

Thus $(\mathcal{X},\tau_1,\tau_2,\tau_3,\mathbb{E})$ is a $soft-\delta^*-T_1$ –space .

Therefore $(\mathcal{X},\tau_1,\tau_2,\tau_3,\mathbb{E})$ is a $soft-\delta^*-T_4$ –space.

Remark 4.10. (1) A $soft-\delta^*-T_4$ –space need not be a $soft-\delta^*-T_3$ –space.

(2) If $(\mathcal{X},\tau_1,\tau_2,\tau_3,\mathbb{E})$ is a $soft-\delta^*-T_4$ –space then $(X,\delta^*.\tau_e)$ may not be a T_4 –space for every parameter $e\in\mathbb{E}$, where $\delta^*.\tau_e=\{F(e)|(F,\mathbb{E})\in S.\delta^*.O(\mathcal{X})\}$ [15].

(3) If $(\mathcal{X},\tau_1,\tau_2,\tau_3,\mathbb{E})$ is a $soft-\delta^*-T_4$ –space and Y is a non-empty subset of X then the $soft$ –subspace $\mathbb{Y}$ need not be a $soft-\delta^*-T_4$ –space.

Example 4.11. Let $X=\{x_1,x_2,x_3,x_4\}$ be a universe set and $\mathbb{E}=\{e_1,e_2\}$ be the set of parameters, then by [5], the $soft-$ set $(X,\mathbb{E})$ cardinality is $|\mathcal{P}(\mathcal{X})|=2^{\sum_{e\in\mathbb{E}}|\mathcal{F}(e)|}$, s.t. $|\mathcal{F}(e)|$ is the cardinality of $\mathcal{F}(e)$. (that is mean $|\mathcal{P}(\mathcal{X})|=2^8=256$ $soft$ –set).

And let $(\mathcal{X},\tau_1,\mathbb{E})$, $(\mathcal{X},\tau_2,\mathbb{E})$ and $(\mathcal{X},\tau_3,\mathbb{E})$ be the three $soft$ –topological spaces on $\mathcal{X}$, and the $soft$ –topologies defined as follows:

$\tau_1=\{\Phi,\mathcal{X},(\mathcal{M}_1,\mathbb{E}),\dots,(\mathcal{M}_8,\mathbb{E})\}$

$\tau_2=\{\Phi,\mathcal{X},(G_1,\mathbb{E}),\dots,(G_8,\mathbb{E})\}$, and

$\tau_3=\{\Phi,\mathcal{X},(\mathcal{H}_1,\mathbb{E}),\dots,(\mathcal{H}_8,\mathbb{E})\}$. Where $(\mathcal{M}_1,\mathbb{E}),\dots,(\mathcal{M}_8,\mathbb{E}),(G_1,\mathbb{E}),\dots,(G_8,\mathbb{E}),(\mathcal{H}_1,\mathbb{E}),\dots,(\mathcal{H}_8,\mathbb{E})$ are $soft-open$ sets in $\mathcal{X}$, defined as follows:

$\mathcal{M}_1(e_1)=\{x_1,x_2,x_4\}$, $\mathcal{M}_1(e_2)=\{x_1,x_2,x_3\}$

$\mathcal{M}_2(e_1)=\{x_1,x_3,x_4\}$, $\mathcal{M}_2(e_2)=\{x_1,x_2,x_3\}$

$\mathcal{M}_3(e_1)=\{x_1,x_4\}$, $\mathcal{M}_3(e_2)=\{x_1,x_2,x_3\}$

$\mathcal{M}_4(e_1)=\{x_2,x_3\}$, $\mathcal{M}_4(e_2)=\{x_1,x_2,x_3\}$

$\mathcal{M}_5(e_1)=\{x_2\}$, $\mathcal{M}_5(e_2)=\{x_1,x_2,x_3\}$

$\mathcal{M}_6(e_1)=\{x_3\}$, $\mathcal{M}_6(e_2)=\{x_1,x_2,x_3\}$

$\mathcal{M}_7(e_1)=\emptyset$, $\mathcal{M}_7(e_2)=\{x_1,x_2,x_3\}$

$\mathcal{M}_8(e_1)=X$, $\mathcal{M}_8(e_2)=\{x_1,x_2,x_3\}$

$G_1(e_1)=\{x_3\}$, $G_1(e_2)=\{x_4\}$

$G_2(e_1)=\{x_2\}$, $G_2(e_2)=\{x_4\}$

$G_3(e_1)=\{x_2,x_3\}$, $G_3(e_2)=\{x_4\}$

$G_4(e_1)=\{x_1,x_4\}$, $G_4(e_2)=\{x_4\}$

$G_5(e_1)=\{x_1,x_3,x_4\}$, $G_5(e_2)=\{x_4\}$

$G_6(e_1)=\{x_1,x_2,x_4\}$, $G_6(e_2)=\{x_4\}$

$G_7(e_1)=X$, $G_7(e_2)=\{x_4\}$

$G_8(e_1)=\emptyset$, $G_8(e_2)=\{x_4\}$

$\mathcal{H}_1(e_1)=\{x_1,x_2,x_4\}$, $\mathcal{H}_1(e_2)=\{x_1,x_2,x_3\}$

$\mathcal{H}_2(e_1)=\{x_1,x_3,x_4\}$, $\mathcal{H}_2(e_2)=\{x_1,x_2,x_3\}$

$\mathcal{H}_3(e_1)=\{x_1,x_4\}$, $\mathcal{H}_3(e_2)=\{x_1,x_2,x_3\}$

$\mathcal{H}_4(e_1)=\{x_2,x_3\}$, $\mathcal{H}_4(e_2)=\{x_1,x_2,x_3\}$

$\mathcal{H}_5(e_1)=\{x_2\}$, $\mathcal{H}_5(e_2)=\{x_1,x_2,x_3\}$

$\mathcal{H}_6(e_1)=\{x_3\}$, $\mathcal{H}_6(e_2)=\{x_1,x_2,x_3\}$

$\mathcal{H}_7(e_1)=\emptyset$, $\mathcal{H}_7(e_2)=\{x_1,x_2,x_3\}$

$\mathcal{H}_8(e_1)=X$, $\mathcal{H}_8(e_2)=\{x_1,x_2,x_3\}$

(if we examine the all 256 $soft$ –sets) , then we get the family of all $soft-\delta^*-open$ sets of the $soft$ –tritopological space $(\mathcal{X},\tau_1,\tau_2,\tau_3,\mathbb{E})$ is,

$S.\delta^*.O(\mathcal{X})=\{\mathcal{X},\Phi,(\mathcal{F}_i,\mathbb{E})\}$, (with i = 1,2,…,8)

$\mathcal{F}_1(e_1)=\{x_1,x_2,x_4\}$, $\mathcal{F}_1(e_2)=\{x_1,x_2,x_3\}$

$\mathcal{F}_2(e_1)=\{x_1,x_3,x_4\}$, $\mathcal{F}_2(e_2)=\{x_1,x_2,x_3\}$

$\mathcal{F}_3(e_1)=\{x_1,x_4\}$, $\mathcal{F}_3(e_2)=\{x_1,x_2,x_3\}$

$\mathcal{F}_4(e_1)=\{x_2,x_3\}$, $\mathcal{F}_4(e_2)=\{x_1,x_2,x_3\}$

$\mathcal{F}_5(e_1)=\{x_2\}$, $\mathcal{F}_5(e_2)=\{x_1,x_2,x_3\}$

$\mathcal{F}_6(e_1)=\{x_3\}$, $\mathcal{F}_6(e_2)=\{x_1,x_2,x_3\}$

$\mathcal{F}_7(e_1)=\emptyset$, $\mathcal{F}_7(e_2)=\{x_1,x_2,x_3\}$

$\mathcal{F}_8(e_1)=X$, $\mathcal{F}_8(e_2)=\{x_1,x_2,x_3\}$

We note that $(\mathcal{X},\tau_1,\tau_2,\tau_3,\mathbb{E})$ is a $soft-\delta^*-T_4$ –space but it is not a $soft-\delta^*-T_3$ –space because $x_1\in X$ and $(\mathcal{F}_3,\mathbb{E})^C$ is a $soft-\delta^*-closed$ set in X such that $x_1\notin(\mathcal{F}_3,\mathbb{E})^C$ but there do not exist $soft-\delta^*-open$ sets $(F,\mathbb{E})$ and $(G,\mathbb{E})$ such that $x_1\in(F,\mathbb{E})$, $(\mathcal{F}_3,\mathbb{E})^C\subseteq(G,\mathbb{E})$ and $(F,\mathbb{E})\cap(G,\mathbb{E})=\Phi$. Thus every $soft-\delta^*-T_4$ –space is not necessarily a $soft-\delta^*-T_3$ –space. Now we have,

$\delta^*.\tau_{e_1}=\{X,\emptyset,\{x_2\},\{x_3\},\{x_2,x_3\},\{x_1,x_4\},\{x_1,x_3,x_4\},\{x_1,x_2,x_4\}\}$

And $\delta^*.\tau_{e_2}=\{X,\emptyset,\{x_1,x_2,x_3\}\}$

Here $(X,\delta^*.\tau_{e_1})$ and $(X,\delta^*.\tau_{e_2})$ are not T_3 –space. This shows that if $(\mathcal{X},\tau_1,\tau_2,\tau_3,\mathbb{E})$ is a $soft-\delta^*-T_4$ –space then $(X,\delta^*.\tau_e)$ need not be a T_4 –space for every parameter $e\in\mathbb{E}$.

Now, let $Y=\{x_1,x_2,x_3\}\subseteq X$. By the definition **2.14**, the $soft$ –sub sets $(\mathcal{F}_{iY},\mathbb{E})$ (with i = 1,2,…,8) of the

$soft-\delta^*-open\ (\mathcal{F}_i,\mathbb{E})$ over $\mathbb{Y}$, results to be defined by:

$\mathcal{F}_{1\mathbb{Y}}(e_1)=\{x_1,x_2\}$, $\mathcal{F}_{1\mathbb{Y}}(e_2)=\mathbb{Y}$

$\mathcal{F}_{2\mathbb{Y}}(e_1)=\{x_1,x_3\}$, $\mathcal{F}_{2\mathbb{Y}}(e_2)=\mathbb{Y}$

$\mathcal{F}_{3\mathbb{Y}}(e_1)=\{x_1\}$, $\mathcal{F}_{3\mathbb{Y}}(e_2)=\mathbb{Y}$

$\mathcal{F}_{4\mathbb{Y}}(e_1)=\{x_2,x_3\}$, $\mathcal{F}_{4\mathbb{Y}}(e_2)=\mathbb{Y}$

$\mathcal{F}_{5\mathbb{Y}}(e_1)=\{x_2\}$, $\mathcal{F}_{5\mathbb{Y}}(e_2)=\mathbb{Y}$

$\mathcal{F}_{6\mathbb{Y}}(e_1)=\{x_3\}$, $\mathcal{F}_{6\mathbb{Y}}(e_2)=\mathbb{Y}$

$\mathcal{F}_{7\mathbb{Y}}(e_1)=\emptyset$, $\mathcal{F}_{7\mathbb{Y}}(e_2)=\mathbb{Y}$

$\mathcal{F}_{8\mathbb{Y}}(e_1)=\mathbb{Y}$, $\mathcal{F}_{8\mathbb{Y}}(e_2)=\mathbb{Y}$

We note that $soft-$subspace $\mathbb{Y}$ is not a $soft-\delta^*-\mathrm{T}_4-$space because $(\mathrm{F}_{3\mathbb{Y}},\mathbb{E})^c$ and $(\mathrm{F}_{4\mathbb{Y}},\mathbb{E})^c$ are $soft-\delta^*-\mathrm{closed}$ sets in $\mathbb{Y}$ such that $(\mathrm{F}_{3\mathbb{Y}},\mathbb{E})^c\cap(\mathrm{F}_{4\mathbb{Y}},\mathbb{E})^c=\Phi$ but there do not exist any $soft-\delta^*-\mathrm{open}$ sets $(\mathrm{F},\mathbb{E})$ and $(\mathrm{G},\mathbb{E})$ in $\mathbb{Y}$ such that $(\mathrm{F}_{3\mathbb{Y}},\mathbb{E})^c\subseteq(\mathrm{F},\mathbb{E})$, $(\mathrm{F}_{4\mathbb{Y}},\mathbb{E})^c\subseteq(\mathrm{G},\mathbb{E})$ and $(\mathrm{F},\mathbb{E})\cap(\mathrm{G},\mathbb{E})=\Phi$

Thus a $soft-$subspace of a $soft-\delta^*-\mathrm{T}_4-$space may not be a $soft-\delta^*-\mathrm{T}_4-$space.

5. CONCLUSIONS

In this work, we have continued the foundations of the theory of a $soft-$tritopological spaces to define and study the separation axioms of a $soft-$tritopological spaces. We defined $soft-\delta^*-\mathrm{T}_0,\mathrm{T}_1,\mathrm{T}_2,\mathrm{T}_3,\mathrm{T}_4$ spaces with respect to ordinary points and studied their behaviors in $soft-$tritopological spaces. These $soft-\delta^*-$separation axioms would be useful for the growth of the set theory of $soft-$tritopology to solve some complex problems. We also discussed some $soft-$transmissible properties with respect to ordinary points. We hope that these results in this paper will help the researchers for strengthening the toolbox of $soft-$tritopology. In the next study, we extend the concept of $soft-\delta^*-$separation axioms in $soft-$tritopological spaces with respect to $soft-$points as well as ordinary.

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