Co-regular Edge Domination in Graphs

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ABSTRACT
An edge dominating set D is a coregular edge dominating set of G. If the induced subgraph $< E(G) - D >$ is regular. The coregular edge domination number $\gamma'_{cr}(G)$ is the minimum cardinality of a coregular edge dominating set. We establish upper and lower bounds on $\gamma'_{cr}(G)$ and compare with other dominating parameters G and elements of G were obtained.

Keywords
Graph, Edge domination number, Coregular edge domination number.

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1. INTRODUCTION
By a graph $G = (V, E)$ be mean of finite undirected graphs without loops or multiple edges.Terms not here are used in the sence of Harary [7].

As usual the maximum degree of a vertex in $V(G)$ is denoted by $\Delta(G)$ and maximum edge degree of edge in $E(G)$ is denoted by $\Delta'(G)$. The notation $\alpha_0(G)\,(\alpha_1(G))$ is the minimum number of vertices(edges) in vertex(edge) cover of G. The notation $\beta_0(G)\,(\beta_1(G))$ is the maximum cardinality of a vertex(edge) independent set in G. A subset D of V is a dominating set of G, if every vertex not in D is adjacent to some vertex in D. The domination number $\gamma(G)$ of G is the minimum cardinality taken over all dominating sets of G. The study of domination in graphs was begun by Ore[15] and Berge[4].

We begin by recalling some standard definitions from domination theory.

A dominating set S of G is said to be a connected dominating set if the subgraph $< S >$ is connected in G. The minimum cardinality of vertices in such a set is called the connected domination number, of G and is denoted by $\gamma_c(G)$.

A dominating set S of G is said to be a total dominating set if the subgraph $< S >$ has no isolated vertices in G. The minimum cardinality of vertices in such a set is called the total domination number, denoted by $\gamma_t(G)$ see [5].

The concept of restrained domination in graphs was introduced by Domke et.al (1999) see [6]. A dominating set $S \subseteq V(G)$ is restrained dominating set of G, if every vertex not in S is adjacent to a vertex in S and to a vertex in $V(G) - S$. The restrained domination number of a graph G is denoted by $\gamma_r(G)$ is the minimum cardinality of a restrained dominating set in G.

A dominating set D of a graph $G = (V, E)$ is a split dominating set if the induced subgraph $< V - D >$ has more than one component. The split domination number $\gamma_s(G)$ of G is the least cardinality of a split dominating set. The concept of domination was introduced in [10].

A restrained dominating set $D \subseteq V(G)$ is a coregular restrained dominating set if the induced subgraph $< V - D >$ is regular. The coregular restrained domination number of G is denoted by $\gamma_{crr}(G)$ is the minimum cardinality of a coregular restrained dominating set. For detail see [13].

A dominating set D of a graph G is a global dominating set if D is also a dominating set of $\overline{G}$. The global domination number $\gamma_g(G)$ is the minimum cardinality of a global dominating set of G.

The concept of Roman domination function (RDF) in a graph $G = (V, E)$ is a function $f: V \to \{0,1,2\}$ satisfying the condition that every vertex u for which $f(u) = 0$ is adjacent to at least one vertex of v for which $f(v) = 2$ in G. The weight of a Roman dominating function is the value $f(V) = \sum_{u \epsilon v} f(u)$. The minimum weight of a Roman domination function of a graph G is called Roman domination number and is denoted by $\gamma_R(G)$.

A dominating set $D \subseteq V(G)$ is a double dominating set of G, if each vertex in V is dominated by at least two vertices in D. The double domination number $\gamma_{dd}(G)$ of G is the minimum cardinality of a double dominating set of G see [9].

Analogously, a split dominating set D of a graph G is a coregular split dominating set if the induced subgraph $< V(G) - D >$ is disconnected and regular. The coregular split dominating number $\gamma_{crs}(G)$ is the minimum cardinality of a coregular split dominating set of G. For details see [12].

A total dominating set D of a graph G is a coregular total dominating set if the induced subgraph $< V - D >$ is regular. The coregular total domination number $\gamma_{crt}(G)$ of G is the minimum cardinality of a coregular total dominating set see [14].

The concept of edge domination was introduced and studied in [2, 11].

In this paper, we obtain many bounds on $\gamma'_{cr}(G)$ in terms of elements of G. Also its relation with other domination parameters were established.

We need the following theorem for our further results.

Theorem A [1]: Let G be a connected graph of order n, then $\gamma'(G) \le \left[\frac{n}{2}\right]$.

2. MAIN RESULTS
Theorem 2.1 a]. For any path P_p with $p \ge 3$ vertices,

$$\gamma'_{cr}(P_p) = \left\lceil \frac{p}{2} \right\rceil - 1.$$

b]. For any cycle C_p with $p \geq 3$ vertices,

$$\gamma'_{cr}(C_p) = \left\lceil \frac{p}{2} \right\rceil.$$

c]. For any star $K_{(1,p)}$ with $p \geq 3$ vertices,

$$\gamma'_{cr}(K_{1,p}) = q - 1.$$

d]. For any wheel W_p with $p \geq 4$ vertices,

$$\gamma'_{cr}(W_p) = p - 1.$$

Theorem 2.2: For any connected (p, q) graph G with $p \geq 3$ vertices,

$\gamma'_{cr}(G) + m \geq \beta_0(G)$ where m be the number of end vertices in G.

Proof: Let $E = \{e_1, e_2, \ldots \ldots, e_k\}$ be the edge set in G. Now consider $E_1 = \{e_1, e_2, \ldots \ldots, e_m\} \subseteq E(G)$ be the set of edges with maximum edge degree and $E_2 = \{e_1, e_2, \ldots \ldots, e_n\} \subseteq E(G)$ be the set of edges with minimum edge degree. Suppose $E_1' \subseteq E_1$ and $E_2' \subseteq E_2$ then $\{E_1' \cup E_2'\}$ forms a minimal edge dominating set of G. Further if induced subgraph $< E(G) - \{E_1' \cup E_2'\} >$ is regular then $\{E_1' \cup E_2'\}$ itself is a coregular edge dominating set of G. On the other hand let $A = \{v_1, v_2, \ldots \ldots, v_n\}$ be the set of all endvertices in G. Let $K = \{v_1, v_2, \ldots \ldots \ldots, v_p\} \subseteq V(G)$ be the maximum set of vertices such that $\deg(v_i, v_j) \geq 2$, and $N(v_i) \cap N(v_j) = x, \forall v_i, v_j \in K$ so that $x \in V(G) - K$. Clearly $|K| = \beta_\circ(G)$. It follows that $|\{E_1' \cup E_2'\}| + |A| \geq |K|$ which gives, $\gamma'_{cr}(G) + m \geq \beta_0(G)$.

Theorem 2.3: For any connected (p, q) graph G with $p \geq 4$ vertices,

$\gamma'_{cr}(G) \geq \gamma_{crt}(G) + \gamma_s(G) - \gamma_c(G)$ with, $G \neq K_p, G \neq P_4$.

Proof: Let $E_1 = \{e_1, e_2, \ldots \ldots, e_k\} \subseteq E(G)$ be the minimal set of edges which covers all the edges in G such that $N[E_1] = E(G)$. Then E_1 is the edge dominating set of G. If the induced subgraph $< E(G) - E_1 >$ is regular then E_1 is a coregular edge dominating set of G. Suppose $A = \{v_1, v_2, \ldots \ldots, v_m\} \subseteq V(G)$ such that $deg(v_j) \geq 2$, $1 \leq j \leq m$. Then there exists at least one vertex v of maximum degree of G in A. Let D be a minimal dominating set of G such that $D \subseteq A$ if the subgraph $< D >$ has exactly one component then D itself is a connected dominating set of G. On the other hand if the induced subgraph $< V(G) - D >= F$ is disconnected then $\{F\}$ is a split dominating set of G. Further $V(G) - D = B, \forall v_i \in B$ if $< D \cup \{v_i\} >$ has no isolates. Then $< D \cup \{v_i\} >$ forms a minimal total dominating set of G. Also if $B_1 = [V(G) - D \cup \{v_i\}]$ and $\forall v_i \in < B_1 >$ has same degree then $\{B_1\}$ is a $\gamma_{crt} - set$ of G. It follows that $|E_1| \geq |B| + |F| - D$, which gives $\gamma'_{cr}(G) \geq \gamma_{crt}(G) + \gamma_s(G) - \gamma_c(G)$.

Theorem 2.4: For any connected (p, q) graph G with $p \geq 3$ vertices,

$\gamma'_{cr}(G) \geq \gamma_{crr}(G) - \gamma(G) + 2$ and $G \neq K_{1,p}$, $G \neq P_p$, $(p \leq 6)$.

Proof: For the graph $G = P_p$ with $p \leq 6$ For $p = 4$, $\gamma'_{cr}(G) = 1 < \gamma_{crr}(G) - \gamma(G) + 2 = 2$. For $p = 5,6$, $\gamma'_{cr}(G) = 2 < (G) - \gamma(G) + 2 = 3,4$ and hence the result not holds for path $p \leq 6$. Let $A = \{v_1, v_2, \ldots \ldots, v_p\} \subseteq V(G)$ be set of vertices with $deg(v_i) \geq 1$, such that $N[A] = V(G)$. Clearly A forms a dominating set of G. Suppose $B = \{v_1, v_2, \ldots \ldots, v_k\} \subseteq V(G)$ be the set of endvertices in G and $A' = V(G) - B$. Then there exists a vertex set $H \subseteq A'$ such that $\forall v_i \in \{V(G) - H \cup B\}$ is adjacent to at least one vertex of $\{H \cup B\}$ and in $V(G) - H \cup B$. Then $\{H \cup B\}$ is a γ_r set of G. If $< V(G) - \{H \cup B\} >$ is regular then $\{H \cup B\}$ itself is a γ_{crr} set of G. Let $\{e_1, e_2, \ldots \ldots, e_p\} = E(G)$ be the edge set in G. Suppose S be the minimal edge dominating set of G. If $< E(G) - S >$ has same degree then S itself is a γ'_{cr} set of G. Hence $|S| \geq |\{H \cup B\}| + |A| + 2$ which gives, $\gamma'_{cr}(G) \geq \gamma_{crr}(G) - \gamma(G) + 2$.

Theorem 2.5: For any connected (p, q) graph G with $p \geq 4$ vertices,

$$\gamma'_{cr}(G) \geq \alpha_0(G) - \gamma_{crs}(G) + 2 \text{ with } G \neq K_p, G \neq P_4$$

Proof: Suppose $G = K_p$. Then by the definition γ_s set does not exists, hence γ_{crs} also does not exists. Let $D = \{v_1, v_2, \ldots \ldots \ldots, v_p\}$ be the minimal set of vertices in G, such that $< V(G) - D >$ is regular and which gives more than one component. Then D forms a minimal coregular split dominating set of G. Suppose $B = \{v_1, v_2, \ldots \ldots, v_n\} \subseteq V(G)$ $\forall e_i \in E(G)$ is incident to at least one vertex B. Then $|B| = \alpha_0(G)$. Further $E(G) = \{e_1, e_2, \ldots \ldots, e_n\}$ be the edge set of G. Let $A = \{e_1, e_2, \ldots \ldots, e_k\} \subseteq E(G)$ which covers all the edges in G. Such that $N[A] = E(G)$, then A is a minimal edge dominating set of G. If the induced subgraph $< E(G) - A >$ has same degree then A is a γ'_{cr} set of G. It follows that $|A| \geq |B| - |D| + 2$. Which gives, $\gamma'_{cr}(G) \geq \alpha_0(G) - \gamma_{crs}(G) + 2$.

Corollary 2.1: For any connected (p, q) graph G $\gamma'_{cr}(G) \geq \gamma'(G)$.

Theorem 2.6: For any connected (p, q) graph G with $p \geq 3$ vertices,

$$2\gamma'_{cr}(G) \geq \gamma'(G) + \gamma_s(G) \quad and\ G \neq K_p, G \neq P_4.$$

Proof: Suppose $G = K_p$ by the definition, γ_s-set does not exists. Also if $G = p_4$, then $2\gamma'_{cr}(G) < \gamma'(G) + \gamma_s(G)$, a contradiction to P_4. Let $A = \{v_1, v_2, \ldots \ldots, v_p\} \subseteq V(G)$ be the set of all endvertices in G and $A' = V(G) - A$. Suppose there exists a vertex set $B \subset A'$, such that $D = [V(G) - B]$ is a dominating set of G. Hence $< D >$ has more than one component then D forms a γ_s - set of G. Further let $E = \{e_1, e_2, \ldots \ldots \ldots, e_p\}$ be the edge set in G. Now consider $E_1 = \{e_1, e_2, \ldots \ldots \ldots, e_m\} \subseteq E(G)$ be the set of edges with maximum edge degree and $E_2 = \{e_1, e_2, \ldots \ldots \ldots, e_n\} \subseteq E(G)$ be the set of edges with with minimum edge degree. Suppose $E_1' \subseteq E_1$ and $E_2' \subseteq E_2$ if every edge in $\{E_1' \cup E_2'\}$ is adjacent to an edge in $\{V(G) - E_1' \cup E_2'\}$ then $\{E_1' \cup E_2'\}$ for a γ' − set of G. Suppose $\{V(G) - E_1' \cup E_2'\} = S$ is regular. Clearly $\{S\}$ is a $\gamma'_{cr} - set$ of G. Thus $2|S| \geq |E_1' \cup E_2'| + |D|$ which gives, $2\gamma'_{cr}(G) \geq \gamma'(G) + \gamma_s(G)$.

Theorem 2.7: For any connected (p, q) graph G with $p \geq 3$ vertices,

$2\gamma'_{cr}(G) \geq \alpha_1(G) - \Delta'(G) + \gamma_g + 1$ with $G \neq P_p$ $(P \leq 4)$

Proof: Let $E = \{e_1, e_2, \dots\dots\dots, e_k\}$ be the edge set of G. Suppose $E' \subseteq E$ then $N[E'] = E(G)$ then E' is an edge dominating set of G. If $< E(G) - \{E'\} >$ is a regular, then $\{E'\}$ itself is a γ'_{cr} set of G. Let e be an edge with degree Δ' and let $D = \{v_1, v_2, \dots\dots, v_n\} \subseteq V(G)$ and $D \subseteq V(\bar{G})$. If $N[D] = V(G)$ and $N[D] = V(\bar{G})$. *Then* D is a dominating set of G *and* $\bar{G}$. Let $A = \{e_1, e_2, \dots\dots, e_m\}$ be the set of all endedges in G. Then $A \cup F$ where $F \subseteq E(G) - A$ be the minimal set of edges which covers all the vertices of G such that $|A \cup F| = \alpha_1(G)$. Thus $2|\{E'\}| \geq |A \cup F| - |e| + |D| + 1$ which gives, $2\gamma'_{cr}(G) \leq \alpha_1(G) - \Delta'(G) + \gamma_g + 1$.

Theorem 2.8: For any connected (p, q) graph G with $p \geq 3$ vertices,

$$\gamma'_{cr}(G) + diam(G) + \gamma(G) \geq \gamma_R(G) + \gamma_t(G).$$

Proof: Let $B \subseteq V(G)$ be the minimal set of vertices. Further, there exists an edge set $J \subseteq J'$ where J' is the set of edges which are incident with the vertices of B constituting the longest path in G such that $|J| = diam(G)$. Let $D = \{v_1, v_2, \dots\dots\dots, v_n\} \subseteq B$ be the minimal set of vertices which covers all the vertices in G. Clearly D forms a dominating set of G. Suppose the subgraph $< D >$ has no isolates.Then D itself is a $\gamma_t(G)$ set. Otherwise if $deg(v_k) < 1$ then attach the vertices $w_i \in N(v_k)$ to make $deg(v_k) \geq 1$ such that $< D \cup \{w_i\} >$ does not contain any isolated vertex. Clearly $D \cup \{w_i\}$ forms a total dominating set of G. Further let function $f: V(G) \to \{0,1,2\}$ and partition the vertex set $V(G)$ into (V_0, V_1, V_2) induced by f with $|V_i| = n_i$ *for* $i = 0,1,2$. Suppose the set V_2 dominates V_0. Then $S = V_1 \cup V_2$ forms a minimal Roman dominating set of G. Further let $A = \{e_1, e_2, \dots\dots\dots, e_p\} \subseteq E(G)$ be the minimal set of edges which covers all the edges in G. Clearly A forms a minimal edge dominating set of G. If $< E(G) - A >$ is regular then A is a coregular edge dominating set of G. Then $|A| + |J| + |D| \geq |S| + |D \cup \{w_i\}|$ which gives, $\gamma'_{cr}(G) + diam(G) + \gamma(G) \geq \gamma_R(G) + \gamma_t(G)$.

In the following theorem we establish the relationship between $\gamma_{dd}(G), \gamma_r(G)$ with coregular edge domination of a graph G.

Theorem 2.9: For any connected (p, q) graph G with $p \geq 3$ vertices,

$$\gamma'_{cr}(G) + \gamma_{dd}(G) \geq \left\lfloor \frac{p}{2} \right\rfloor + \gamma_r(G) - 1.$$

Proof: Let $S = \{e_1, e_2, \dots\dots\dots, e_m\}$ be an edge dominating set of G. Let $D_1 = \{v_1, v_2, \dots\dots\dots, v_k\}$ which is dominating set of G. Suppose $V_1 \subseteq V(G) - D_1$ be the set of vertices which are neighbours of the elements of D_1. Further $D_2 \subseteq V_2$ and $D_2 \in N(D_1)$. Then $D^d = D_1 \cup D_2$ forms double dominating set of G such that any vertex $v \in V(G) - D^d$ has at least two neighbours in $D_1 \cup D_2$. Further let $A = \{e_1, e_2, \dots\dots\dots, e_p\} \subseteq E(G)$ be the minimal set of edges which covers all the edges in G. Such that $N[E_1] = E(G)$. Then E_1 is an edge dominating set of G. If $< E(G) - E_1 >$ is regular then $\{E_1\}$ itself is a γ'_{cr} set of G. Let $B = \{v_1, v_2, \dots\dots\dots, v_p\} \subseteq V(G)$ be the set of endvertices in G and $B' = V(G) - B$. Then there exists vertex set $H \subseteq B'$ such that $\forall v_i \in \{V(G) - \{H \cup B\}$ is adjacent to at least one vertex of $\{H \cup B\}$ and in $V(G) - \{H \cup B\}$. Then $\{H \cup B\}$ is a γ_r set of G. Also by theorem $A, \gamma'(G) \leq \left\lfloor \frac{p}{2} \right\rfloor$. Thus $|E_1| + |D^d| \geq \left\lfloor \frac{V(G)}{2} \right\rfloor + |H \cup B| - 1$ which gives, $\gamma'_{cr}(G) + \gamma_{dd}(G) \geq \left\lfloor \frac{p}{2} \right\rfloor + \gamma_r - 1$.

Theorem 2.10: For any connected (p, q) graph G with $p \geq 3$ vertices,

$$\gamma'_{cr}(G) + \gamma'_e(G) + 1 \leq 2(p - 1).$$

Proof: Suppose D be a minimal edge dominating set of G and $E(G) - D$ be the set of all edges which are adjacent to the edges in D. Then $D' = [E(G) - D]$ has same degree then $\{D'\}$ is a coregular edge dominating set of G. Now let $E_1 = \{e_1, e_2 \dots\dots\dots, e_j\}$ denote the set of all endedges in G and $E_2 = E(G) - E_1$. Further if $F \subseteq E_2$ is edge dominating set of subgraph $< E_2 >$ then $E_1 \cup F$ forms an endedge dominating set of G. Clearly it follows that $|D'| + |E_1 \cup F| + 1 \leq 2(p - 1)$ and hence $\gamma'_{cr}(G) + \gamma'_e(G) + 1 \leq 2(p - 1)$.

An edge dominating set X is called a connected edge dominating set if the edge induced subgraph $< X >$ is connected. The minimum cardinality of a connected edge dominating set of G is called the connected edge domination number of G and is denoted by $\gamma'_c(G)$. For detail see [3].

Theorem 2.11: For any connected graph G with $p \geq 4$ vertices,

$$\gamma'_{cr}(G) + \gamma'_c(G) \geq \alpha_1(G) + \gamma_s(G) \quad \text{and} \quad G \neq K_p, G \neq P_p \ (p \leq 5)$$

Proof: For the graph $G = P_p$ with $p \leq 5$ if $p = 3,4,5$ then $\gamma'_{cr}(G) + \gamma'_c(G) = 2,2,4 \ngeq \alpha_1(G) + \gamma_s(G) = 3,4,5$. Hence $G \neq P_p$ *with* $p \leq 5$. Suppose $D = \{e_1, e_2, \dots\dots, e_n\}$ be the set of all endedges in G. Then $D \cup J$ where $J \subseteq E(G) - D$ be the minimal set of edges which covers all the vertices of G such that $|D \cup J| = \alpha_1(G)$. Let $D_1 = \{e_1, e_2, \dots\dots\dots, e_j\}$ be the set of nonendedges which covers all the edges in G. If the induced subgraph $< E(G) - D_1 >$ is regular then $\{D_1\}$ is a coregular edge dominating set of G. Now consider $S = \{e_1, e_2, \dots\dots, e_i\}$ be the minimal edge dominating set then $< S >$ does not contain more than one component. Then S itself is a connected edge dominating set of G. Otherwise if the subgraph $< S >$ has more than one component then attach the minimum number of edges $\{e_k\} \in E(G) - S$ with $deg(e_k) \geq 2$ such that $S_1 = S \cup \{e_k\}$ forms exactly one component clearly S_1 forms a γ'_c set of G. On the other hand let $F = \{v_1, v_2, \dots\dots\dots, v_n\}$ be a minimal dominating set G if the $< V(G) - F >$ is disconnected then clearly F forms a split dominating set of G. $|D_1| + |S_1| \geq |D \cup J| + |F|$ which gives, $\gamma'_{cr}(G) + \gamma'_c(G) \geq \alpha_1(G) + \gamma_s(G)$.

Theorem 2.12: For any connected (p, q) graph G with $p \geq 3$ vertices,

$$\gamma'_{cr}(G) + \gamma(G) + 1 \leq p + \gamma'_c(G).$$

Proof: Let D be a dominating set of G and let $E = \{e_1, e_2, \dots\dots, e_n\} \subseteq E(G)$ be the set of all nonendedges in G. Suppose there exist a minimal set of edges such that $N[e_i] = E(G), \forall e_i \in E_1, 1 \leq i \leq n$ then E_1 forms a minimal edge dominating set of G. Further if subgraph $< E_1 >$ has exactly one component then E_1 itself is a connected edge dominating set of G. Further $E_2 \subseteq E_1$ such that the induced subgraph $< E(G) - E_2 >$ is regular clearly E_2 is a coregular edge dominating set of G. Hence $|E_2| + |D| + 1 \leq |V(G)| + |E_1|$ which gives, $\gamma'_{cr}(G) + \gamma(G) + 1 \leq p + \gamma'_c(G)$.

Theorem 2.13: For any graph (p, q) with $p \geq 3$ vertices,

$$\gamma'_{cr}(G) \geq \left\lceil \frac{diam\,(G) + 1}{2} \right\rceil - 1.$$

Proof: Let $E = \{e_1, e_2, \dots\dots, e_n\} \subseteq E(G)$ be the set of edges which constitute the longest path between two distinct vertices $u, v \in V(G)$ such that $d(u,v) = diam(G)$. Now $E_1 \subseteq E(G), \forall e_i \in E_1$ since E_1 be the minimal set of edges which covers all the edges in G then E_1 is a minimal edge dominating set of G. Further if $deg(e_j) \geq 1, e_j \in E(G) - E_1$ then $< E(G) - E_1 >$ is regular then $\{E_1\}$ is a coregular edge dominating set. It follows that $|E_1| \geq \left\lceil \frac{diam\ (G)+1}{2} \right\rceil - 1$. Hence $\gamma'_{cr}(G) \geq \left\lceil \frac{diam\ (G)+1}{2} \right\rceil - 1$.

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