Stochastic Analysis of Reliability Indices for a Redundant System under Poisson Shocks

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ABSTRACT
This paper studies the effectiveness of repairman on system consisting of two operating repairable units. The system fail due to external factor like Poisson shocks that occur in different times. The arrivals of the shocks follow a Poisson process and the magnitude of a shock is an independent random variable following a known distribution. Repair time, the length of repairman's vacation and recall time are arbitrary distributions. Certain important results have been derived: the reliability, mean time to failure, steady-state availability and steady-state frequency of the system using supplementary variable technique. Special case is derived from the system.

Keywords
Poisson shock, Steady-state availability, Steady-state frequency, Mean time to failure, Supplementary variable technique.

1. INTRODUCTION
A Poisson process is the simplest stochastic process that arises in many applications for arrival processes. It is of great importance to know the properties of a Poisson process and to learn how to apply the process to reliability models. Idea of shock model has been studied in the last decades. For example, Li et al. [1] deal with the study of complex systems consisting of n i.i.d. units with a δ-shock model. M. Salah EL-Sherbeny [2] studied stochastic analysis of a two non-identical unit parallel system with different types of failures subject to preventive maintenance and repairs. Cost (benefit) models in the machine repair problem have been investigated by several authors including M. Salah EL-Sherbeny et al. [3]. Mahmoud and Moshrefa [4] studied the stochastic analysis of a two-component cold standby system considering hardware failure, human error failure and preventive maintenance. Optimal system for series systems have been proposed, for example, optimal system for series systems with mixed standby components by M. Salah EL-Sherbeny [5, 6]. Doshi [7] studied a comprehensive survey on vacation system models. Ke and Wang [8] investigated a machine repair problem with two vacation policies (single vacation and multiple vacations), both of which were based on a queuing theory viewpoint. Goel & Shrivastava [9] studied comparison reliability characteristics of two systems with bivariate exponential lifetimes. Jia and Wu [10] developed a replacement policy for a repairable system with its repairman taking multiple vacations. Yuan and Xu [11] provided a repairable system with a repairman, who can take multiple vacations. Wu and Wu [12] investigated a two-component cold standby repairable system with its repairman taking single vacation and the system might be attached by cumulative shocks. Yutian Chen et al [13] studied two-component cold standby system, in which the operating component may fail due to the intrinsic factors or external factors; besides, the repairman can take vacation. Systems studied are useful for engineers and interested in maintenance in construction models to avoid unnecessary expenses and reduction of safety standards.

This paper interested in studying the effect of the repairman on repairable system consisting of two operating units. Special case is derived and makes comparison with the main system. In the main system the repairman has a choice between go vacation or stay in initial state of this system and when he is on vacation we must recall him in case of the presence of the failure . In special case system the repairman already on vacation in initial state and we must recall him when the failure occurs. The contributions of this paper are twofold. The first is to derive some measures of system effectiveness like the reliability, Mean time to failure, the steady-state failure frequency of the system and the steady-state availability. The second is to study the performance of the repairman on systems is shown graphically.

2. ASSUMPTIONS
The following assumptions are associated with the system

1. The system consists of two operating non-identical parallel units.
2. The system remains operating even if a single unit operates.
3. The system suffers from an external factor like Poisson shocks that occur in different times.
4. The distributions of vacation length of the repairman, recall time of repairman and repair time are arbitrary while the failure time follows the exponential distribution.
5. In the initial state the repairman has a choice between go on vacation or stay idle until the emergence of the first failure.
6. We must recall the repairman from vacation when the failure occurs for any unit.
7. The failure unit will be repaired immediately with the presence of the repairman.
8. The repair rule is first- come- first- service.
9. The arrivals of the shocks follow a Poisson process $\{N(t),\ t \geq 0\}$ with the intensity $\lambda_i > 0$.
10. The quantity of each shock, $\widehat{X}$, is an independent random variable with distribution function *F*.
11. The shock affects the units with manly the unit will fail when values of the shocks exceeds a threshold.
12. The threshold of unit *i* is a non-negative random

variable τ_i with a distribution function ψ_i , (i=1,2) .

13. Suppose $Y_i\ (i=1,\ 2)$ as unit i is repair time, Z as the vacation length of the repairman and K as the recall time of the repairman. their distributions are:

$$H_i(t)=\int_0^t h_i(x)dx = 1-\ell^{-\int_0^t \mu_i(x)dx},$$

$$E(Y_i)=\frac{1}{\mu_i},\quad \overline{H}_i(t)=1-H_i(t)\ \ (i=1,2),$$

$$V_1(t)=\int_0^t v_1(x)dx = 1-\ell^{-\int_0^t \varphi(x)dx}$$

$$E(Z)=\frac{1}{\varphi},\quad \overline{V}_1(t)=1-V_1(t)$$

$$V_2(t)=\int_0^t v_2(x)dx = 1-\ell^{-\int_0^t \alpha(x)dx}$$

$$E(K)=\frac{1}{\alpha},\quad \overline{V}_2(t)=1-V_2(t)$$

respectively.

14. The system is down if the two units fail,
15. After the repair, the unit is as good as new.

3. NOTATION AND STATES OF THE SYSTEM

3.1 Notations

$X_1(t)$	Random variable denoting the elapsed vacation time when the repairman is taking a vacation at time *t*
$X_2(t)$	Random variable denoting the recalling time from vacation at time *t*
$Y_i(t)$	Random variable denoting the elapsed repair time at time *t*, $\forall i = 1,2$.
$h(t),\ H(t)$	p.d.f. and c.d.f. of the repair time
$v_1(t),\ V_1(t)$	p.d.f. and c.d.f. of the vacation time of a repairman
$v_2(t), V_2(t)$	p.d.f. and c.d.f. of the recall time of a repairman
$f(t),\ F(t)$	p.d.f. and c.d.f. of the magnitude of each shock
$P_i(t,x)$, $Q_i(t,x)$	p.d.f. and c.d.f. of the system is in state i=2,4,9 at period t and has an elapsed repair time of x for unit 1
$P_i(t,y)$, $Q_i(t,y)$	p.d.f. and c.d.f. of the system is in state i=1,8,11 at period t and has an elapsed repair time of y for unit 2
$P_0(t,u)$, $Q_0(t,u)$	p.d.f. and c.d.f. of the system is in state S_0 at period *t* and has an elapsed vacation time of *u*
$P_i(t,z)$, $Q_i(t,z)$	p.d.f. and c.d.f. of the system is in state *i*=5,6,7,10 at period *t* and has recalling time of *z*
$\varphi(u)$	Vacation time distribution function
$\mu_1(x)$	The repair rate of unit A
$\mu_2(y)$	The repair rate of unit B
$\alpha(z)$	Recall time distribution function
λ_i	Intensity of shock on unit *i* $\forall i = 1,2$.
r_i	The probability that one shock causes unit i to fail is: $r_i = P(\hat{X} > \tau_i) = \int_0^\infty P(\tau_i < \hat{X} / \hat{X} = \hat{x})dP(\hat{X} \le \hat{x}) = \int_0^\infty \psi_i(\hat{x})dF(\hat{x})$, $(i=1,2)$
$h^*(s)$	Laplace transform of $h(t)$
$p_i(t)$	Probability that the system is in state *i* at time *t*

3.2 Symbols for the states of the system

These symbols are common for two systems.

A_N	Component *A* in normal and operative mode.
B_N	Component *B* in normal and operative mode.
A_r	Component *A* in failure mode due to shock failure and under repair.
B_r	Component *B* in failure mode due to shock failure and under repair.
A_R	Component *A* in repair of failed unit is continued from earlier state.
B_R	Component *B* in repair of failed unit is continued from earlier state.
A_{wr}	Component *A* in failed unit waiting for the repair.

B_{wr}	Component *B* in failed unit waiting for the repair.
A_{WR}	Component *A* in waiting for the repair unit is continued from earlier state.
B_{WR}	Component *B* in waiting for the repair unit is continued from earlier state.

Considering these symbols, the system can be in any of the following states which are the description of system

Up states: $S_0 = (A_N, B_N)$, $S_1 = (A_N, B_r)$, $S_2 = (A_r, B_N)$, $S_5 = (A_N, B_{wr})$, $S_6 = (A_{wr}, B_N)$

Idle state: $S_3 = (A_N, B_N)$

Down states: $S_4 = (A_R, B_{wr})$, $S_7 = (A_{WR}, B_{wr})$, $S_8 = (A_{wr}, B_R)$, $S_9 = (A_r, B_{WR})$, $S_{10} = (A_{wr}, B_{WR})$, $S_{11} = (A_{WR}, B_r)$.

Figure 1 describes the state transition of the system according to a Markov chain, where the states 0, 1, 2, 5 and 6 are working states; while the states 4, 7,…, 11 are failure states and the state 3 is idle state.

4. CALCULATIONS OF SYSTEM

4.1 The steady-state availability

This section investigates availability behavior of the system. Using supplementary variable technique

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial u} + \lambda_1 r_1 + \lambda_2 r_2 + \varphi(u)\right) P_0(u,t) = 0, \quad (1)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \lambda_1 r_1 + \mu_2(y)\right) P_1(y,t) = 0, \quad (2)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \lambda_2 r_2 + \mu_1(x)\right) P_2(x,t) = 0, \quad (3)$$

$$\left(\frac{d}{dt} + \lambda_1 r_1 + \lambda_2 r_2\right) p_3(t) = \int_0^\infty P_0(u,t)\varphi(u)du, \quad (4)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_1(x)\right) P_4(x,t) = \lambda_2 r_2 P_2(x,t), \quad (5)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + \lambda_1 r_1 + \alpha(z)\right) P_5(z,t) = 0, \quad (6)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + \lambda_2 r_2 + \alpha(z)\right) P_6(z,t) = 0, \quad (7)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + \alpha(z)\right) P_7(z,t) = \lambda_2 r_2 P_6(z,t) \quad (8)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \mu_2(y)\right) P_8(y,t) = \lambda_1 r_1 P_1(y,t) \quad (9)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_1(x)\right) P_9(x,t) = 0 \quad (10)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + \alpha(z)\right) p_{10}(z,t) = \lambda_1 r_1 p_5(z,t) \quad (11)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \mu_2(y)\right) P_{11}(y,t) = 0 \quad (12)$$

The boundary conditions are:

$$P_0(t,0) = \int_0^\infty P_1(y,t)\mu_2(y)dy + \int_0^\infty P_2(x,t)\mu_1(x)dx + \delta(t) \quad (13)$$

$$\begin{aligned} P_1(t,0) = &\int_0^\infty P_0(u,t)\lambda_2 r_2 du + \int_0^\infty P_4(x,t)\mu_1(x)dx \\ &+ \int_0^\infty P_5(z,t)\alpha(z)dz + \int_0^\infty P_9(x,t)\mu_1(x)dx \end{aligned} \quad (14)$$

$$\begin{aligned} P_2(t,0) = &\int_0^\infty P_0(u,t)\lambda_1 r_1 du + \int_0^\infty P_8(y,t)\mu_2(y)dy \\ &+ \int_0^\infty P_{11}(y,t)\mu_2(y)dy + \int_0^\infty P_6(z,t)\alpha(z)dz \end{aligned} \quad (15)$$

$$P_5(t,0) = p_3(t)\lambda_2 r_2 \quad (16)$$

$$P_6(t,0) = p_3(t)\lambda_1 r_1 \quad (17)$$

$$P_9(t,0) = \int_0^\infty P_7(z,t)\alpha(z)dz \quad (18)$$

$$P_{11}(t,0) = \int_0^\infty P_{10}(z,t)\alpha(z)dz \quad (19)$$

$$P_i(t,0) = 0, \quad (i = 4, 7, 8, 10) \quad (20)$$

Using the following normalizing condition:

$$\sum_{i=0}^{2}\int_0^\infty P_i(t,m)dm + p_3(t) + \sum_{i=4}^{11}\int_0^\infty P_i(t,m)dm = 1, \quad \forall m = (u, x, y, z) \quad (21)$$

Initial conditions are defined as

$$P_0(0,u) = \delta(u) = \begin{cases} 1 & u = 0 \\ 0 & u \neq 0 \end{cases},$$

$P_i(0,w)=0, \quad w \neq 0, \forall w = (x, y, z),$
$i = 1, \ldots, 11, \quad p_3(0)=0$

Taking the limit $t \rightarrow \infty$ in the equations (1) – (21), we obtain the following equations:

$$\left(\frac{d}{du}+\lambda_1 r_1+\lambda_2 r_2+\varphi(u)\right) g_0(u)=0 \tag{22}$$

$$\left(\frac{d}{dy}+\lambda_1 r_1+\mu_2(y)\right) g_1(y)=0 \tag{23}$$

$$\left(\frac{d}{dx}+\lambda_2 r_2+\mu_1(x)\right) g_2(x)=0 \tag{24}$$

$$(\lambda_1 r_1+\lambda_2 r_2) p_3=\int_0^\infty g_0(u)\varphi(u)du \tag{25}$$

$$\left(\frac{d}{dx}+\mu_1(x)\right) g_4(x)=\lambda_2 r_2 g_2(x) \tag{26}$$

$$\left(\frac{d}{dz}+\lambda_1 r_1+\alpha(z)\right) g_5(z)=0 \tag{27}$$

$$\left(\frac{d}{dz}+\lambda_2 r_2+\alpha(z)\right) g_6(z)=0 \tag{28}$$

$$\left(\frac{d}{dz}+\alpha(z)\right) g_7(z)=\lambda_2 r_2 g_6(z) \tag{29}$$

$$\left(\frac{d}{dy}+\mu_2(y)\right) g_8(y)=\lambda_1 r_1 g_1(y) \tag{30}$$

$$\left(\frac{d}{dx}+\mu_1(x)\right) g_9(x)=0 \tag{31}$$

$$\left(\frac{d}{dz}+\alpha(z)\right) g_{10}(z)=\lambda_1 r_1 g_5(z) \tag{32}$$

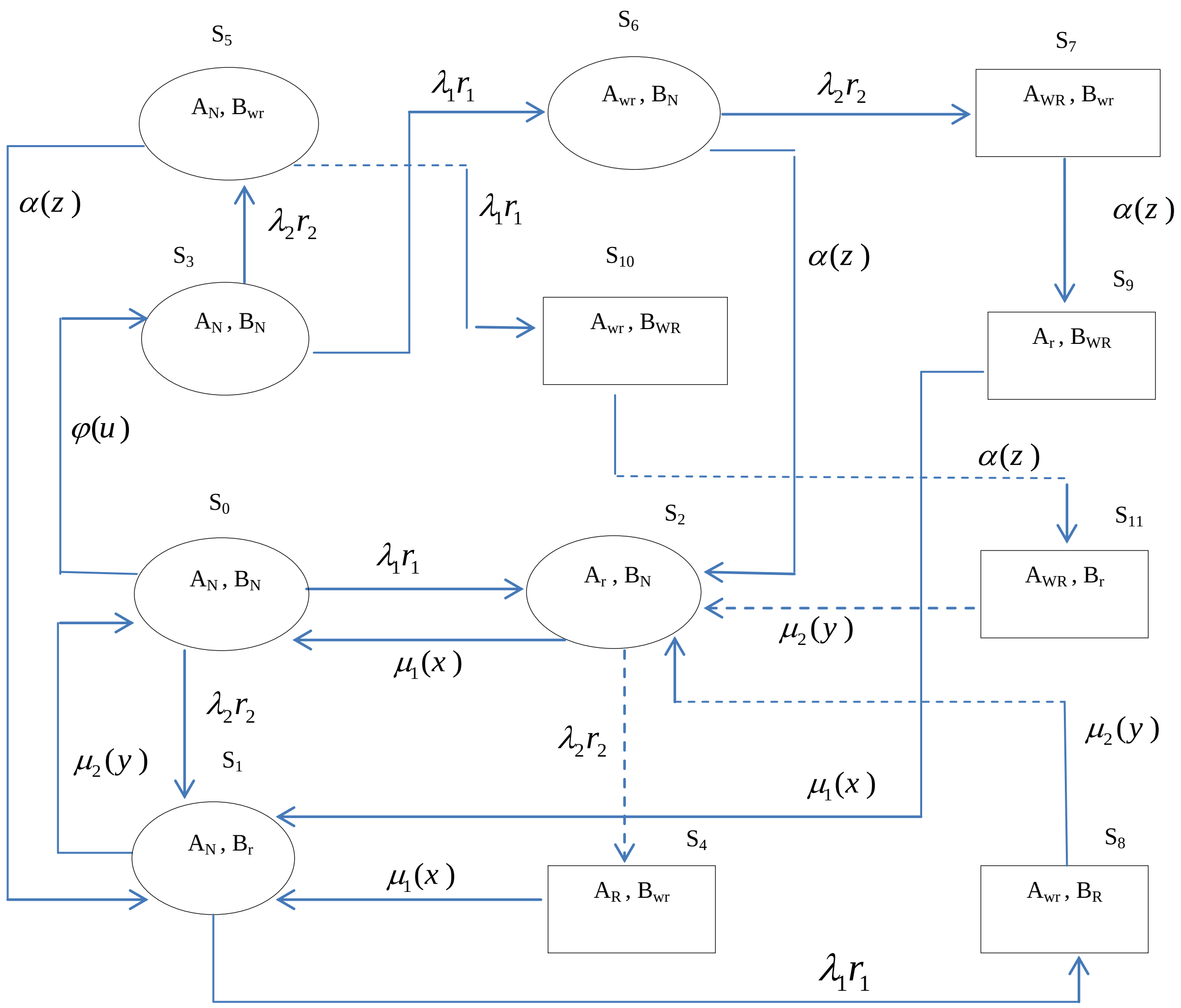


Fig 1: State transition diagram of the system

$$\left(\frac{d}{dy}+\mu_2(y)\right)g_{11}(z)=0 \tag{33}$$

The boundary conditions are:

$$g_0(0)=\int_0^\infty g_1(y)\mu_2(y)dy+\int_0^\infty g_2(x)\mu_1(x)dx \tag{34}$$

$$g_1(0)=\int_0^\infty g_0(u)\lambda_2 r_2\,du+\int_0^\infty g_4(x)\mu_1(x)dx$$
$$+\int_0^\infty g_5(z)\alpha(z)dz+\int_0^\infty g_9(x)\mu_1(x)dx \tag{35}$$

$$g_2(0)=\int_0^\infty g_0(u)\lambda_1 r_1 du+\int_0^\infty g_6(z)\alpha(z)dz$$
$$+\int_0^\infty g_8(y)\mu_2(y)dy+\int_0^\infty g_{11}(y)\mu_2(y)dy \tag{36}$$

$$g_9(0)=\int_0^\infty g_7(z)\alpha(z)dz\,,\quad g_{11}(0)=\int_0^\infty g_{10}(z)\alpha(z)dz \tag{37}$$

$$g_5(0)=p_3\lambda_2 r_2,\quad g_6(0)=p_3\lambda_1 r_1, g_i(0)=0,$$
$$\forall i = 4,\ 7,\ 8,\ 10. \tag{38}$$

$$\sum_{i=0}^{2}\int_0^\infty g_i(m)dm+p_3+\sum_{i=4}^{11}\int_0^\infty g_i(m)dm=1.$$
$$\forall m = (u,\ x,\ y,\ z) \tag{39}$$

where, $g_i(m)=\lim_{t\to\infty} p_i(t,m),\ i = 0,1,2,4,...,11$ which follows the following relations: $p_i = \int_0^\infty g_i(m)\ dm$.

We get the solutions $g_i(m), p_3$ of the above equations (22) - (38). Using equation (39) we get $p_0,..., p_{11}$. (see Appendix A)

Hence, the steady-state availability of the system is

$$A(\infty)=\sum_{i=0}^{3}p_i+\sum_{i=5}^{6}p_i \tag{40}$$

4.2 The steady-state probability that the repair man is on vacation is

$$P_{rv}=\sum_{i=5}^{7}p_i+p_{10} \tag{41}$$

4.3 The steady-state probability that the system is waiting for being repaired is:

$$P_{rw}=p_7+p_{10} \tag{42}$$

4.4 The steady-state failure frequency is

$$M=\lambda_1 r_1\ p_1+\lambda_2 r_2\ p_2+\lambda_1 r_1\ p_5+\lambda_2 r_2\ p_6 \tag{43}$$

4.5 Mean time to failure of the system

4.5.1 The reliability of the system

Let the failure states 4, 7, 8, 9, 10, 11 be the absorbing states, and then we have another vector Markov process $\left\{\tilde{S}(t),\ \tilde{X}_1(t),\ \tilde{X}_2(t),\ \tilde{Y}_1(t),\ \tilde{Y}_2(t),\ t\ge 0\right\}$.

Let: $\pi_0(t,u)=\frac{d}{du}P(\tilde{S}(t)=0,\ \ \tilde{X}_1(t)\le u)$,

$$\pi_1(t,y)=\frac{d}{dy}P(\tilde{S}(t)=1,\quad \tilde{Y}_2(t)\le y),$$

$$\pi_2(t,x)=\frac{d}{dx}P(\tilde{S}(t)=2,\quad \tilde{X}_2(t)\le x),$$

$$\pi_3(t)=P(\tilde{S}(t)=3),$$

$$\pi_i(t,z)=\frac{d}{dz}P(\tilde{S}(t)=i,\ \tilde{X}_2\le z),\ \ (i = 5,\ 6)$$

Using the same method in section 4.1, we have the following partial- differential equations:

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial u}+\lambda_1 r_1+\lambda_2 r_2+\varphi(u)\right)\pi_0(t,u)=0 \tag{44}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\lambda_1 r_1+\mu_1(y)\right)\pi_1(t,y)=0 \tag{45}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_2 r_2+\mu_2(x)\right)\pi_2(t,x)=0 \tag{46}$$

$$\left(\frac{d}{dt}+\lambda_1 r_1+\lambda_2 r_2\right)\pi_3(t)=\int_0^\infty \pi_0(t,u)\varphi(u)du \tag{47}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\lambda_1 r_1+\alpha(z)\right)\pi_5(t,z)=0 \tag{48}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\lambda_2 r_2+\alpha(z)\right)\pi_6(t,z)=0 \tag{49}$$

The boundary conditions are:

$$\pi_0(t,0)=\int_0^\infty \pi_1(t,y)\mu_1(y)dy+\int_0^\infty \pi_2(t,x)\mu_2(x)dx+\delta t \tag{50}$$

$$\pi_1(t,0)=\int_0^\infty \pi_0(t,u)\lambda_2 r_2 du+\int_0^\infty \pi_5(t,z)\alpha(z)dz \tag{51}$$

$$\pi_2(t,0)=\int_0^\infty \pi_0(t,u)\lambda_1 r_1 du+\int_0^\infty \pi_6(t,z)\alpha(z)dz \tag{52}$$

$$\pi_5(t,0)=\pi_3(t)\lambda_2 r_2\ ,\ \ \pi_6(t,0)=\pi_3(t)\lambda_1 r_1 \tag{53}$$

The initial conditions are:

$$\pi_0(0,u)=\delta(u)=\begin{cases}1, & \text{u=0}\\ 0, & \text{u}\neq 0\end{cases},\ \text{otherwise is 0}$$

$$\frac{d}{du}\pi_0^*(s,u)+\left(s+\lambda_1 r_1+\lambda_2 r_2+\varphi(u)\right)\pi_0^*(s,u)=0 \tag{54}$$

$$\frac{d}{dy}\pi_1^*(s,y)+\left(s+\lambda_1 r_1+\mu_1(y)\right)\pi_1^*(s,y)=0 \tag{55}$$

$$\frac{d}{dx}\pi_2^*(s,x)+\left(s+\lambda_2 r_2+\mu_2(x)\right)\pi_2^*(s,x)=0 \tag{56}$$

$$(s+\lambda_1 r_1+\lambda_2 r_2)\pi_3^*(s)=\int_0^\infty \pi_0^*(s,u)\varphi(u)du \tag{57}$$

$$\frac{d}{dz}\pi_5^*(s,z)+\left(s+\lambda_1 r_1+\alpha(z)\right)\pi_5^*(s,z)=0 \tag{58}$$

$$\frac{d}{dz}\pi_6^*(s,z)\left(s+\lambda_2 r_2+\alpha(z)\right)\pi_6^*(s,z)=0 \tag{59}$$

$$\pi_0^*(s,0)=\int_0^\infty \pi_1^*(s,y)\mu_2(y)dy+\int_0^\infty \pi_2^*(s,x)\mu_1(x)dx+1 \tag{60}$$

$$\pi_1^*(s,0)=\int_0^\infty \pi_0^*(s,u)\lambda_2 r_2 du+\int_0^\infty \pi_5^*(s,z)\alpha(z)dz\ , \tag{61}$$

$$\pi_2^*(s,0)=\int_0^\infty \pi_0^*(s,u)\lambda_1 r_1 du,\ \pi_5^*(s,0)=\pi_3^*(s)\lambda_2 r_2,\quad \pi_6^*(s,0)=\pi_3^*(s)\lambda_1 r_1 \tag{62}$$

According to the initial conditions, we obtain: $\pi_3^*(0)=0$.

From the solutions of the equations (54-62), We obtain $\pi_0^*(s,u)$, $\pi_1^*(s,y)$, $\pi_2^*(s,x)$, $\pi_3^*(s)$, $\pi_5^*(s,z)$, $\pi_6^*(s,z)$. (see Appendix A):

The reliability of the system can be defined as:

$$R(t)=\int_0^\infty \pi_0(t,u)du+\int_0^\infty \pi_1(t,y)dy+\int_0^\infty \pi_2(t,x)dx+\pi_3(t)+\sum_{i=5}^{6}\int_0^\infty \pi_i(t,z)dz\ . \tag{63}$$

In order to get the reliability of the system, we take The Laplace transformation formula of equation (62) :

$$R^*(s)=\int_0^\infty \pi_0^*(s,u)du+\int_0^\infty \pi_1^*(s,y)dy+\int_0^\infty \pi_2^*(s,x)dx+\pi_3^*(s)+\sum_{i=5}^{6}\int_0^\infty \pi_i^*(s,z)dz\ . \tag{64}$$

4.5.2 The mean time to failure of the system as follows:

$$MTTF=\int_0^\infty R(t)dt=\lim_{s\to 0^+}R^*(s) \tag{65}$$

5. SPECIAL CASES

In this section, we assume that the Vacation time distribution function is not found, $\varphi(u)=0$,

1. The Vacation time distribution function is not found, $\varphi(u)=0$
2. The state S_0 is neglected from the system, therefore the transpose from S_1 and S_2 move to S_3

That is mean we have a new system which characterizes by the repairman already on vacation when two units are operating. We can describe the system's equations like the other system.

From these changes and using the same method, we get $p_0,\dots,p_{10}$ and some reliability indices

The steady-state availability is

$$A_s(\infty)=\sum_{i=1}^{3}p_i+\sum_{i=5}^{6}p_i \tag{66}$$

The steady-state probability that the repair man is on vacation is

$$P_{vs}=\sum_{i=5}^{7}p_i+p_{10} \tag{67}$$

The steady-state probability that the system is waiting for being repaired is

$$P_{ws}=P_7+P_{10} \tag{68}$$

The steady-state failure frequency is

$$M_s=\lambda_1 r_1 p_1+\lambda_2 r_2 p_2+\lambda_1 r_1 p_5+\lambda_2 r_2 p_6 \tag{69}$$

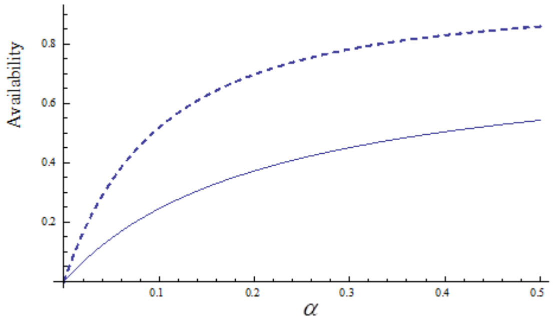
The reliability of the system can be defined as:

$$R_s^*(s)=\sum_{i=1}^{2}\int_0^\infty L_i^*(s,x)dx+L_3^*(s,y)+\sum_{i=5}^{6}\int_0^\infty L_i^*(s,z)dz \tag{70}$$

The mean time to failure of system as follows:

$$MTTF_s=\int_0^\infty R_s(t)dt=\lim_{s\to 0^+}R_s^*(s) \tag{71}$$

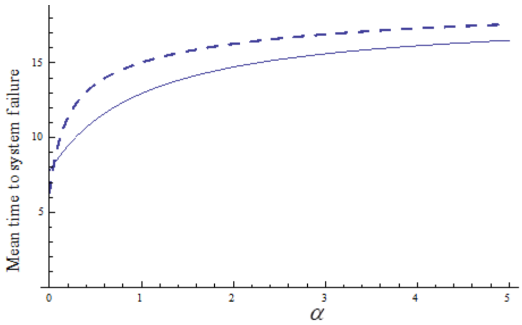
Finally, we make comparative analysis between two systems (see Figs. 2 and 3)



---- System where the repairman has choice between stay or take vacation in initial state

—— Special case where the repairman is on vacation in initial state

Fig 2: Effect of α on availability



---- System where the repairman has choice between stay or take vacation in initial state

—— Special case where the repairman is on vacation in initial state

Fig 3: Effect of α on Mean time to system failure

6. CONCLUSION

In this paper we studied two repairable systems which influenced by repairman. The repairman has multiple choices either stays or take vacation in initial state, therefore we note the effect of repairman on perform of the system. Explicit expressions for the system availability, MTSF, The steady-state probability that the repair man is on vacation and the steady-state failure frequency for two the systems were derived. After comparative analysis via Figs. 2 and 3, we deduced the system that the repairman stay in initial state is better than the system whose repairman takes vacation in initial state. We recommend that the repairman have to exist in the beginning of the operation.

7. APPENDIX A

This section investigates some reliability indices of the system using supplementary variable technique. From equations (22) -(38) and (39) we get $p_0,\dots,p_{11}$.

$$p_0=((r_1\lambda_1+r_2\lambda_2)\ C_0\ \bar{V}_2^*(r_1\lambda_1+r_2\lambda_2)\ (h_1^*(r_2\lambda_2)$$
$$(-1+h_2^*(r_1\lambda_1))-h_2^*(r_1\lambda_1)))$$
$$/((r_1\lambda_1+r_2\lambda_2)(-1+r_1\lambda_1\ \bar{V}_2^*(r_1\lambda_1+r_2\lambda_2)h_1^*(r_2\lambda_2))$$
$$+h_1^*(r_2\lambda_2)(-r_2\lambda_2(-1+v_1^*(r_1\lambda_1))+r_1\lambda_1\ v_1^*(r_2\lambda_2))$$
$$v_2^*(r_1\lambda_1+r_2\lambda_2))$$

$$p_1=C_0\ \bar{H}_2^*(r_1\lambda_1)$$

$$p_2=\ C_0\ \bar{H}_1^*(r_2\lambda_2)((-r_1\lambda_1-r_2\lambda_2)(1+(-1$$
$$+r_1\lambda_1\ \bar{V}_2^*(r_1\lambda_1+r_2\lambda_2))\ h_2^*(r_1\lambda_1))-h_2^*(r_1\lambda_1)(r_2\lambda_2$$
$$(1-v_1^*(r_1\lambda_1))+r_1\lambda_1\ v_1^*(r_2\lambda_2))\ v_2^*(r_1\lambda_1+r_2\lambda_2))$$
$$/((r_1\lambda_1+r_2\lambda_2)(-1+r_1\lambda_1\ \bar{V}_2^*(r_1\lambda_1+r_2\lambda_2)$$
$$(h_1^*(r_2\lambda_2))+h_1^*(r_2\lambda_2)(r_2\lambda_2(1-v_1^*(r_1\lambda_1))$$
$$+r_1\lambda_1 v_1^*(r_2\lambda_2))\ v_2^*(r_1\lambda_1+r_2\lambda_2))$$

$$p_3=C_0(h_1^*(r_2\lambda_2)(-1+h_2^*(r_1\lambda_1))-h_2^*(r_1\lambda_1))v_2^*(r_1\lambda_1$$
$$+r_2\lambda_2)\ \ /((r_1\lambda_1+r_2\lambda_2)(-1+r_1\lambda_1\bar{V}_2^*(r_1\lambda_1+r_2\lambda_2)$$
$$h_1^*(r_2\lambda_2))+h_1^*(r_2\lambda_2)(-r_2\lambda_2(-1+v_1^*(r_1\lambda_1))+r_1\lambda_1 v_1^*$$
$$(r_2\lambda_2))v_2^*(r_1\lambda_1+r_2\lambda_2))$$

$$p_4=C_0\ \ (-1+\mu_1\ \bar{H}_1^*(r_2\lambda_2))((r_1\lambda_1+r_2\lambda_2)(1+(-1$$
$$+r_1\lambda_1\ \bar{V}_2^*(r_1\lambda_1+r_2\lambda_2))h_2^*(r_1\lambda_1))+h_2^*(r_1\lambda_1)(-r_2\lambda_2$$
$$(-1+v_1^*(r_1\lambda_1))+r_1\lambda_1\ v_1^*(r_2\lambda_2))\ v_2^*(r_1\lambda_1+r_2\lambda_2))$$
$$/\mu_1((r_1\lambda_1+r_2\lambda_2)(-1+r_1\lambda_1\ \bar{V}_2^*(r_1\lambda_1+r_2\lambda_2)$$
$$h_1^*(r_2\lambda_2))+h_1^*(r_2\lambda_2)(-r_2\lambda_2(-1+v_1^*(r_1\lambda_1))$$
$$+r_1\lambda_1 v_1^*(r_2\lambda_2))\ v_2^*(r_1\lambda_1+r_2\lambda_2))$$

$$p_5=r_2\lambda_2\ C_0\ \bar{V}_1^*(r_1\lambda_1)(-h_1^*(r_2\lambda_2)(-1+h_2^*(r_1\lambda_1))+h_2^*(r_1\lambda_1))$$
$$v_2^*(r_1\lambda_1+r_2\lambda_2)\ \ /(-(r_1\lambda_1+r_2\lambda_2)+r_1\lambda_1\ \bar{V}_2^*(r_1\lambda_1+r_2\lambda_2)$$
$$(-1\ h_1^*(r_2\lambda_2))+h_1^*(r_2\lambda_2)(r_2\lambda_2(-1+v_1^*(r_1\lambda_1))$$
$$-r_1\lambda_1\ v_1^*(r_2\lambda_2))\ v_2^*(r_1\lambda_1+r_2\lambda_2))$$

$$p_6=r_1\lambda_1\ C_0\ \bar{V}_1^*(r_2\lambda_2)\ v_2^*(r_1\lambda_1+r_2\lambda_2)(h_1^*(r_2\lambda_2)$$
$$(-1+h_2^*(r_1\lambda_1))-h_2^*(r_1\lambda_1))\ \ /((r_1\lambda_1+r_2\lambda_2)$$
$$(-1+r_1\lambda_1\ \bar{V}_2^*(r_1\lambda_1+r_2\lambda_2)\ h_1^*(r_2\lambda_2))+h_1^*(r_2\lambda_2)$$
$$(-r_2\lambda_2(-1+v_1^*(r_1\lambda_1))+r_1\lambda_1\ v_1^*(r_2\lambda_2)\ v_2^*(r_1\lambda_1+r_2\lambda_2))$$

$$p_7=-r_1\lambda_1\ C_0\ \ (-1+\alpha_1\ \bar{V}_1^*(r_2\lambda_2))(h_1^*(r_2\lambda_2)(-1$$
$$+h_2^*(r_1\lambda_1))-h_2^*(r_1\lambda_1))\ v_2^*(r_1\lambda_1+r_2\lambda_2)$$
$$/\alpha_1((r_1\lambda_1+r_2\lambda_2)(-1+r_1\lambda_1\ \bar{V}_2^*(r_1\lambda_1+r_2\lambda_2)\,h_1^*(r_2\lambda_2))$$
$$+h_1^*(r_2\lambda_2)(-r_2\lambda_2(-1+\ v_1^*(r_1\lambda_1))+r_1\lambda_1\ v_1^*(r_2\lambda_2))$$
$$v_2^*(r_1\lambda_1+r_2\lambda_2))$$

$$p_8 = C_0\left(\frac{1}{\mu_2} - \bar{H}_2^*(r_1\lambda_1)\right)$$

$$p_9 = -r_1\lambda_1\ C_0\ (h_1^*(r_2\lambda_2)(-1+h_2^*(r_1\lambda_1)) - h_2^*(r_1\lambda_1))(-1 + v_1^*(r_2\lambda_2))\ v_2^*(r_1\lambda_1+r_2\lambda_2)\ /\mu_1((r_1\lambda_1+r_2\lambda_2)(-1+r_1\lambda_1\ \bar{V}_2^*(r_1\lambda_1+r_2\lambda_2)\ h_1^*(r_2\lambda_2)) + h_1^*(r_2\lambda_2)(-r_2\lambda_2(-1 + v_1^*(r_1\lambda_1)) + r_1\lambda_1\ v_1^*(r_2\lambda_2))\ v_2^*(r_1\lambda_1+r_2\lambda_2))$$

$$p_{10} = -r_2\lambda_2\ C_0(-1+\alpha_1\ \bar{V}_1^*(r_1\lambda_1))(h_1^*(r_2\lambda_2)(-1 + h_2^*(r_1\lambda_1)) - h_2^*(r_1\lambda_1))\ v_2^*(r_1\lambda_1+r_2\lambda_2)\ /\alpha_1((r_1\lambda_1 + r_2\lambda_2)(-1+r_1\lambda_1\ \bar{V}_2^*(r_1\lambda_1+r_2\lambda_2)\ h_1^*(r_2\lambda_2)) + h_1^*(r_2\lambda_2)(-r_2\lambda_2(-1+v_1^*(r_1\lambda_1)) + r_1\lambda_1\ v_1^*(r_2\lambda_2))\ v_2^*(r_1\lambda_1+r_2\lambda_2))$$

$$p_{11} = -r_2\lambda_2\ C_0\ (h_1^*(r_2\lambda_2)(-1+h_2^*(r_1\lambda_1)) - h_2^*(r_1\lambda_1))(-1+v_1^*(r_1\lambda_1))\ v_2^*(r_1\lambda_1+r_2\lambda_2)\ /\mu_2((r_1\lambda_1+r_2\lambda_2)(-1 + r_1\lambda_1\ \bar{V}_2^*(r_1\lambda_1+r_2\lambda_2)\ h_1^*(r_2\lambda_2)) + h_1^*(r_2\lambda_2)(-r_2\lambda_2(-1 + v_1^*(r_1\lambda_1)) + r_1\lambda_1\ v_1^*(r_2\lambda_2))\ v_2^*(r_1\lambda_1+r_2\lambda_2))$$

$$C_0 = \Big\{(r_1\lambda_1+r_2\lambda_2)\ (-1+r_1\ \lambda_1\ \bar{V}_2^*(r_1\lambda_1+r_2\lambda_2)h_1^*(r_2\lambda_2)) + h_1^*(r_2\lambda_2)\ (-r_2\lambda_2(-1+v_1^*(r_1\lambda_1)) + r_1\lambda_1\ v_1^*(r_2\lambda_2))\ v_2^*(r_1\lambda_1+r_2\lambda_2)\Big\} \Big/ \Big\{\frac{1}{\mu_1\mu_2}(r_1\lambda_1+r_2\lambda_2)\ (-\mu_2+\mu_1(-1 + (r_1\lambda_1-\mu_2)\bar{V}_2^*(r_1\lambda_1+r_2\lambda_2)\ h_1^*(r_2\lambda_2)) + \mu_2(1 - \bar{V}_2^*(r_1\lambda_1+r_2\lambda_2)(r_1\lambda_1-\mu_1(-1+h_1^*(r_2\lambda_2))))\ h_2^*(r_1\lambda_1)) - \left(-\frac{r_2\lambda_2 h_2^*\ (r_1\lambda_1)(-1+v_1^*(r_1\lambda_1))}{\mu_1} + (1-h_1^*(r_2\lambda_2))h_2^*\ (r_1\lambda_1)\ (r_1\lambda_1(\frac{1}{\alpha_1}+\frac{1}{\mu_1}) + \frac{\alpha_1\mu_2 + r_2\lambda_2(\alpha_1+\mu_2-\alpha_1\ v_2^*(r_1\lambda_1))}{\alpha_1\mu_2}\right) + \frac{h_1^*(r_2\lambda_2)(\alpha_1+r_2\lambda_2)\mu_1}{\alpha_1\mu_1} + \frac{h_1^*(r_2\lambda_2)r_1\lambda_1(\mu_1-\alpha_1(-1+v_1^*(r_2\lambda_2)))}{\alpha_1\mu_1} + r_1\lambda_1\ h_1^*(r_2\lambda_2)\left(-\frac{1}{\mu_2}+\frac{h_2^*(r_1\lambda_1)}{\mu_1}\right)\ v_1^*(r_2\lambda_2))\ v_2^*(r_1\lambda_1+r_2\lambda_2))\Big\}$$

The steady-state availability of the system is

$$A(\infty) = \sum_{i=0}^{3} p_i + \sum_{i=5}^{6} p_i$$

$$A(\infty) = (C_0((h_1^*(r_2\lambda_2)(-1+h_2^*(r_1\lambda_1)) - h_2^*(r_1\lambda_1))v_2^*(r_1\lambda_1 + r_2\lambda_2) + r_1^2\lambda_1^2\ \bar{V}_2^*(r_1\lambda_1+r_2\lambda_2)(-h_2^*(r_1\lambda_1)\ \bar{H}_1^*(r_2\lambda_2) + h_1^*(r_2\lambda_2)\bar{H}_2^*(r_1\lambda_1)) + r_2\lambda_2(\ \bar{V}_2^*(r_1\lambda_1+r_2\lambda_2)\ (h_1^*(r_2\lambda_2)(-1+h_2^*(r_1\lambda_1)) - h_2^*(r_1\lambda_1)) + \bar{V}_1^*(r_1\lambda_1)\ (-h_1^*(r_2\lambda_2) + (-1+h_1^*(r_2\lambda_2))\ h_2^*(r_1\lambda_1))\ v_2^*(r_1\lambda_1 + r_2\lambda_2) + (-1+h_2^*(r_1\lambda_1)(1+(-1+v_1^*(r_1\lambda_1))\ v_2^*(r_1\lambda_1 + r_2\lambda_2)))\ \bar{H}_1^*(r_2\lambda_2) + (-1-h_1^*(r_2\lambda_2)(-1+v_1^*(r_1\lambda_1))\ v_2^*(r_1\lambda_1+r_2\lambda_2))\ \bar{H}_2^*(r_1\lambda_1)) + r_1\lambda_1(\ \bar{V}_1^*(r_2\lambda_2)(-h_1^*(r_2\lambda_2) + (-1+h_1^*(r_2\lambda_2))\ h_2^*(r_1\lambda_1))\ v_2^*(r_1\lambda_1+r_2\lambda_2) + (-1 + h_2^*(r_1\lambda_1)(1-v_1^*(r_2\lambda_2)\ v_2^*(r_1\lambda_1+r_2\lambda_2)))\bar{H}_1^*(r_2\lambda_2) + (-1 + h_1^*(r_2\lambda_2)\ v_1^*(r_2\lambda_2)\ v_2^*(r_1\lambda_1+r_2\lambda_2))\ \bar{H}_2^*(r_1\lambda_1) + \bar{V}_2^*(r_1\lambda_1 + r_2\lambda_2)(-h_2^*(r_1\lambda_1)(1+r_2\lambda_2\ \bar{H}_1^*(r_2\lambda_2)) + h_1^*(r_2\lambda_2)(-1 + h_2^*(r_1\lambda_1) + r_2\lambda_2\ \bar{H}_2^*(r_1\lambda_1))))))\Big/\Big\{((r_1\lambda_1+r_2\lambda_2)(-1+r_1\lambda_1\ \bar{V}_2^*(r_1\lambda_1+r_2\lambda_2)\ h_1^*(r_2\lambda_2)) + h_1^*(r_2\lambda_2)(-r_2\lambda_2(-1 + v_1^*(r_1\lambda_1)) + r_1\lambda_1\ v_1^*(r_2\lambda_2))\ v_2^*(r_1\lambda_1+r_2\lambda_2)\Big\}$$

Taking the Laplace transform with respect to t to equations (54 - 62), we get

$$\pi_0^*(s,u) = e^{-u(s+r_1\lambda_1+r_2\lambda_2)}\pi_0^*(s,0)\ \bar{V}_2(u),$$

$$\pi_1^*(s,y) = e^{-y(s+r_1\lambda_1)}\pi_1^*(s,0)\bar{H}_2(y),$$

$$\pi_2^*(s,x) = e^{-x(s+r_2\lambda_2)}\pi_2^*(s,0)\bar{H}_1(x),$$

$$\pi_3^*(s) = \frac{\pi_0^*(s,0)v_2(s+r_1\lambda_1+r_2\lambda_2)}{s+r_1\lambda_1+r_2\lambda_2}$$

$$\pi_5^*(s,z) = e^{-z(s+r_1\lambda_1)}\pi_5^*(s,0)\ \bar{V}_1(z),$$

$$\pi_6^*(s,z) = e^{-z(s+r_2\lambda_2)}\pi_6^*(s,0)\ \bar{V}_1(z)$$

$$\pi_0^*(s,0) = ((s+r_1\lambda_1+r_2\lambda_2)(1+C_1(s)h_2^*(s+r_1\lambda_1))) /(s+r_2\lambda_2+r_1\lambda_1(1-(v_1(s+r_2\lambda_2)v_2(s+r_1\lambda_1+r_2\lambda_2) + (s+r_1\lambda_1+r_2\lambda_2)\bar{V}_2(s+r_1\lambda_1+r_2\lambda_2))h_1^*(s+r_2\lambda_2)(1 + C_1(s)h_2^*(s+r_1\lambda_1))))$$

$$\pi_1^*(s,0) = ((s+r_1\lambda_1+r_2\lambda_2)\ C_1(s))\big/(s+r_2\lambda_2+r_1\lambda_1 (1-(v_1(s+r_2\lambda_2)\ v_2(s+r_1\lambda_1+r_2\lambda_2) + (s+r_1\lambda_1+r_2\lambda_2) \bar{V}_2(s+r_1\lambda_1+r_2\lambda_2))\ h_1^*(s+r_2\lambda_2)(1+C_1(s)\ h_2^*(s+r_1\lambda_1))))$$

$$\pi_2^*(s,0) = (r_1\lambda_1(-\frac{v_1(s+r_2\lambda_2)\ v_2(s+r_1\lambda_1+r_2\lambda_2)}{s+r_1\lambda_1+r_2\lambda_2} - \bar{V}_2(s+r_1\lambda_1+r_2\lambda_2))(1+C_1(s)\ h_2^*(s+r_1\lambda_1)))$$

$$\Big/ \left(-1+\frac{1}{s+r_1\lambda_1+r_2\lambda_2} r_1\lambda_1 (v_1(s+r_2\lambda_2) v_2(s+r_1\lambda_1+r_2\lambda_2)\right.$$
$$+(s+r_1\lambda_1+r_2\lambda_2)\ \bar{V}_2(s+r_1\lambda_1+r_2\lambda_2))h_1^*(s+r_2\lambda_2)$$
$$(1+C_0(s)\ h_2^*(s+r_1\lambda_1)))$$

$$\pi_3^*(s,0)=(v_2(s+r_1\lambda_1+r_2\lambda_2)(1+C_1(s)h_2^*(s+r_1\lambda_1)))$$
$$/(s+r_2\lambda_2+r_1\lambda_1(1-(v_1(s+r_2\lambda_2)$$
$$v_2(s+r_1\lambda_1+r_2\lambda_2)+(s+r_1\lambda_1+r_2\lambda_2)\ \bar{V}_2(s+r_1\lambda_1+r_2\lambda_2))$$
$$h_1^*(s+r_2\lambda_2)(1+C_1(s)\ h_2^*(s+r_1\lambda_1))))$$

$$\pi_5^*(s,0)=(r_2\lambda_2 v_2(s+r_1\lambda_1+r_2\lambda_2)(1+C_1(s)h_1^*(s+r_1\lambda_1)))$$
$$/(s+r_2\lambda_2+r_1\lambda_1(1-(v_1(s+r_2\lambda_2)\ v_2(s+r_1\lambda_1+r_2\lambda_2)$$
$$+(s+r_1\lambda_1+r_2\lambda_2)\ \bar{V}_2(s+r_1\lambda_1+r_2\lambda_2))(1+C_1(s)\ h_1^*(s$$
$$+r_1\lambda_1))\ h_2^*(s+r_2\lambda_2)))$$

$$\pi_6^*(s,0)=(r_1\lambda_1\ v_2(s+r_1\lambda_1+r_2\lambda_2)(1+C_1(s)\ h_1^*(s$$
$$+r_1\lambda_1)))/(s+r_2\lambda_2+r_1\lambda_1(1-(v_1(s+r_2\lambda_2)v_2(s+r_1\lambda_1$$
$$+r_2\lambda_2)+(s+r_1\lambda_1+r_2\lambda_2)\bar{V}_2(s+r_1\lambda_1+r_2\lambda_2))(1+C_1(s)$$
$$h_1^*(s+r_1\lambda_1))h_2^*(s+r_2\lambda_2)))$$

$$C_1(s)=(r_2\lambda_2(v_1(s+r_1\lambda_1)\ v_2(s+r_1\lambda_1+r_2\lambda_2)$$
$$+(s+r_1\lambda_1+r_2\lambda_2)\ \bar{V}_2(s+r_1\lambda_1+r_2\lambda_2)))$$
$$/(s+r_1\lambda_1+r_2\lambda_2-r_2\lambda_2(v_1(s+r_1\lambda_1)v_2(s+r_1\lambda_1+r_2\lambda_2)$$
$$+(s+r_1\lambda_1+r_2\lambda_2)\bar{V}_2(s+r_1\lambda_1+r_2\lambda_2))h_2^*(s+r_1\lambda_1))$$

The reliability of the system can be defined as:

$$R(t)=\int_0^\infty \pi_0(t,u)du+\int_0^\infty \pi_1(t,y)dy+\int_0^\infty \pi_2(t,x)dx$$
$$+\pi_3(t)+\sum_{i=5}^{6}\int_0^\infty \pi_i(t,z)dz\,.$$

The Laplace transformation formula of the system is:

$$R^*(s)=\int_0^\infty \pi_0^*(s,u)du+\int_0^\infty \pi_1^*(s,y)dy+\int_0^\infty \pi_2^*(s,x)dx$$
$$+\pi_3^*(s)+\sum_{i=5}^{6}\int_0^\infty \pi_i^*(s,z)dz\,.$$

$$R^*(s)=(v_2^*(s+r_1\lambda_1+r_2\lambda_2)(1+r_2\lambda_2\ \bar{V}_1^*(s+r_1\lambda_1)$$
$$+r_1\lambda_1(v_1^*(s+r_2\lambda_2)\ \bar{H}_2^*(s+r_2\lambda_2)+\bar{V}_1^*(s+r_2\lambda_2)))$$
$$(1+C_1(s)h_2^*(s+r_1\lambda_1))+(s+r_1\lambda_1+r_2\lambda_2)((1+r_1\lambda_1\bar{H}_2^*(s$$
$$+r_2\lambda_2))\bar{V}_2^*(s+r_1\lambda_1+r_2\lambda_2)+C_1(s)(\bar{H}_1^*(s+r_1\lambda_1)$$
$$+(1+r_1\lambda_1\bar{H}_2^*(s+r_2\lambda_2))\bar{V}_2^*(s+r_1\lambda_1+r_2\lambda_2)h_2^*(s+r_1\lambda_1))))$$
$$/(s+r_2\lambda_2+r_1\lambda_1(1-(v_1^*(s+r_2\lambda_2)\ v_2^*(s+r_1\lambda_1+r_2\lambda_2)$$
$$+(s+r_1\lambda_1+r_2\lambda_2)\ \bar{V}_2^*(s+r_1\lambda_1+r_2\lambda_2))\ h_1^*(s+r_2\lambda_2)$$
$$(1+C_1(s)\ h_2^*(s+r_1\lambda_1))))$$

The mean time to failure of the system as follow

$$MTTF=\int_0^\infty R(t)dt=\lim_{s\to 0^+} R^*(s)$$

$$MTTF=-(v_2(r_1\lambda_1+r_2\lambda_2)(1+r_2\lambda_2(v_1(r_1\lambda_1)\ \bar{H}_1(r_1\lambda_1)$$
$$+\bar{V}_1(r_1\lambda_1))+r_1\lambda_1(v_1(r_2\lambda_2)\ \bar{H}_2(r_2\lambda_2)+\bar{V}_1(r_2\lambda_2)))$$
$$+(r_1\lambda_1+r_2\lambda_2)(1+r_2\lambda_2\bar{H}_1(r_1\lambda_1)+r_1\lambda_1\bar{H}_2(r_2\lambda_2))$$
$$\bar{V}_2(r_1\lambda_1+r_2\lambda_2))/(-r_2\lambda_2+r_1\lambda_1(-1+(v_1(r_2\lambda_2)$$
$$v_2(r_1\lambda_1+r_2\lambda_2)+(r_1\lambda_1+r_2\lambda_2)\bar{V}_2(r_1\lambda_1+r_2\lambda_2))$$
$$h_1^*(r_2\lambda_2))+r_2\lambda_2(v_1(r_1\lambda_1)v_2(r_1\lambda_1+r_2\lambda_2)+(r_1\lambda_1$$
$$+r_2\lambda_2)\bar{V}_2(r_1\lambda_1+r_2\lambda_2))h_2^*(r_1\lambda_1))$$

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