Simultaneous Weak Singularity and Strong Curvature Singularity in Tolman-Bondi Model with $k(r) = 0$

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ABSTRACT

The study of continues type of shell-crossing singularity and strong shell-focusing singularity in dust collapse in absence of cosmological constant. We find that for change of the scaling function singularity can change, physically initial data can lead to weak singularity. Although the free rescaling choice is simplest method for proving simultaneous singularity and being purely mathematical analysis.

Keywords

Shell-crossing, Shell-focusing

1. INTRODUCTION

Tolman-Bondi model describe the gravitational collapse of spherically symmetric dust matter distribution. Tolaman-Bondi model matched to Schwarzschild exterior where all g^{ij} are functions of C^{∞} type. Since Tolaman-Bondi model is spherically symmetric that implies that the initial density and velocity profile are only functions of radial co-ordinate r. Collapse in Tolman-Bondi model is pressureless, that mean every particular shell of dust with finite radius will collapse through its Schwarzschild radius.

For homogeneous matter(dust)- Oppenheimer-Snyder [1] all the matter shells become singular at the same time and thus there is no shell-crossing at all. The proper time for inhomogeneous matter distribution depends on radius (comoving co-ordinate) r, As shell-crossings are not genuine curvature singularities, nearby shell of matter operate developing momentary density singularity, where Kretschmann curvature scalar cloud blow up, this can be removed through extension of spacetime. However, shell-crossing are invertible in continual collapse of weakly charge spherically dust matter distributions [2]. The inner shells are more weakly bound then outer shells-and thus the inner collapse is slower then outer shell collapse, thereby leading weak singularity where volume element dose not converge.

In real objects density and pressure are very large, this may be reason of occurrence of weak singularity. In Tolman-Bondi model shell-crossing singularities is not general singularity and it is removable. However, detailed analysis by Szekeres and Lum [3] considered that newtonian and relativistic spherically symmetric matter distribution, and they suggested that following notes;

(1) Jacobi fields approach the singularity having finite limits,
(2) The boundary region can be transformed by a C^1 transformations.

The purpose here to show that shell-crossing singularity also depends on scaling function.

2. TOLMAN-BONDI SPACETIME

As already mentioned, the Tolman model represents a distribution of pressureless matter(dust) that is spherically symmetric, but inhomogeneous in the radial direction. It is written in comoving coordinate, so that $g_{tt} = -1$, and $G_{ti} = 0 (i = 1, 2, 3)$, and the tangent vector of the particles of matter is $u^{\alpha} \equiv (1, 0, 0, 0)$, which means that coordinate time,t, is also the proper time of the particles. The cosmological constant, Λ, will be neglected throughout this paper. In addition, geometric units such that $G = 1$ and $c = 1$ will be used throughout. Thus the metric is,

$$ds^2 = -dt^2 + e^{-2\nu(t,r)} dr^2 + R^2 d\Omega^2, \tag{1}$$
$$d\Omega^2 \equiv d\theta^2 + sin^2\theta d\phi^2, \tag{2}$$

together with the stress-energy tensor for dust:

$$T_{ij} = \rho(t, r) u_i u_j = \rho \delta^t{}_i \delta^t{}_j, \tag{3}$$

where$u^i = \delta^i{}_t$ is 4-velocity of dust element and $\rho(t, r)$ the energy density.

With the metric (1), the independent non-vanishing Einstein tensor components are

$$G_{tt} = \frac{[-Re^{2\nu}(2R'\nu' + 2R'' + R'^2R^{-1}) - 2\dot{R}\nu R + 1 + \dot{R}^2]}{R^2}, \tag{4}$$

$$G_{rt} = \frac{-2(\dot{R}' + R'\dot{\nu})}{R}, \tag{5}$$

$$G_{rr} = \frac{e^{-2\nu}(2\ddot{R}R + \dot{R}^2 + 1) - R'^2}{R^2}, \tag{6}$$

$$G_{\theta\theta} = sin^{-2}\theta G_{\phi\phi} = R(\dot{R}\dot{\nu} + R'\nu''e^{2\nu}, + R''e^{2\nu} - \ddot{R} + \ddot{\nu}R - \dot{\nu}^2 R), \tag{7}$$

where dot and prime denote partial differentiation with respect to t and r, respectively.

Introduction new auxiliary functions

$$k(t,r) = e^{2\nu}R'^2 - 1, \tag{8}$$

$$F(t,r) = R(\dot{R}^2 + k). \tag{9}$$

Einstein's equations simplify greatly to

$$\dot{R}^2 = \frac{2m}{R} + k, \tag{10}$$

$$\dot{k} = 0, \tag{11}$$

$$\dot{F} = 0, \tag{12}$$

with the restriction

$$F' = 4\pi R^2 R' T_{tt}. \tag{13}$$

The metric (1) with $e^{2\nu} = [1 + k(r)]/R'^2$, together with Equations (10) and (13), fully determine the Tolman-Bondi family of solutions, and has following parametric solutions;

elliptic, $k < 0$

$$(t - a_0) = \frac{F}{2(-k^{3/2})}(\beta - sin\beta), \quad R = \frac{F}{2(-k)}(1 - cos\beta);$$

parabolic,$k = 0$

$$R = \left[\frac{9F(t - a_0)^2}{4}\right]^{1/3};$$

hyperbolic, $k > 0$

$$R = \frac{F}{2k}(cosh\beta - 1), \quad (sinh\beta - \beta) = \frac{2k^{3/2}(t - a_0)}{F}.$$

They all emerge from the big bang at $t = a(r)$, and expansion rate is positive ,$\dot{R} > 0$, thus the areal radius of each shell of matter at $r = const$ is increasing. In elliptic, expansion reach to the maximum size and then suddenly start collapsing, and terminating in big crunch. in hyperbolic, expansion is continuous and indenfitely, in parabolic model, that is marginal cases, since their extension in vague way decreases to zero at maximum(infinite) time. The hyperbolic and elliptic cases can easily be shown to reduce to the parabolic model form for $\beta \to 0$, i.e. as $t \to a_0$, so that all three cases have the same demeanor at very early times. alike, near the big crunch in elliptic models, when $\beta \to 2\pi$, the behavior approaches that of a collapsing parabolic model. It is entirely possible for all three types of evolution to obtain within different domain in the same model.

The density is given by

$$\rho = \frac{F'}{R^2 R'}, \tag{14}$$

and the Kretschmann scalar is[4]

$$K = R^{hijk}R_{hijk} = -\frac{3F'^2}{R^4 R'^2} - \frac{8F'F}{R^5 R'} + \frac{24m^2}{R^6}, \tag{15}$$

where R_{hijk} is the Riemann tensors.

The functions, F, k, and a_0, are all arbitrary functions of the coordinate radius r. The function $F(r)$ is equal to twice the efficacious mass of matter, m.

3. SHELL-CROSSING SINGULARITY

In the Tolman-Bondi model metric, shell-crossing(weak) singularities are defined by [4]

$$R' = 0 \quad and \quad R > 0. \tag{16}$$

Also, it is known as at $R = 0 = R'$, as shell-focusing singularity occurs, different from shell-crossing singularity, this is central shell focusing singularity and it dose not admit any metric extension through in and the spactime is therefor geodesically incomplete. It has been shown that shell-focusing singularity can be naked [2, 6] and gravitationally strong as finite physical volume are crushed to zero at center singularity [7]. Here the discussion on weak shell-crossing singularity as defined by equation (16). As shell-crossing on which $R' = 0$, and where the density ρ, diverges.

In this section the author use the coordinate freedom left in rescaling the radial coordinate, and we define new scaling function R. Let $a[t]$ be an increasing function with constant C satisfying the condition $C \geq max \mid a[t] \mid^{2n+1}$, therefore new scaling function,

$$R(r,t) = \frac{1}{2n+1}[C + (r - a[t])^{2n+1}], \qquad n = 1, 2, 3, ... \tag{17}$$

For parabolic region $k = 0$, The boundary between an elliptic and a hyperbolic region deserves special consideration since the parameter β is not valid there. For the radial derivative of R with respect to r and t are

$$R'(r,t) = (r - a[t])^{2n}, \tag{18}$$

$$\dot{R}(r,t) = -(r - a[t])^{2n}\dot{a}[t]. \tag{19}$$

Here $a[t]$ is an increasing function and in equation (19), $\dot{R} < 0$, this is collapse condition throughout the evolution with $a[t] > 0$. With the formalism for spherical collapse, we can consider now continual collapse with

$$R(r,t) = \frac{1}{2n+1}[C + (r - a[t])^{2n+1}],$$
$$R'(r,t) = (r - a[t])^{2n}.$$

At t_{max} and $R(r,0) = [C + r^{2n}]$ is always positive at initial epoch.

and

$$r \to 0 \quad as \quad Max \mid a[t] \mid^{2n+1} \to C,$$

this gives genuine strong curvature singularity that can not be removed with any kind of space extension.

Then what we show here by changing scaling function, for arbitrary stage of collapse, corresponding to $R' = (r - a[t])^{2n}$ with $a[t]$ an increasing function, there exists a continues type of shell-crossing singularity.

In the case of marginally bound collapse, the Einstein field equations (10) to (13) they are solved to obtain, $P_\theta = P_r = 0$ and there are solve obtain

$$\dot{R}^2 = \frac{2m}{R} \quad and \quad R' = (r - a[t])^{2n},$$

where F is mass function which is always positive throughout the collapse. Thus the continuous type of shell-crossing singularity and central shell-focusing singularity because from (15), the density keeps diverging continuously.

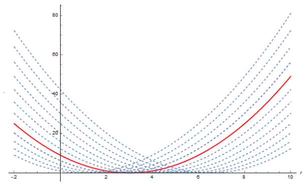

Fig. 1. The behaviour of R', defined in equation (18).

4. CONCLUSION

In the present study of spherically symmetric collapse it was observ that both the shell-crossing and shell-focusing singularities occur simultaneously. The shell-focusing curvature singularity is genuine singularity, where as the shell-crossing singularity can be prevented by excluding cosmological constant. Also the rescaling dose not prevent shell-crossing singularity, that is it occur continuously with the parameter $R(r, t)$. However this is a purely mathematical interpretation can be attached.

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