

# The impact of scaling on Support Vector Machine in Breast Cancer Diagnosis

Elsayed Badr

Scientific Computing Department,  
Faculty of Computers and  
Informatics, Benha University,  
Benha, Egypt

Mustafa Abdulsalam

Scientific Computing Department,  
Faculty of Computers and  
Informatics, Benha University,  
Benha, Egypt

Hagar Ahmed

Scientific Computing Department,  
Faculty of Computers and  
Informatics, Benha University,  
Benha, Egypt

## ABSTRACT

By using support vector machine (SVM) and the grid technique Badr *et al.* [1] introduced new scaling techniques on the data set Wisconsin from UCI machine learning with a total 569 rows and 33 columns. These scaling techniques overcame the standard normalization techniques. In this paper, three new scaling techniques are proposed by using SVM and the grid technique on the the data set Wisconsin from UCI machine learning with a total 569 rows and 32 columns. These scaling techniques are: (i) de Buchet for  $p = (\infty)$  (ii) Lp-norm for  $p = (\infty)$  (iii) Entropy . Experimental results show that SVM with new scaling techniques achieves 98.60 % , 98.42 % and 98.42 % accuracy against to the standard normalization by 96.49 %.

## General Terms

Data Mining, Classification

## Keywords

Machine Learning, Breast Cancer, Support vector machine, scaling techniques.

## 1. INTRODUCTION

Improving the accuracy of identifying the breast cancer disease is very important task. Breast cancer disease is the second most common type of cancer after lung cancer. Breast cancer is the most widespread by 12.3% of all cancer for males and females of all ages. It is the most spreading in women worldwide, accounting 25.4% of the whole cases diagnosed in 2018 [1]. Defects in breast cancer diagnosis by experts can be avoided by expert systems and artificial intelligent techniques. These expert systems can examine the medical data in shorter time and help junior physicians.

Tomlin [2] performed a computational study comparing arithmetic mean, geometric mean, equilibration, Curtis and Reid scaling technique [3], Fulkerson and Wolfe scaling technique [4], and various combinations on six test problems. The conclusion of Tomlin's comparative study was that geometric mean scaling method, optionally followed by equilibration or Curtis and Reid scaling technique are the best combined scaling techniques. Larsson [5] extended Tomlin's study by comparing entropy [5], Lpnorm [6] and de Buchet [7] scaling techniques over a dataset of 135 randomly generated LPs. Larsson concluded that the entropy scaling technique can improve the conditioning number of the constraint matrix. Elble and Sahinidis [8] expanded on Tomlin's and Larsson's studies by conducting a computational study comparing arithmetic mean, de Buchet, entropy, equilibration, geometric mean, IBM MPSX [9], Lpnorm, binormalization, and various combinations of the aforementioned scaling techniques over Netlib and

Kennington set. They used four measures to evaluate each scaling technique: (i) scaling time, (ii) solution time, (iii) number of iterations for the solution of the LP, and (iv) maximum conditioning number of the constraint matrix. Elble and Sahinidis concluded that equilibration is the best scaling technique.

In a previous paper [10], the authors reviewed and compared both the CPU- and GPU-based implementations of seven scaling techniques, namely: (i) arithmetic mean, (ii) de Buchet, (iii) entropy, (iv) equilibration, (v) geometric mean, (vi) IBM MPSX, and (vii) Lp-norm scaling methods. They have performed a computational study over Netlib and Kennington set and concluded that arithmetic mean, equilibration and geometric mean are the best serial scaling techniques. In this paper a computational study is performed over a set of sparse randomly generated LPs in order to highlight the impact of scaling prior to the application of IPM, EPSA and simplex algorithms. To the best of our knowledge, this is the first paper that investigates the effect of scaling on IPM, EPSA and simplex algorithms.

The scaling techniques can improve the accuracy of classifiers. Elsayed Badr et al. [11] proposed ten efficient scaling techniques for optimizing SVM. These scaling techniques are efficient for linear programming approach [12-20]. The scaling techniques that they applied with SVM on WDBC dataset are arithmetic mean, de Buchet for three cases ( $p=1, 2$ ), equilibration, geometric mean, IBM MPSX, Lp-norm for three cases ( $p=1$  or  $2$ ).

The rest of this paper is organized as follows. The algorithms that are used in the study: SVM described in Section 2. The proposed model is introduced in section 3. In Section 4, detailed descriptions of new scaling techniques, de Buchet for  $p=(\infty)$ , entropy and Lp-norm for  $p=(\infty)$  are proposed. Experimental design which has data description, experimental setup, measure for performance evaluation and a comparative study are introduced in section 5. In Section 6 the main results and discussion are proposed. Finally, conclusions and future works are introduced in section 7.

## 2. PRELIMINARIES

In this section, Support vector machine (SVM), and grew wolf optimizer (GWO) are presented and discussed.

### Support Vector Machine (SVM)

Support vector machine SVM developed by Vapnik [9], the support vector machine (SVM) was primarily intended for binary classification. Its main objective is to determine the optimal hyperplane  $f(w, x) = w \cdot x + b$  separating two classes in a given dataset having input features  $x \in R^p$ , and labels  $y$

$\in \{-1, +1\}$ . SVM learns by solving the following constrained optimization problem:

$$\begin{aligned} \min \quad & \frac{1}{p} w^T w + c \sum_{i=1}^p \xi_i \\ \text{subject to} \quad & y_i (w \cdot x + b) \geq 1 - \xi_i \\ & \xi_i \geq 0 \quad i=1, \dots, p \end{aligned}$$

where  $w^T w$  is the Manhattan norm,  $\xi$  is a cost function, and  $C$  is the penalty parameter (may be an arbitrary value or a selected value using hyper-parameter tuning). Its corresponding unconstrained optimization problem is the following:

$$\min \frac{1}{p} w^T w + c \sum_{i=1}^p \max(0, 1 - y_i (w \cdot x_i + b)) \quad (1)$$

where  $w \cdot x + b$  is the predictor function. The objective of Eq. 1 is known as the primal form problem of L1-SVM, with the standard hinge loss. The problem with L1-SVM is the fact that it is not differentiable[18], as opposed to its variation, the L2-SVM:

$$\min \frac{1}{p} \|w\|_2^2 + c \sum_{i=1}^p \max(0, 1 - y_i (w \cdot x_i + b))^2$$

The L2-SVM is differentiable and provides more stable results than its L1 counterpart.

### 3. THE PROPOSED CLASSIFICATION MODEL

A grid search algorithm must be guided by some performance metric, typically measured by cross-validation on the training set [21] or evaluation on a held-out validation set [22]. A typical soft-margin SVM classifier equipped with an RBF kernel has at least two hyperparameters that need to be tuned for good performance on unseen data: a regularization constant  $C$  and a kernel hyperparameter  $\gamma$ . Both parameters are continuous, so to perform grid search, one selects a finite set of "reasonable" values for each, say:  $C \in \{10, 100, 1000\}$  and  $\gamma \in \{0.01, 0.1, 1, 10\}$ . Grid search then trains an SVM with each pair  $(C, \gamma)$  in the Cartesian product of these two sets and evaluates their performance on a held-out validation set (or by internal cross-validation on the training set, in which case multiple SVMs are trained per pair). Finally, the grid search algorithm outputs the settings that achieved the highest score in the validation procedure.

### 4. SCALING TECHNIQUES

Here, we introduce the mathematical notations of ten scaling techniques in addition to the normalization scaling techniques with ranges  $[0, 1]$  and  $[-1, 1]$ . First of all, we introduce the following mathematical preliminaries as shown in Table 1.

The scaled matrix is expressed as  $RAS$ , such that  $R = \text{diag}(r_1, \dots, r_m)$  and  $S = \text{diag}(s_1, \dots, s_n)$ . All scaling techniques proposed in this section apply first rows scaling and after that columns scaling. Then, the matrix after full scaling (row and column) is given by:

$$A^R = RA; A^{RS} = A^R S \quad (2)$$

**Table 1. Mathematical preliminaries for scaling techniques**

Symbol	Description
$A (a_{ij})$ :	$m \times n$ matrix (with $m$ (observations) and $n$ (attributes)).
$r_i$ :	The scaling agent of row $i$
$s_j$ :	The scaling agent of column $j$
$R$ :	Diagonal matrix such that $R = \text{diag}(r_1, \dots, r_m)$
$S$ :	Diagonal matrix such that $S = \text{diag}(s_1, \dots, s_n)$
$N_i$ :	$N_i = \{j : A_{ij} \neq 0\}$ , such that $1 \leq i \leq m$
$M_j$ :	$M_j = \{i : A_{ij} \neq 0\}$ such that $1 \leq j \leq n$
$n_i$ :	The number of elements for the set $N_i$
$m_j$ :	The number of elements for the set $M_j$
$A^R (a_{ij}^R)$	The scaled matrix by row $R$ scaling agent.
$A^{RS} (a_{ij}^{RS})$	The final scaled matrix.

1) **de Buchet scaling technique:** The de Buchet scaling model is based on the relative divergence and is formulated as shown in Equation (3):

$$\min_{r, s > 0} \left( \sum_{i, j \in \bar{Z}} \{a_{ij} r_i s_j + 1/a_{ij} r_i s_j\}^p \right)^{1/p} \quad (3)$$

where  $p$  is a positive integer and  $\bar{Z}$  is the number of the nonzero elements of  $A$ . We focus on the case  $p = \infty$ .

**Case  $p = \infty$ ,** Equation (3) is formulated as shown in Equation (4):

$$\min_{r, s > 0} \max_{i, j \in \bar{Z}} |\log(a_{ij} r_i s_j)| \quad (4)$$

The row scaling factors are described in Equation (5):

$$R_i = 1 / \left\{ \left( \max_{j \in N_i} |a_{ij}| \right) \left( \min_{j \in N_i} |a_{ij}| \right) \right\}^{1/2} \quad (5)$$

Similarly, the column scaling factors are presented in Equation (6):

$$s_j = 1 / \left\{ \left( \max_{i \in M_j} |a_{ij}^R| \right) \left( \min_{i \in M_j} |a_{ij}^R| \right) \right\}^{1/2} \quad (6)$$

The de Buchet for the case  $p = \infty$  scaling technique corresponds to the geometric mean scaling technique that will be described later.

2)  **$L_p$ -norm scaling technique:** The  $L_p$ -norm scaling model is formulated as shown in Equation (7):

$$\min_{r, s > 0} \sum_{i, j \in \bar{Z}} (|\log(a_{ij} r_i s_j)|)^p)^{1/p} \quad (7)$$

where  $p$  is a positive integer and  $\bar{Z}$  is the cardinality number of the nonzero elements of the constraint matrix  $A$ .

We focus now our attention on the case  $p = \infty$ .

**Case  $p = \infty$ ,** the model is equivalent to the de Buchet method (case  $p = \infty$ ) and geometric mean scaling techniques which were proposed in [11].

3) **Entropy scaling technique** is equivalent to the arithmetic scaling technique that was introduced in [11].

**4) Normalization scaling technique [0, 1]:** Equation (8) is used for normalization scaling method with range [0, 1] such that  $a$ ,  $a'$ ,  $max_k$  and  $min_k$  are the original value, the scaled value, the maximum value and the minimum value of feature  $k$  respectively.

$$a' = \frac{a - min_k}{max_k - min_k} \quad (8)$$

## 5. EXPERIMENTAL DESIGN

In this section, we introduce data description, measure for performance evaluation and the comparative study.

### 5.1 Data description

In this work, we have run the proposed model on the Wisconsin diagnosis Breast Cancer (WDBC) dataset that available the UCI Machine Learning Repository [23]. The dataset consists of 569 instances divided into two classes. The two classes malignant and benign have 357 and 212 cases respectively. Each record in the database has thirty-three attributes.

### 5.2. Experimental setup

The proposed model was developed by MATLAB. SVM, implementation was enhanced, which is originally developed by Chang and Lin [24]. Table 3 describes the experiments computing environment.

**Table 2. Description of the computing environment**

CPU	Intel (R) Core (TM) i5- 7200U CPU@ 2.70 GHz
RAM Size	4 GB RAM
MATLAB version	R2015a (8.5.0.197613)

Salzberg [25] introduced the k-fold CV which is used to guarantee the valid results. In this paper,  $k = 10$ .

### 5.3. Measure for performance evaluation

In order to test the performance of the proposed model, we use accuracy. According to the confusion matrix, accuracy is defined as follows:

$$Acc = (TruPos + TruNeg) / [TruPos + FlsPos + TruNeg + FlsNeg] \times 100\% \quad (9)$$

Where: Acc: Accuracy; TruPos: true positive; TruNeg: true negative; FlsPos: false positive and FlsNeg.: false negative.

### 5.4. Comparative study

In this study, we compare the performance of the proposed SVM using grid search technique with different scaling techniques. The best  $C$  and  $\gamma$  are computed by grid search.

The searching space of parameters  $C$  and  $\gamma$  are set to  $C = \{2^5, 2^3, \dots, 2^{15}\}$  and  $\gamma = \{2^{-15}, 2^{-13}, \dots, 2^1\}$ , respectively.

## 6. EXPERIMENTAL RESULTS AND DISCUSSIONS

Table 3 and Table 4 show a comparison among classification accuracies of SVM with normalization scaling [0, 1], de Buchet scaling for  $p=(\infty)$ ,  $L_p$ -norm scaling for  $p=(\infty)$  and entropy scaling technique. It is apparent from these tables that the average accuracy rates achieved by SVM with de Buchet scaling technique for  $p = \infty$  (98.60 %),  $L_p$ -norm scaling technique for  $p = \infty$  (98.42) and entropy scaling technique

(98.42 %) are better than that obtained by SVM with normalization scaling techniques (96.49%)

**Table 3: Accuracy for WBCD database using SVM with C and  $\gamma$  which were calculated by grid search technique**

**(Without scaling and Normalization scaling [0,1])**

Fold	Normalization scaling [0,1] (S1)			de Buchet p =inf (S2)		
	C	$\gamma$	Acc.%	C	$\gamma$	Acc.%
1	2 <sup>13</sup>	2 <sup>-7</sup>	100	2 <sup>5</sup>	2 <sup>-5</sup>	100
2	2 <sup>15</sup>	2 <sup>-9</sup>	98.25	2 <sup>11</sup>	2 <sup>-9</sup>	100
3	2 <sup>15</sup>	2 <sup>1</sup>	92.98	2 <sup>11</sup>	2 <sup>-9</sup>	96.49
4	2 <sup>15</sup>	2 <sup>-1</sup>	94.74	2 <sup>11</sup>	2 <sup>-9</sup>	96.49
5	2 <sup>15</sup>	2 <sup>-1</sup>	94.74	2 <sup>3</sup>	2 <sup>-11</sup>	100
6	2 <sup>15</sup>	2 <sup>1</sup>	96.49	2 <sup>11</sup>	2 <sup>-9</sup>	100
7	2 <sup>15</sup>	2 <sup>-3</sup>	98.25	2 <sup>11</sup>	2 <sup>-9</sup>	100
8	2 <sup>15</sup>	2 <sup>-13</sup>	96.49	2 <sup>11</sup>	2 <sup>-9</sup>	94.74
9	2 <sup>15</sup>	2 <sup>1</sup>	94.74	2 <sup>13</sup>	2 <sup>-11</sup>	98.25
10	2 <sup>15</sup>	2 <sup>1</sup>	98.25	2 <sup>13</sup>	2 <sup>-11</sup>	100
Avg.	<b>30310.4</b>	<b>0.91</b>	<b>96.49</b>	2871.2	0.0044	98.60
CPU Time	7.263212			15.417493		

**Table 4: Accuracy for WBCD database using SVM with C and  $\gamma$  which were calculated by grid search technique**

**(Normalization scaling [-1,1] and de Buchet scaling(p=1))**

Fold	Entropy (S3)			Lp-norm p = inf (S4)		
	C	$\gamma$	Acc. %	C	$\gamma$	Acc. %
1	2 <sup>3</sup>	2 <sup>-7</sup>	100.00	2 <sup>11</sup>	2 <sup>-9</sup>	98.21
2	2 <sup>15</sup>	2 <sup>-9</sup>	98.25	2 <sup>9</sup>	2 <sup>-9</sup>	98.25
3	2 <sup>9</sup>	2 <sup>-5</sup>	96.49	2 <sup>11</sup>	2 <sup>-9</sup>	98.25
4	2 <sup>-1</sup>	2 <sup>-5</sup>	96.49	2 <sup>3</sup>	2 <sup>-3</sup>	98.25
5	2 <sup>9</sup>	2 <sup>-9</sup>	100.00	2 <sup>11</sup>	2 <sup>-9</sup>	98.25
6	2 <sup>5</sup>	2 <sup>-5</sup>	98.25	2 <sup>11</sup>	2 <sup>-9</sup>	96.49
7	2 <sup>7</sup>	2 <sup>-7</sup>	98.25	2 <sup>11</sup>	2 <sup>-9</sup>	100
8	2 <sup>-1</sup>	2 <sup>-3</sup>	98.25	2 <sup>11</sup>	2 <sup>-9</sup>	100
9	2 <sup>9</sup>	2 <sup>-9</sup>	100.00	2 <sup>11</sup>	2 <sup>-9</sup>	98.25
10	2 <sup>15</sup>	2 <sup>-9</sup>	98.25	2 <sup>9</sup>	2 <sup>-11</sup>	98.25
Avg.	<b>6724.1</b>	<b>0.024</b>	<b>98.42</b>	1536.8	0.014	98.42
CPU Time	12.516496			15.03757		

Table 5 summarize the results of all scaling techniques that obtained by SVM according the accuracies and CPU times. It is apparent from Table 5 that the normalization scaling technique [0, 1] overcomes all other scaling techniques according to CPU time only. On the other hand, the de Buchet

( $p = \infty$ ) scaling technique outperforms all scaling techniques according to the accuracy.

**Table 5: Accuracy and CPU Time for WBCD database using SVM with C and  $\gamma$  which were calculated by grid search technique**

No.	Techniques	Accuracy%	CPU Time
S1	Normalization [0,1]	96.49	7.263212
S2	de Buchet $p = \infty$	98.60	15.417493
S3	Entropy	98.42	12.516496
S4	Lp-norm $p = \infty$	98.42	15.03757

## 7. CONCLUSION AND FUTURE WORK

In this paper, three new scaling techniques were proposed by using SVM and the grid technique on the the data set Wisconsin from UCI machine learning with a total 569 rows and 33 columns. These scaling techniques are: (i) de Buchet for  $p = (\infty)$  (ii) Lp-norm for  $p = (\infty)$  (iii) Entropy. Experimental results showed that SVM with new scaling techniques achieved 98.60 % , 98.42 % and 98.42 % accuracy against to the standard normalization by 96.49 %.

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