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A Comprehensive Study of Intuitionistic Fuzzy Soft Matrices and its Applications in Selection of Laptop by using Score Function

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ABSTRACT
This paper is being carried out to discuss Intuitionistic Fuzzy Soft Matrices and their operations have been described employing decisive issues by using Score Function of Intuitionistic Fuzzy soft matrices resulting in the efficiency of Intuitionistic Matrices over fuzzy matrices. Finally at the end we have presented a case study for the best selection of laptop.

Keywords
Intuitionistic Fuzzy Soft Matrices Score Function

1. INTRODUCTION
Uncertainty is impreciseness and fuzziness in our daily lives. In real life, most of fields deals with imprecise and vague data set. Latterly, many theories have been presented to work on inconsistency, fuzziness and indefiniteness. Many theories have been established to deal with various kinds of inconsistency and fuzziness that is enclosed in a system. In this regard, the first theory which was presented is probability theory. In 1965 Zadeh introduced fuzzy soft sets and then IVF sets [2] and rough set theory [33] in later years which proved to be more accurate. In 1986, Intuitionistic Fuzzy soft sets are proposed by Atanassov. Later on Florentine presented the concept of Neutrosophic soft sets which give more precise information. Molodtsov [31] noticed that these theories have some intensive complications. Lack of configured mechanism of thesis is basically the main cause of these difficulties. Thereafter he established the concept of soft set theory dealing with various kinds of uncertainty and many other fascinating consequences of soft set theory have been attained by proposals of fuzzy sets, intuitionistic sets and etc. For instance, fuzzy soft set [26], rough soft set etc. These theories have been established and appropriate in various aspects of life for example on soft decision making [5], and the relation in intuitionistic fuzzy soft sets [13,32] etc.

Many analysts put out various research papers on fuzzy and intuitionistic fuzzy soft matrices, and it is applicable in various real life disciplines [19, 21, 20, and 24]. Soft matrices and its issues in decision making was latterly presented by Cagman et al [6]. Fuzzy soft matrices [8] was also latterly presented by them. Intuitionistic fuzzy soft matrices and various products and characteristics of these products was explained by Chetia and Das. Moreover properties the concept of fuzzy soft matrices and four distinct products of intuitionistic fuzzy soft matrix and their implementation in medical field was presented by Saikia et al [35]. Further Broumi et al [4] deliberated fuzzy soft matrix and introduced some modified operations such as fuzzy soft complement matrix etc. Few years ago Mondal et al [28, 29, and 30] established intuitionistic and fuzzy soft matrices and its purpose in decision making problems established from 3 fundamental t-norm operators. In various real life disciplines [3, 32, and 34] matrices appear in many administrations. The researchers[18,19,20,36,38,41-44] observe in their study soft set and some relations with soft sets are the best tools for decision making, medical diagnosis, MCDM Problems.

Basically the theory of intuitionistic fuzzy matrices and some operations on these matrices are defined and its implementation in decisive issue is our main intention. The extended portion is ordered as: Portion 2 describes fundamental description and symbolizations that are used in this paper. Portion 3 describes some redefined intuitionistic soft set, operations and comparison between fundamental definitions of intuitionistic soft set. Portion 4 contains the concept of intuitionistic fuzzy matrices and present their fundamental properties. Portion 5 describes 2 types of products defined on intuitionistic fuzzy matrices. Portion 6 explained soft decisive issues procedures established from Score Function of IS-Matrices. At the end in the last step conclusion is drawn.

2. PRELIMINARIES
This portion describes some essential definitions and conclusions of Intuitionistic Set Theory, SMT [6] and SST [28] that helps in following analysis.

DEFINITION. 1
Let Ʊ be a universal set and elements of this universal sets i.e. Ʊ can be represented by ʋ. An intuitionistic set Ä in Ʊ can be regarded a truthiness function by $\mathcal{T}_{Ä}$ and falseness function by $\mathcal{F}_{Ä}$ Where $\mathcal{T}_{Ä}(ʋ)$ and $\mathcal{F}_{Ä}(ʋ)$the real usual and unusual sets of [0, 1] and can be inscribed as follows:

$$Ä=\begin{Bmatrix} (ʋ, < (\mathcal{T}_{Ä}(ʋ)), \mathcal{F}_{Ä}(ʋ) >) \\ such\ that\ ʋ \in Ʊ, \mathcal{T}_{Ä}(ʋ), \mathcal{F}_{Ä}(ʋ) \in [0,1] \end{Bmatrix}$$

The sum of $\mathcal{T}_{Ä}(ʋ)$ and $\mathcal{F}_{Ä}(ʋ)$ have no limit. Such that

$0 \le \text{Sup}\, \mathcal{T}_{Ä}(ʋ) + \text{Sup}\, \mathcal{F}_{Ä}(ʋ) \le 2.$

DEFINITION. 2: [31] If a universal set is denoted as Ʊ and Ë indicates the parameters set or attributes with respect to Ʊ. Let Ä be a set such that Ä ⊑ Ë. So that $\mathcal{F}_{Ä}$ is a soft set with respect to the universal set Ʊ and can be described as a function $f_{Ä}$ and represented as given below:

$$f_{Ä}:\ Ë \to P(Ʊ)\ \text{as}\ f_{Ä}(\mathfrak{x}) = \phi\ \ if\ \ \mathfrak{x} \in Ë - Ä$$

$f_{Ä}$ is known as estimated function of $\mathcal{F}_{Ä}$. In addition, this soft set is related to the subsets of the universal set Ʊ, and therefore it can be described as well-defined set given below:

$\mathcal{F}_{Ä} = \{ (x, f_{Ä}(x))$ as $x \in Ë, f_{Ä}(x) = \phi$ if $x \in Ë - Ä \}$

Subscript Ä of $f_{Ä}$ implies that $f_{Ä}$ is an estimated function of $\mathcal{F}_{Ä}$. Where $f_{Ä}(x)$ is the set of x-elements of $\mathcal{F}_{Ä}$ $\forall x \in Ë$.

DEFINITION.3: [6]
Suppose that a soft set of universal set Ʊ be $\mathcal{F}_{Ä}$.Then $\mathcal{R}_{Ä}$ be a subset of Ʊ × Ë and commonly can be described as:

$\mathcal{R}_{Ä} = \{< (\upsilon, x)/(\upsilon, x) >$ Such that $(\upsilon, x) \in Ʊ \times Ë\}$

$\mathcal{R}_{Ä}$ is a relation of soft set $\mathcal{F}_{Ä}$ The distinctive function of $\mathcal{R}_{Ä}$ can be written as $\chi_{\mathcal{R}_{Ä}} : Ʊ \times Ë \to [0,1]$ such that

$$\chi_{\mathcal{R}_{Ä}}(\upsilon, x) = \begin{cases} 1 & \text{if } (\upsilon, x) \in \mathcal{R}_{Ä} \\ 0 & \text{if } (\upsilon, x) \notin \mathcal{R}_{Ä} \end{cases}$$

Let $Ʊ = \{\upsilon_1, \upsilon_2, \ldots, \upsilon_m\}$ be a universal *set* $Ë = \{x_1, x_2, \ldots, x_n\}$ be a set of parameters and $Ä \sqsubseteq Ë$ so that the relation $\mathcal{R}_{Ä}$is represented by corresponding table.

$\mathcal{R}_{Ä}$	x_1	x_2	x_3	⋯	x_n
υ_1	$\chi_{\mathcal{R}_{Ä}}(\upsilon_1, x_1)$	$\chi_{\mathcal{R}_{Ä}}(\upsilon_1, x_2)$	$\chi_{\mathcal{R}_{Ä}}(\upsilon_1, x_3)$	⋯	$\chi_{\mathcal{R}_{Ä}}(\upsilon_1, x_n)$
υ_2	$\chi_{\mathcal{R}_{Ä}}(\upsilon_2, x_1)$	$\chi_{\mathcal{R}_{Ä}}(\upsilon_2, x_2)$	$\chi_{\mathcal{R}_{Ä}}(\upsilon_2, x_3)$	⋯	$\chi_{\mathcal{R}_{Ä}}(\upsilon_2, x_n)$
⋮	⋮	⋮	⋮	⋱	⋮
υ_m	$\chi_{\mathcal{R}_{Ä}}(\upsilon_m, x_1)$	$\chi_{\mathcal{R}_{Ä}}(\upsilon_m, x_2)$	$\chi_{\mathcal{R}_{Ä}}(\upsilon_m, x_3)$	⋯	$\chi_{\mathcal{R}_{Ä}}(\upsilon_m, x_n)$

Let $\dot{a}_{ij} = \chi_{\mathcal{R}_{Ä}}(\upsilon_i, x_j)$.It can be represented in the following matrix form:

$$[\dot{a}_{ij}]_{m \times n} = \begin{bmatrix} x_{11} & x_{12} & x_{13} & \cdots & x_{1n} \\ x_{21} & x_{22} & x_{23} & \cdots & x_{2n} \\ x_{31} & x_{32} & x_{33} & \cdots & x_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & x_{m3} & \cdots & x_{mn} \end{bmatrix}$$

Where $[\dot{a}_{ij}]$ is known as $m \times n$ square matrix of soft set$\mathcal{F}_{Ä}$ with respect to a universal set Ʊ. Next by deleting the subscripts m ×n of$[\dot{a}_{ij}]_{m \times n}$, and by using $[\dot{a}_{ij}]$ in place of $[\dot{a}_{ij}]_{m \times n}$, as i = 1, 2, 3 ... m & j = 1, 2, 3 ... n.

DEFINITION. 4[6]
If $[\dot{a}_{ij}]$ and $[\dot{b}_{i\kappa}]$ are two matrices such that$[\dot{a}_{ij}]$, $[\dot{b}_{i\kappa}] \in \mathcal{FSM}_{m \times n}$. So AND Product of $[\dot{a}_{ij}]$ and $[\dot{b}_{i\kappa}]$ can be defined as

$\wedge : \mathcal{FSM}_{m \times n} \times \mathcal{FSM}_{m \times n} \to \mathcal{FSM}_{m \times n^2}$

Where $[\dot{a}_{ij}] \wedge [\dot{b}_{i\kappa}] = [\dot{c}_{ip}]$

as $\dot{c}_{ip} = max\{\dot{a}_{ij}, \dot{b}_{i\kappa}\}$ where

$p = n(j - 1) + \kappa$.

DEFINITION. 5 [6]
If $[\dot{a}_{ij}]$ and $[\dot{b}_{i\kappa}]$ are two matrices such that$[\dot{a}_{ij}]$, $[\dot{b}_{i\kappa}] \in \mathcal{FSM}_{m \times n}$.So the OR Product of $[\dot{a}_{ij}]$ & $[\dot{b}_{i\kappa}]$ can be defined as:

$\vee : \mathcal{FSM}_{m \times n} \times \mathcal{FSM}_{m \times n} \to \mathcal{FSM}_{m \times n^2}$ Where $[\dot{a}_{ij}] \vee [\dot{b}_{i\kappa}] = [\dot{c}_{ip}]$

Whereas $\dot{c}_{ip} = max\{\dot{a}_{ij}, \dot{b}_{i\kappa}\}$ and

$p = n\ (j - 1) + \kappa$.

1) DEFINITION. 6
If Ʊ be a universal set and I(Ʊ) be a set of all intuitionistic sets on Ʊ, Ë be a set of parameters that define the elements of Ʊ where $Ä \sqsubseteq Ë$. So, an intuitionistic soft set I over Ʊ is a set defined by a respected function f_I represented by following mapping

$$f_I : Ä \to I(Ʊ)$$

Where f_I is known as estimated function of the intuitionistic soft set I. Similarly, the intuitionistic soft set is a parametrized family of elements of the set P (Ʊ), and therefore it can be written a set of ordered pairs

$$I = \{(x, f_I\ (x)) : x \in Ä\}$$

DEFINITION.7
Let I_1 and I_2 be two intuitionistic soft sets over intuitionistic soft universes (Ʊ, Ä) and (Ʊ, B), respectively.

I_1 is said to be Intuitionistic soft subset of I_2 if $Ä \sqsubseteq B$ and $\mathcal{T}_{f_{I_1}(x)}(\upsilon) \le \mathcal{T}_{f_{I_2}(x)}(\upsilon)$, $F_{f_{I_1}(x)}(\upsilon) \ge F_{f_{I_2}(x)}(\upsilon)$,for all $x \in Ä$ and $\upsilon \in Ʊ$. I_1 and $I_2$are said to be equal if I_1 intuitionistic soft subset of $I_2$and I_2 intuitionistic soft subset of I_1.

2) DEFINITION. 8
If A set of parameters or attributes be $Ë = \{e_1, e_2, \ldots\}$ with respect to the universal set Ʊ then the NOT set of Ë is represented as ¬ Ë and can defined by $\neg Ë = \{\neg e_1, \neg e_2, \ldots\}$ as $\neg e_i$ = NOT e_i , for all i.

DEFINITION. 9 If I_1 and I_2 areintuitionistic softsets with respect to two universal sets (Ʊ, Ä) and(Ʊ, B).

3. COMPLEMENT

The complement of an intuitionistic soft set I_1 is denoted as $I_1'$and defined by a respected function f_{I_1}' and represented by a corresponding mapping

$$F_{I_1}' : \neg Ë \to I\ (Ʊ)$$

Where

$$F_{I_1}' = \left\{ \left(\upsilon, < \mathcal{F}_{f_{I_1}(x)}(\upsilon), \mathcal{T}_{f_{I_1}(x)}(\upsilon) > \right) : x \in \neg Ë, \upsilon \in Ʊ \right\}.$$

a. UNION

The union of two ISS I_1 and I_2 is represented as $I_1 \cup I_2$ and is defined in the following way I_3(C = Ä ∪ Ɓ), whereas the truthiness and falseness of I_3 are as follows: for all ʋ ∈ Ʊ,

$$\mathcal{T}_{f_{I_3}(x)}(\upsilon) = \begin{cases} \mathcal{T}_{f_{I_1}(x)}(\upsilon) & if\ x \in Ä - Ɓ \\ \mathcal{T}_{f_{I_2}(x)}(\upsilon) & if\ x \in Ɓ - Ä \\ \dot{s}\left\{T_{f_{I_1}(x)}(\upsilon), T_{f_{I_2}(x)}(\upsilon)\right\}, & if\ x \in Ä \cap Ɓ \end{cases}$$

$$\mathcal{F}_{f_{I_3}(x)}(\upsilon) = \begin{cases} \mathcal{F}_{f_{I_1}(x)}(\upsilon) & if\ x \in Ä - Ɓ \\ \mathcal{F}_{f_{I_2}(x)}(\upsilon) & if\ x \in Ɓ - Ä \\ \dot{t}\left\{I_{f_{I_1}(x)}(\upsilon), I_{f_{I_2}(x)}(\upsilon)\right\}, & if\ x \in Ä \cap Ɓ \end{cases}$$

b. INTERSECTION

The intersection of two ISS I1 and I2 is represented as I1∩I2 and defined in the following way I3(C = Ä ∩ Ɓ), whereas the truthiness and falseness of N3 are defined as follows for all: ∀ ʋ ∈ Ʊ.

$$\mathcal{T}_{f_{I_3}(x)}(\upsilon) = \dot{t}\left\{\mathcal{T}_{f_{I_1}(x)}(\upsilon), \mathcal{T}_{f_{I_2}(x)}(\upsilon)\right\} \quad \text{and}$$

$$\mathcal{F}_{f_{I_3}(x)}(\upsilon) = \dot{s}\left\{\mathcal{F}_{f_{I_1}(x)}(\upsilon), \mathcal{F}_{f_{I_2}(x)}(\upsilon)\right\}, \forall\ x \in C$$

DEFINITION. 10 [14]

The ṫ -norm is basically a binary operation of two valued function which defines a mapping:

$\dot{t}$:[0, 1] × [0, 1]⇢ [0, 1]

Further ṫ -norm is a continuous, associative, commutative and monotonic function. The properties

Can be created with the help of following requirements such that ∀ ȧ,ḃ,ċ,ḋ

∈ [0, 1],

$\dot{t}(0,0) = 0$ and

$\dot{t}(\dot{a},1) = \dot{t}(1,\dot{a}) = \dot{a},$

$If\ \dot{a} \le \dot{c}$ and $\dot{b} \le \dot{d}$, then $\dot{t}(\dot{a},\dot{b}) \le \dot{t}(\dot{c},\dot{d})$

$$\dot{t}(\dot{a},\dot{b}) = \dot{t}(\dot{b},\dot{a})$$

$$\dot{t}(\dot{a},\dot{t}(\dot{b},\dot{c})) = \dot{t}(\dot{t}(\dot{a},\dot{b},\dot{c}))$$

DEFINITION .11 [14]

The t-conorm that is s norm is basically a binary operation of two valued function which defined by a mapping

s: [0,1] × [0,1] ⇢[0,1]

Further t co-norm is a continuous, associative, commutative and monotonic function. The properties can be created with the help of following requirements such that

∀ ȧ, ḃ, ċ, ḋ ∈ [0, 1]:

$\dot{s}(1,1) = 1$ and

$\dot{s}(\dot{a},0) = \dot{s}(0,\dot{a}) = \dot{a}$

$if\ \dot{a} \le \dot{c}$ and $\dot{b} \le \dot{d}$,

then $\dot{s}(\dot{a},\dot{b}) \le \dot{s}(\dot{c},\dot{d})$

$$\dot{s}(\dot{a},\dot{b}) = \dot{s}(\dot{b},\dot{a})$$

$\dot{s}(\dot{a},\dot{s}(\dot{b},\dot{c})) = (\dot{s}(\dot{a},\dot{b},\dot{c}))$

These two norms are interconnected on the basis of the logics used behind them. The list of standard not parametrized norms and co-norm is listed below:

3) **Drastic Sum**

$$\dot{s}_w(\dot{a},\dot{b}) = \begin{cases} \dot{s}\{\dot{a},\dot{b}\}, & \dot{t}\{\dot{a}\dot{b}\} = 0 \\ 1, & \text{otherwise} \end{cases}$$

4) **Drastic Product**

$$\dot{t}_w(\dot{a},\dot{b}) = \begin{cases} \dot{t}\{\dot{a},\dot{b}\}, & \dot{t}\{\dot{a}\dot{b}\} = 1 \\ 0, & \text{otherwise} \end{cases}$$

5) **Bounded Sum**

$\dot{s}_1\ (\dot{a},\dot{b}) = \dot{t}\ \{1, \dot{a} + \dot{b}\}$

6) **Bounded Product**

$\dot{t}_1\ (\dot{a},\dot{b}) = \dot{s}\{0, \dot{a} + \dot{b} - 1\}$

7) **Einstein Sum**

$\dot{s}_{1.5}\ (\dot{a},\dot{b}) = \frac{\dot{a}+\dot{b}}{[1+\dot{a}\,.\,\dot{b}]}$

8) **Einstein Product**

$\dot{t}_{1.5}\ (\dot{a},\dot{b}) = \frac{\dot{a}\,.\,\dot{b}}{2-[\dot{a}+\dot{b}-\dot{a}\,.\,\dot{b}]}$

9) **Algebraic Product**

$\dot{t}_2\ (\dot{a},\dot{b}) = \dot{a}\,.\,\dot{b}$

10) **Algebraic Sum**

$\dot{s}_2\ (\dot{a},\dot{b}) = \dot{a} + \dot{b} - \dot{a}\,.\,\dot{b}$

11) **Hamacher Product**

$\dot{t}_{2.5}\ (\dot{a},\dot{b}) = \frac{\dot{a}\,.\,\dot{b}}{\dot{a}+\dot{b}-\dot{a}\,.\,\dot{b}}$

12) **Hamacher Sum**

$\dot{s}_{2.5}\ (\dot{a},\dot{b}) = \frac{a+b-2.\dot{a}\,.\,\dot{b}}{1-\dot{a}\,.\,\dot{b}}$

13) **Maximum**

$$\dot{s}_3(\dot{a},\dot{b}) = \dot{s}\{\dot{a},\dot{b}\}$$

14) **Minimum**

$$\dot{t}_3\ (\dot{a},\dot{b}) = \dot{t}\{\dot{a},\dot{b}\}$$

4. INTUITIONISTIC SOFT SET AND SOME MODIFIED OPERATIONS

This portion deals with some redefined operations and definitions of intuitionistic soft set. Some of these are estimated from [7, 13].

DEFINITION. 12

If Ʊ be a universal set and I(Ʊ) be the set of all intuitionistic sets on Ʊ and Ë is a set of attributes that defines the elements of Ʊ. Therefore, an intuitionistic soft set I with respect to Ʊ is a set described with an estimated function f_I represented by following mapping: f_I : Ë → I (Ʊ)

Here f_I is called estimated function of intuitionistic soft set I. For x ∈ Ë, this set f_I (x) is known as set of x -elements of the intuitionistic soft set I which may be arbitrary, some of them have empty and non-empty intersections. Similarly, the intuitionistic soft set is a parametrized family of elements of the set I(Ʊ). Hence can be write in the following ordered pairs:

$$I = \{(x, \{< \upsilon, \mathcal{T}_{f_I(x)}(\upsilon), \mathcal{F}_{f_I(x)}(\upsilon) >: x \in Ʊ\} : x \in Ë\}$$

As $\mathcal{T}_{f_I(x)}(\upsilon)\, and\, \mathcal{F}_{f_I(x)}(\upsilon) \in [0,1]$

DEFINITION.13

If I1 and I2 be two intuitionistic soft sets with respect to a universal set Ʊ. So, the complement of an intuitionistic soft set I1is denoted by I′ and can be defined as:

$$I_1' = \{(ғ, \{< υ, \mathcal{F}_{f_{I_1}(ғ)}(υ), \mathcal{T}_{f_{I_1}(ғ)}(υ) >$$

DEFINITION. 14

If I1 and I2 be two intuitionistic soft sets with respect to a universal set Ʊ. The Union of two intuitionistic soft sets I1 and I2 can be represented as I3 = I1∪˜ I2 and can be described as follows:

$$I_3 = \{(ғ, \{< υ, \mathcal{T}_{f_{I_3}(ғ)}(υ), \mathcal{F}_{f_{I_3}(ғ)}(υ) > : ғ \in Ʊ\} : ғ \in Ë$$

Since $\mathcal{T}_{f_{I_3}(ғ)}(υ) = \dot{s}\left(\mathcal{T}_{f_{I_1}(ғ)}(υ), \mathcal{T}_{f_{I_2}(ғ)}(υ)\right)$

$$\mathcal{F}_{f_{I_3}(ғ)}(υ) = \dot{t}\left(\mathcal{F}_{f_{I_1}(ғ)}(υ), \mathcal{F}_{f_{I_2}(ғ)}(υ)\right)$$

DEFINITION.15

If I1 and I2 be two intuitionistic soft sets with respect to a universal set Ʊ. The intersection I1 and I2 can be represented as I4 = $I_1 \cap I_2$ and can be described as follows:

$$I_4 = \{(ғ, \{< υ, \mathcal{T}_{f_{I_4}(ғ)}(υ), \mathcal{F}_{f_{I_4}(ғ)}(υ) >$$

Where

$ғ \in Ʊ\} : ғ \in Ë\}$, Since

$$\mathcal{T}_{f_{I_4}(ғ)}(υ) = \dot{t}\left(\mathcal{T}_{f_{I_1}(ғ)}(υ), \mathcal{T}_{f_{I_2}(ғ)}(υ)\right)$$

$$\mathcal{F}_{f_{I_4}(ғ)}(υ) = \dot{s}\left(\mathcal{F}_{f_{I_1}(ғ)}(υ), \mathcal{F}_{f_{I_2}(ғ)}(υ)\right)$$

EXAMPLE. 1

Consider a universal set $Ʊ = \{υ_1, υ_2, υ_3\}$ and set of parameters can be defined as $Ë = \{ғ_1, ғ_2, ғ_3\}$ and I_1 and I_2 represents two intuitionistic soft set with respect to I(Ʊ) defined as:

I_1={$(ғ_1, \{< υ_1, (0.1,0.2) >, < υ_2, (0.2,0.3) >, <$ υ3,0.3,0.4>,ғ2,<υ1,0.2,0.3>,<υ2,0.3,0.1>,<υ3,0.1,0.2>,ғ3, <υ1,0.1,0.4>,<υ2,0.5,0.6>,<υ3,0.5,0.6>

I_2={$(ғ_1, \{< υ_1, (0.2,0.3) >, < υ_2, (0.3,0.4) >, < υ_3, (0.3,0.5) >\}), (ғ_2, \{< υ_1, (0.3,0.5) >, < υ_2, (0.5,0.7) >, < υ_3, (0.8,0.9) >\}), (ғ_3, \{< υ_1, (0.1,0.7) >, < υ_2, (0.6,0.1) >, < υ_3, (0.1,0.2) >\})\}$

I_1'={$(ғ_1, \{< υ_1, (0.2,0.1) >, < υ_2, (0.3,0.2) >, < υ_3, (0.4,0.3) >\}), (ғ_2, \{< υ_1, (0.3,0.2) >, < υ_2, (0.1,0.3) >, < υ_3, (0.2,0.1) >\}), (ғ_3, \{< υ_1, (0.4,0.1) >, < υ_2, (0.6,0.5) >, < υ_3, (0.6,0.5) >\})\}$

I_1Ṵ I_2={$(ғ_1, \{< υ_1, (0.2,0.3) >, < υ_2, (0.3,0.1) >, < υ_3, (0.3,0.4) >\}), (ғ_2, \{< υ_1, (0.3,0.2) >, < υ_2, (0.5,0.3) >, < υ_3, (0.5,0.1) >\}), (ғ_3, \{< υ_1, (0.4,0.1) >, υ_2, (0.6,0.5) >, < υ_3, (0.6,0.5) >\})\}$

$I_1 \cap$ I_2={$(ғ_1, \{< υ_1, (0.1,0.3) >, < υ_2, (0.2,0.4) >, < υ_3, (0.3,0.5) >\}), (ғ_2, \{< υ_1, (0.2,0.3) >, < υ_2, (0.3,0.3) >, < υ_3, (0.1,0.2) >\}), (ғ_3, \{< υ_1, (0.1,0.4) >, < υ_2, (0.5,0.6) >, < υ_3, (0.5,0.6) >\})\}$

5. DIFFERENCE BETWEEN DEFINITIONS

This section we related our description of intuitionistic soft to that presented by Maji [32] by getting motivation from [14].

By relating our description of intuitionistic soft to that presented by Maji [32] given a table below

PROPOSITION. 1

Consider three Intuitionistic soft sets over the same universal set Ʊ. Then the following propositions are listed below:

1. I_1 Ṵ I_2 = I_2Ṵ I_1
2. $I_1 \cap I_2 = I_2 \cap I_1$
3. I_1 Ṵ (I_2 Ṵ I_3) = (I_1 Ṵ I_2)ṴI_3
4. $I_1 \cap (I_2 \cap I_3) = (I_1 \cap I_2) \cap I_3$

PROOF.

The solution of these proofs obtained by using t-norm and co-norm operators as they satisfied the Commutative and Associative properties.

Methodology used in this paper	Methodology presented by Maji
$\boldsymbol{I = \{(ғ, f_I(ғ)) : ғ \in Ë\}}$	$I = \{(ғ, f_I(ғ)) : ғ \in \ddot{A}\}$
Whereas	$I = \{(ғ, f_I(ғ)) : ғ \in \ddot{A}\}$
Ë set of attributes &	$\ddot{A} \sqsubseteq Ë$
$\mathbf{f_I : Ë \dashrightarrow I(Ʊ)}$	$f_I : \ddot{A} \dashrightarrow I(Ʊ)$

Table.1

Methodology used in this paper	Methodology presented by Maji
$\boldsymbol{\mathcal{F}_1'}$	I_1°
$\mathbf{f_1'}: Ë \dashrightarrow \mathbf{I(Ʊ)}$	$f_{I1}^{\circ} : \neg Ë \dashrightarrow I(Ʊ)$
$\boldsymbol{\mathcal{T}_{fI'_{1(ғ)(υ)}} = \mathcal{F}_{f_{I_1}(ғ)}}(υ)$	$\mathcal{T}_{f_{I_1^{\circ}}(ғ)(υ)} = \mathcal{F}_{f_{I_1}(ғ)}(υ)$
$\boldsymbol{\mathcal{F}_{fI'_{1(ғ)(υ)}} = \mathcal{T}_{f_{I_1}(ғ)}}(υ)$	$\mathcal{F}_{f_{I_1^{\circ}}(ғ)(υ)} = \mathcal{T}_{f_{I_1}(ғ)}(υ)$

Table.2

In this table we relate our definition of Compliment with that presented by Maji [32]

In this table we relate our definition of Union with that presented by Maji [32]

Methodology used in this pap	Methodology presented by Maji
$\mathbf{I_3}=(\mathbf{I_1} \cup \mathbf{I_2})$	$I_3=(I_1 \cup I_2)$
$\mathbf{f_{I_3}}: \ddot{E} \dashrightarrow \mathbf{I}(\mho)$	$f_{I_3}: \ddot{A} \dashrightarrow I(\mho)$
$\boldsymbol{\mathcal{T}}_{\mathbf{f_{I_3}}(x)}(\upsilon) = \dot{s}\left(\boldsymbol{\mathcal{T}}_{\mathbf{f_{I_1}}(x)}(\upsilon), \boldsymbol{\mathcal{T}}_{\mathbf{f_{I_2}}(x)}(\upsilon)\right)$	$\mathcal{T}_{f_{I_3}(x)}(\upsilon) = \begin{cases} \mathcal{T}_{f_{I_1}(x)}(\upsilon), & x \in \ddot{A} - B \\ \mathcal{T}_{f_{I_2}(x)}(\upsilon), & x \in B - \ddot{A} \\ \max\left\{\mathcal{T}_{f_{I_1}(x)}(\upsilon), \mathcal{T}_{f_{I_2}(x)}(\upsilon)\right\} & \ddot{A} \cap B \end{cases}$
$\boldsymbol{\mathcal{F}}_{\mathbf{f_{I_3}}(x)}(\upsilon) = \dot{t}\left(\boldsymbol{\mathcal{F}}_{\mathbf{f_{I_1}}(x)}(\upsilon), \boldsymbol{\mathcal{F}}_{\mathbf{f_{I_2}}(x)}(\upsilon)\right)$	$\mathcal{F}_{f_{I_3}(x)}(\upsilon) = \begin{cases} \mathcal{F}_{f_{I_1}(x)}(\upsilon), & x \in \ddot{A} - B \\ \mathcal{F}_{f_{I_2}(x)}(\upsilon), & x \in B - \ddot{A} \\ \min\left\{\mathcal{F}_{f_{I_1}(x)}(\upsilon), \mathcal{F}_{f_{I_2}(x)}(\upsilon)\right\} & \ddot{A} \cap B \end{cases}$

Table. 3

In this table we relate our definition of Intersection with that presented by Maji [32]

Methodology used in this paper	Methodology presented by Maji
$\mathbf{I_3}=(\mathbf{I_1} \cap \mathbf{I_2})$	$I_3=(I_1 \cap I_2)$
$\mathbf{f_{I_3}}: \ddot{E} \dashrightarrow \mathbf{I}(\mho)$	$f_{I_3}(x): \ddot{A} \dashrightarrow I(\mho)$
$\boldsymbol{\mathcal{T}}_{\mathbf{f_{I_3}}(x)}(\upsilon) = \dot{t}\left(\boldsymbol{\mathcal{T}}_{\mathbf{f_{I_1}}(x)}(\upsilon), \boldsymbol{\mathcal{T}}_{\mathbf{f_{I_2}}(x)}(\upsilon)\right)$	$\mathcal{T}_{f_{I_3}(x)}(\upsilon) = \min\left\{\mathcal{T}_{f_{I_1}(x)}(\upsilon), \mathcal{T}_{f_{I_2}(x)}(\upsilon)\right\}$
$\boldsymbol{\mathcal{F}}_{\mathbf{f_{I_3}}(x)}(\upsilon) = \dot{s}\left(\boldsymbol{\mathcal{F}}_{\mathbf{f_{I_1}}(x)}(\upsilon), \boldsymbol{\mathcal{F}}_{\mathbf{f_{I_2}}(x)}(\upsilon)\right)$	$\mathcal{F}_{f_{I_3}(x)}(\upsilon) = \max\left\{\mathcal{F}_{f_{I_1}(x)}(\upsilon), \mathcal{F}_{f_{I_2}(x)}(\upsilon)\right\}$

Table.4

6. INTUITIONISTIC SOFT MATRICES

In general, Intuitionistic soft matrices are characteristics of Intuitionistic soft sets. These matrices are effective for saving Intuitionistic soft sets in Computers Memory that was very helpful appropriate in many purposes. Several of them are mentioned from [8, 6].

This describes a struggle to expand the idea of intuitionistic fuzzy soft matrices, soft matrices [6], and fuzzy soft matrices [8]

DEFINITION. 16
Consider an Intuitionistic soft set I with respect to a universal set I(Ʊ). The subset $\mathcal{R}_I$ of cartesian product of I(Ʊ) × Ë is described as:

$$\mathcal{R}_I = \{(f_I(x), x): x \in \ddot{E}, f_I(x) \in I(\mho)\}$$

This represented the relation of (I, Ë). The characteristic function of R_I is written by R_1 Can be uniquely written in the form given below.

$$\Theta\mathcal{R}_I : I(\mho) \times \ddot{E} \to [0,1] \times [0,1] \times [0,1],$$

$$\Theta\mathcal{R}_I(\upsilon, x) = \left(\mathcal{T}_{f_{I_1}(x)}(\upsilon), \mathcal{F}_{f_{I_1}(x)}(\upsilon)\right)$$

Such that

$\mathcal{T}_{f_{I_1}(x)}(\upsilon)$ & $\mathcal{F}_{f_{I_1}(x)}(\upsilon)$ Represents the truthiness, indeterminacy and falseness value of universal set Ʊ such that ʊ are the elements of Ʊ.

DEFINITION. 17
Consider a universal set $\mho = \{\upsilon_1, \upsilon_2, \ldots, \upsilon_m\}$ and set of parameters can be defined as $\ddot{E} = \{x_1, x_2, \ldots, x_n\}$ and

$\mathcal{R}_I$	$f_I(x_1)$	$f_I(x_2)$	$f_I(x_3)$	…	$f_I(x_n)$
υ_1	$\Theta_{\mathcal{R}_I}(\upsilon_1, x_1)$	$\Theta_{\mathcal{R}_I}(\upsilon_1, x_2)$	$\Theta_{\mathcal{R}_I}(\upsilon_1, x_3)$	…	$\Theta_{\mathcal{R}_I}(\upsilon_1, x_n)$
υ_2	$\Theta_{\mathcal{R}_I}(\upsilon_2, x_1)$	$\Theta_{\mathcal{R}_I}(\upsilon_2, x_2)$	$\Theta_{\mathcal{R}_I}(\upsilon_2, x_3)$	…	$\Theta_{\mathcal{R}_I}(\upsilon_2, x_n)$
⋮	⋮	⋮	⋮	⋱	⋮
υ_m	$\Theta_{\mathcal{R}_I}(\upsilon_m, x_1)$	$\Theta_{\mathcal{R}_I}(\upsilon_m, x_2)$	$\Theta_{\mathcal{R}_I}(\upsilon_m, x_3)$	…	$\Theta_{\mathcal{R}_I}(\upsilon_m, x_n)$

Let $\dot{a}_{ij} = \Theta_{\mathcal{R}_I}(\upsilon_i, \mathfrak{x}_j)$ then a matrix can represented as follows:

$$[\dot{a}_{ij}] = \begin{bmatrix} \dot{a}_{11} & \dot{a}_{12} & \dot{a}_{13} & \cdots & \dot{a}_{1n} \\ \dot{a}_{21} & \dot{a}_{22} & \dot{a}_{23} & \cdots & \dot{a}_{2n} \\ \dot{a}_{31} & \dot{a}_{32} & \dot{a}_{33} & \cdots & \dot{a}_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \dot{a}_{m1} & \dot{a}_{m2} & \dot{a}_{m3} & \cdots & \dot{a}_{mn} \end{bmatrix}$$

Whereas $\dot{a}_{ij} = \left(\mathcal{T}_{f_I(\mathfrak{x}_j)}(\upsilon_i), \mathcal{F}_{f_I(\mathfrak{x}_j)}(\upsilon_i)\right) = \left(\mathcal{T}^{\dot{a}}_{ij}, \mathcal{F}^{\dot{a}}_{ij}\right)$ this can be termed as an $m \times n$ Intuitionistic soft matrix of the intuitionistic soft set I with respect to I($\mho$).

Corresponding to this illustration we can relate an intuitionistic soft set I to a matrix of the form $[\dot{a}_{ij}]_{m\times n}$.So that we can characterized any intuitionistic soft set by an IS-Matrix and similarly can used both these ideas. All $m \times n$ IS-Matrix with respect to I($\mho$) can represented as $I_{m\times n}$.By deleting subscript of matrix form $[\dot{a}_{ij}]_{m\times n}$., and using $[\dot{a}_{ij}]$ in place of $[\dot{a}_{ij}]_{m\times n}$.

EXAMPLE. 2
Consider a universal set $\mho = \{\upsilon_1, \upsilon_2\}$, and set of parameters can be defined as $\ddot{E} = \{\mathfrak{x}_1, \mathfrak{x}_2\}$, and I represents an intuitionistic soft set with respect to I($\mho$) defined as:

I=$\{(\mathfrak{x}_1, \{< \upsilon_1, (0.2,0.3) >, < \upsilon_2, (0.3,0.4) >\}), (\mathfrak{x}_2, \{< \upsilon_1, (0.9,0.3) >, < \upsilon_2, (0.2,0.4) >\})\}$

The IS -Matrix written as:

$$[\dot{a}_{ij}] = \begin{bmatrix} (0.2,0.3) & (0.3,0.4) \\ (0.9,0.3) & (0.2,0.4) \end{bmatrix}$$

DEFINITION. 18
An Intuitionistic soft matrix of order $1 \times n$ that is of a single row is known as a Row Intuitionistic soft matrix. Actually, a Row intuitionistic soft matrix is an Intuitionistic soft set in which the universal set have one element.

EXAMPLE. 4
Consider a universal set $\mho = \{\upsilon_1\}$, and set of parameters can be defined as

$\ddot{E} = \{\mathfrak{x}_1, \mathfrak{x}_2, \mathfrak{x}_3\}$, and I represents an intuitionistic soft set with respect to I($\mho$) defined as:

I=$\{(\mathfrak{x}_1, \{< \upsilon_1, (0.7,0.8) >\}), (\mathfrak{x}_2, \{< \upsilon_1, (0.9,0.5) >\}), (\mathfrak{x}_3, \{< \upsilon_1, (0.4,0.2) >\})\}$

The IS -Matrix written as:

$$[\dot{a}_{ij}] = [(0.7,0.8) \quad (0.9,0.5) \quad (0.4,0.2)]$$

DEFINITION. 19
An Intuitionistic soft matrix of order m × 1 that is of a single column is known as Column Intuitionistic soft matrix. Actually, a column Intuitionistic soft matrix is an Intuitionistic soft set in which set of parameters have one element.

EXAMPLE. 3
Consider a universal set $\mho = \{\upsilon_1, \upsilon_2, \upsilon_3, \upsilon_4\}$„ and set of parameters can be defined as $\ddot{E} = \{\mathfrak{x}_1\}$ and I represents an intuitionistic soft set with respect to I($\mho$) defined as:

I=$\{(\mathfrak{x}_1, \{< \upsilon_1, (0.2,0.2) >, < \upsilon_2, (0.4,0.3) >, < \upsilon_3, (0.7,0.5) >, < \upsilon_3, (0.9,0.6) >\})\}$

$$[\dot{a}_{ij}] = \begin{bmatrix} (0.2,0.2) \\ (0.4,0.3) \\ (0.7,0.5) \\ (0.9,0.6) \end{bmatrix}$$

DEFINITION. 20
A square Intuitionistic soft matrix can be defined as an $m \times n$ intuitionistic soft matrix when $m = n$ this means when the rows and columns are equal the matrix is a square Intuitionistic soft matrix. In other words, square Intuitionistic soft matrix is an Intuitionistic soft set in which number of elements of universal set $\mho$ and number of attributes are equal.

DEFINITION. 21
A Diagonal intuitionistic soft matrix can be defined as a Square IS-matrix if it has (0,1) in its non -diagonal places.

EXAMPLE 6.
Consider a universal set $\mho = \{\upsilon_1, \upsilon_2, \upsilon_3, \upsilon_4\}$„ and set of parameters can be defined as $\ddot{E} = \{\mathfrak{x}_1\}$ and I represents an Intuitionistic soft set with respect to I($\mho$) defined as:

I=$\{(\mathfrak{x}_1, \{< \upsilon_1, (0.2,0.1) >, < \upsilon_2, (0.0,0.1) >, < \upsilon_3, (0.0,0.1) >\}), (\mathfrak{x}_2, \{< \upsilon_1, (0.0,0.1) >, < \upsilon_2, (0.0,0.1) >, < \upsilon_3, (0.0,0.1) >\}), (\mathfrak{x}_3, \{< \upsilon_1, (0.2,0.4) >, < \upsilon_2, (0.0,0.1) >, < \upsilon_3, (0.0,0.1) >\})\}$

$$[\dot{a}_{ij}] = \begin{bmatrix} (0.2,0.1) & (0.0,0.1) & (0.0,0.1) \\ (0.0,0.1) & (0.0,0.1) & (0.0,0.1) \\ (0.0,0.1) & (0.0,0.1) & (0.4,0.5) \end{bmatrix}$$

DEFINITION. 22
The transpose of a square intuitionistic soft matrix $[\dot{a}_{ij}]$ of order $m \times n$ can be obtained from square intuitionistic soft matrix of order $n \times m$ by exchanging the Rows and Columns of this IS Matrix. And can be represented as $[\dot{a}^{\mathcal{T}}_{ij}]$. Therefore the intuitionistic soft set associated with $[\dot{a}^{\mathcal{T}}_{ij}]$ becomes a new intuitionistic soft set over the same universe and over the same set of parameters. So that the transpose of an IS-matrix can be regarded as a modified form of ISS with respect to the same universal set $\mho$ and same parameters set $\ddot{E}$.

EXAMPLE. 7
By using Example 2. If the I$\mathcal{S}$-matrix $[\dot{a}_{ij}]$ can be written as:

$$[\dot{a}_{ij}] = \begin{bmatrix} (0.2,0.3) & (0.4,0.5) \\ (0.7,0.5) & (0.9,0.7) \end{bmatrix}$$

The transpose of this matrix can be written as:

$$[\dot{a}_{ij}] = \begin{bmatrix} (0.2,0.3) & (0.7,0.5) \\ (0.4,0.5) & (0.9,0.7) \end{bmatrix}$$

DEFINITION. 23
A Square Intuitionistic soft matrix $[\dot{a}_{ij}]$ of order $n \times n$ is said to be A symmetric Intuitionistic soft matrix is a Square IS Matrix $[\dot{a}_{ij}]$ of order $n \times n$ if transpose of this matrix is equal to that matrix $[\dot{a}^{\mathcal{T}}_{ij}] = [\dot{a}_{ij}]$. Therefore an IS matrix $[\dot{a}_{ij}]$ is symmetric whenever $[\dot{a}_{ij}] = [\dot{a}_{ij}]\ \forall\ i, j$.

EXAMPLE. 8
Consider a universal set $\mho = \{\upsilon_1, \upsilon_2, \upsilon_3\}$, and set of parameters can be defined as $\ddot{E} = \{\mathfrak{x}_1, \mathfrak{x}_2, \mathfrak{x}_3\}$ and I represents an Intuitionistic soft set with respect to I($\mho$) defined as:

I=$\{(\mathfrak{x}_1, \{< \upsilon_1, (0.2,0.1) >, < \upsilon_2, (0.1,0.3) >, < \upsilon_3, (0.1,0.2) >\}), (\mathfrak{x}_2, \{< \upsilon_1, (0.1,0.9) >, < \upsilon_2, (0.7,0.6) >, < \upsilon_3, (0.2,0.3) >\}), (\mathfrak{x}_3, \{< \upsilon_1, (0.2,0.4) >, < \upsilon_2, (0.5,0.6) >, < \upsilon_3, (0.4,0.5) >\})\}$

Symmetric IS-matrix $[\dot{a}_{ij}]$ can be written as:

$$[\dot{a}_{ij}] = \begin{bmatrix} (0.2,0.1) & (0.1,0.3) & (0.1,0.2) \\ (0.1,0.9) & (0.7,0.6) & (0.2,0.3) \\ (0.2,0.4) & (0.5,0.6) & (0.4,0.5) \end{bmatrix}$$

DEFINITION 24.
If $[\dot{a}_{ij}] \in I_{m\times n}$. The matrix $[\dot{a}_{ij}]$ is said to be:

1. ZERO IS-MATRIX
Let $[\dot{a}_{ij}]$ be a Zero I$\mathcal{S}$-matrix represented as [0], when $\dot{a}_{ij}$ = (0, 1) ∀ i,j.

2. UNIVERSAL IS-MATRIX
Let $[\dot{a}_{ij}]$ be a Universal I$\mathcal{S}$-matrix represented as [1], when $\dot{a}_{ij}$ = (1, 0) ∀ i , j.

EXAMPLE. 9
Consider a universal set $\mho = \{\upsilon_1, \upsilon_2, \upsilon_3\}$, and set of parameters can be defined as $\ddot{E} = \{x_1, x_2, x_3\}$ .So, a zero IS-matrix $[\dot{a}_{ij}]$ can be written as:

$$[\dot{a}_{ij}] = \begin{bmatrix} (1,0) & (1,0) & (1,0) \\ (1,0) & (1,0) & (1,0) \\ (1,0) & (1,0) & (1,0) \end{bmatrix}$$

Universal IS-matrix $[\dot{a}_{ij}]$ can be written as:

$$[\dot{a}_{ij}] = \begin{bmatrix} (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) \end{bmatrix}$$

DEFINITION. 25
Suppose that $[\dot{a}_{ij}]$ and $[\dot{b}_{ij}] \in I_{m\times n}$. Therefore

1. IS-SUBMATRIX
Let $[\dot{a}_{ij}]$ and $[\dot{b}_{ij}]$ are two matrices then $[\dot{a}_{ij}]$ is said to be IS-submatrix of $[\dot{b}_{ij}]$ represented as $[\dot{a}_{ij}] \sqsubseteq [\dot{b}_{ij}]$, if $\mathcal{T}_{ij}^{\dot{b}} \geq \mathcal{T}_{ij}^{\dot{a}}$, $\mathcal{F}_{ij}^{\dot{a}} \geq \mathcal{F}_{ij}^{\dot{b}}$,∀ i ,j.

2. PROPER IS-SUBMATRIX
Let $[\dot{a}_{ij}]$ and $[\dot{b}_{ij}]$ are two matrices then $[\dot{a}_{ij}]$is said to be a proper IS-submatrix of represented as $[\dot{a}_{ij}] \sqsubseteq [\dot{b}_{ij}]$, if $\mathcal{T}_{ij}^{\dot{a}} \geq \mathcal{T}_{ij}^{\dot{b}}$, $\mathcal{F}_{ij}^{\dot{a}} \leq \mathcal{F}_{ij}^{\dot{b}}$for at least $\mathcal{T}_{ij}^{\dot{a}} > \mathcal{T}_{ij}^{\dot{b}}$ and $\mathcal{F}_{ij}^{\dot{a}} < \mathcal{F}_{ij}^{\dot{b}}$ ∀ i, j.

3. EQUAL MATRICES
Let $[\dot{a}_{ij}]$ and $[\dot{b}_{ij}]$ are two matrices then $[\dot{a}_{ij}]$and $[\dot{b}_{ij}]$ is said to be an IFS equal matrices, represented as $[\dot{a}_{ij}] = [\dot{b}_{ij}]$, if $\dot{a}_{ij}$ = $\dot{b}_{ij}$ ∀ i, j.

DEFINITION. 26
Consider two matrices $[\dot{a}_{ij}]$and $[\dot{b}_{ij}]$ such that $[\dot{a}_{ij}]$, $[\dot{b}_{ij}] \in I_{m\times n}$. So

I. UNION
Let $[\dot{a}_{ij}]$and $[\dot{b}_{ij}]$ be two I$\mathcal{S}$-Matrices then the union of $[\dot{a}_{ij}]$and$[\dot{b}_{ij}]$, represented as $[\dot{a}_{ij}] \underset{\sim}{\cup} [\dot{b}_{ij}]$ where $\dot{c}_{ij} = (\mathcal{T}_{ij}^{\dot{c}} , \mathcal{F}_{ij}^{\dot{c}})$, as

$\mathcal{T}_{ij}^{\dot{c}} = \max\{\mathcal{T}_{ij}^{\dot{a}} , \mathcal{T}_{ij}^{\dot{b}}\}$,and $\mathcal{F}_{ij}^{\dot{c}} = \min\{\mathcal{F}_{ij}^{\dot{a}} , \mathcal{F}_{ij}^{\dot{b}}\}$ ∀i ,j.

II. INTERSECTION
Let $[\dot{a}_{ij}]$and $[\dot{b}_{ij}]$ be two I$\mathcal{S}$-Matrices then the Intersection of $[\dot{a}_{ij}]$and $[\dot{b}_{ij}]$ represented as $[\dot{a}_{ij}] \cap [\dot{b}_{ij}]$ where $\dot{c}_{ij} = (\mathcal{T}_{ij}^{\dot{c}} , \mathcal{F}_{ij}^{\dot{c}})$ as $\mathcal{T}_{ij}^{\dot{c}} = \min\{\mathcal{T}_{ij}^{\dot{a}} , \mathcal{T}_{ij}^{\dot{b}}\}$ and $\mathcal{F}_{ij}^{\dot{c}} = \max\{\mathcal{F}_{ij}^{\dot{a}} , \mathcal{F}_{ij}^{\dot{b}}\}$ ∀ i ,j.

III. COMPLEMENT
Let $[\dot{a}_{ij}]$ is an I$\mathcal{S}$ matrix then the complement of $[\dot{a}_{ij}]$ represented as $[\dot{a}_{ij}]'$ such that $\dot{c}_{ij} = (\mathcal{F}_{ij}^{\dot{a}} , \mathcal{T}_{ij}^{\dot{a}})$ ∀ i ,j.

EXAMPLE.10 Consider the Example 1.Let two matrices $[\dot{a}_{ij}]$ and $[\dot{b}_{ij}]$ defined as:

$$[\dot{a}_{ij}] = \begin{bmatrix} (0.2,0.3) & (0.4,0.5) & (0.6,0.7) \\ (0.1,0.4) & (0.6,0.7) & (0.8,0.9) \\ (0.8,0.9) & (0.9,0.6) & (0.5,0.6) \end{bmatrix}$$

$$[\dot{b}_{ij}] = \begin{bmatrix} (0.3,0.4) & (0.2,0.1) & (0.1,0.3) \\ (0.5,0.6) & (0.4,0.6) & (0.4,0.5) \\ (0.2,0.1) & (0.9,0.7) & (0.6,0.7) \end{bmatrix}$$

$$[\dot{a}_{ij}] \underset{\sim}{\cup} [\dot{b}_{ij}] = \begin{bmatrix} (0.3,0.3) & (0.4,0.1) & (0.6,0.3) \\ (0.5,0.4) & (0.6,0.6) & (0.8,0.9) \\ (0.8,0.1) & (0.9,0.6) & (0.6,0.6) \end{bmatrix}$$

$$[\dot{a}_{ij}] \cap [\dot{b}_{ij}] = \begin{bmatrix} (0.2,0.4) & (0.2,0.5) & (0.1,0.7) \\ (0.1,0.4) & (0.4,0.7) & (0.4,0.9) \\ (0.2,0.9) & (0.9,0.7) & (0.5,0.7) \end{bmatrix}$$

$$[\dot{a}_{ij}]' = \begin{bmatrix} (0.4,0.2) & (0.5,0.2) & (0.7,0.1) \\ (0.4,0.1) & (0.7,0.4) & (0.9,0.4) \\ (0.9,0.2) & (0.7,0.9) & (0.7,0.5) \end{bmatrix}$$

7. PRODUCTS OF IS-MATRICES

In order to solve decisive issues 2 different types of product of I$\mathcal{S}$ Matrix is introduced in this section.

DEFINITION. 27
Suppose$[\dot{a}_{ij}]$,$[\dot{b}_{ij}]$ are two IS matrices so that$[\dot{a}_{ij}]$, $[\dot{b}_{ij}] \in I_{m\times n}$. So the AND product of $[\dot{a}_{ij}]$ and $[\dot{b}_{ij}]$ defined as follows:

$\wedge : I_{m\times n} \times I_{m\times n} \rightarrow I_{m\times n^2}$ *Such that* $[\dot{a}_{ij}] \wedge [\dot{b}_{ij}] = [\dot{c}_{ip}] = (\mathcal{T}_{ip}^{\dot{c}}, \mathcal{F}_{ip}^{\dot{c}})$

Since

$\mathcal{T}_{ip}^{\dot{c}} = \min(\mathcal{T}_{ij}^{\dot{a}} , \mathcal{T}_{j\kappa}^{\dot{b}})$, and $\mathcal{F}_{ip}^{\dot{c}} = \max(\mathcal{F}_{ij}^{\dot{a}} , \mathcal{F}_{j\kappa}^{\dot{b}})$ such that $p = n(j-1)+\kappa$

DEFINITION. 28 Suppose$[\dot{a}_{ij}]$, $[\dot{b}_{ij}]$ are two IS matrices so that $[\dot{a}_{ij}]$, $[\dot{b}_{ij}] \in I_{m\times n}$. Then OR -product of $[\dot{a}_{ij}]$and $[\dot{b}_{ij}]$ and defined as follows:

$\vee : I_{m\times n} \times I_{m\times n} \rightarrow I_{m\times n^2}$ such that $[\dot{a}_{ij}] \vee [\dot{b}_{ij}] = [\dot{c}_{ip}] = (\mathcal{T}_{ip}^{\dot{c}}, \mathcal{F}_{ip}^{\dot{c}})$

Since

$\mathcal{T}_{ip}^{\dot{c}} = \dot{s}(\mathcal{T}_{ij}^{\dot{a}} , \mathcal{T}_{j\kappa}^{\dot{b}})$ & $\mathcal{F}_{ip}^{\dot{c}} = \dot{t}(\mathcal{F}_{ij}^{\dot{a}}, \mathcal{F}_{j\kappa}^{\dot{b}})$ where $p = n(j-1)+\kappa$

EXAMPLE. 11 suppose $[\dot{a}_{ij}]$, $[\dot{b}_{ij}]$ are two IS matrices so that $[\dot{a}_{ij}]$, $[\dot{b}_{ij}] \in I_{2\times 2}$ defined as follows:

$$[\dot{a}_{ij}] = \begin{bmatrix} (0.2,0.3) & (0.3,0.4) \\ (0.5,0.6) & (0.7,0.8) \end{bmatrix}$$

$$[\dot{b}_{ij}] = \begin{bmatrix} (0.1,0.2) & (0.3,0.5) \\ (0.6,0.7) & (0.2,0.4) \end{bmatrix}$$

$$[\dot{a}_{ij}] \wedge [\dot{b}_{ij}] = \begin{bmatrix} (0.1,0.3) & (0.2,0.5) & (0.1,0.4) & (0.3,0.5) \\ (0.5,0.7) & (0.2,0.6) & (0.6,0.8) & (0.2,0.8) \end{bmatrix}$$

$$[\dot{a}_{ij}] \vee [\dot{b}_{ij}] = \begin{bmatrix} (0.2,0.2) & (0.3,0.3) & (0.3,0.2) & (0.3,0.4) \\ (0.6,0.6) & (0.5,0.6) & (0.7,0.7) & (0.7,0.4) \end{bmatrix}$$

8. MATHODOLOGY SOLUTION OF DECISIVE ISSUES WITH THE HELP OF SCORE FUNCTION OF IS MATRICES

DEFINITION. 29

1) Suppose a matrix $\ddot{A} = [\dot{a}_{ij}] \in I_{m\times n}$ such that $\dot{a}_{ij} = (\mathcal{T}_{ij}^{\dot{a}}, \mathcal{F}_{ij}^{\dot{a}})$.Value of this matrix $\ddot{A}$ can be represented as $V(\ddot{A})$ & described as follows $V(\ddot{A}) = [v_{ij}^{\dot{a}}]_{m\times n}$ such that $v_{ij}^{\dot{a}} = \mathcal{T}_{ij}^{\dot{a}} - \mathcal{F}_{ij}^{\dot{a}}$ for all i and j.
2) Score function of two intuitionistic soft matrices $\ddot{A}$ and $\mathcal{B}$ can be described as $\mathcal{S}(\ddot{A}, \mathcal{B}) = [\dot{s}_{ij}]_{m\times n}$ such that $\dot{s}_{ij} = v_{ij}^{\dot{a}} + v_{ij}^{\dot{b}}$ .Then$\mathcal{S}(\ddot{A}, \mathcal{B}) = V(\ddot{A}) + V(\mathcal{B})$.
3) Total Score Function can be calculated for every object of Ʊ is as follows $\sum_{j=1}^{n} \dot{s}_{ij}$

9. ALGORITHM

The procedure to solve a decisive problem is mentioned below:

Step.1
Select the suitable subset to each set of attributes.

Step.2
Construct IS Matrices corresponding to each attribute.

Step.3
With the help of Value Matrices we can calculate Score Matrix and total Score of all object that contained in Ʊ.

Step.4
Obtain the object of Maximum Score whereas this is the optimal solution of given problem.

Step.5
If the Score is Maximum for greater than one object so we can obtain $\sum_{j=1}^{n}(\dot{s}_{ij})^{\kappa}$ such that $\kappa \geq 2$.Then by selecting an object which have Maximum score and then we can find the Optimal Solution.

10. CASE STUDY

Let's suppose that a shop have laptops of three different kinds $Ʊ = \{\upsilon_1, \upsilon_2, \upsilon_3\}$ and the parameter set or attributes are $\ddot{E} = \{e_1, e_2, e_3\}$ such that e_1 , e_2 and $e_3$stands for and for size, memory and color respectively of laptops. Assume that two friends want to buy a laptop. Before buying a laptop they must have their own parameters set. They can buy a laptop according to their parameters set with the help of Score Function of IS matrices.

Step.1
At first, both person will select the parameters set $\ddot{A} = \{e_1, e_2, e_3\}$ & $Ƀ = \{e_1, e_2, e_3\}$.

Step.2
So according to parameters set Ä and Ƀ we make IS-matrices and are given below:

$$[\dot{a}_{ij}] = \begin{bmatrix} (0.3,0.4) & (0.4,0.5) & (0.1,0.2) \\ (0.9,0.1) & (0.6,0.8) & (0.2,0.4) \\ (0.2,0.3) & (0.5,0.9) & (0.8,0.6) \end{bmatrix}$$

$$[\dot{b}_{ij}] = \begin{bmatrix} (0.9,0.6) & (0.5,0.2) & (0.8,0.1) \\ (0.7,0.9) & (0.3,0.2) & (0.3,0.6) \\ (0.9,0.5) & (0.1,0.4) & (0.6,0.4) \end{bmatrix}$$

Step. 3
With the help of Value Matrices we can calculate Score Matrix and total Score of all object that contained in Ʊ. So that the respective Value Matrices are given below:

$$V(\ddot{A}) = \begin{pmatrix} -0.1 & -0.1 & -0.1 \\ 0.8 & -0.2 & -0.2 \\ -0.1 & -0.4 & -0.3 \end{pmatrix}$$

$$V(Ƀ) = \begin{pmatrix} 0.3 & 0.3 & 0.7 \\ -0.2 & 0.1 & -0.3 \\ 0.4 & -0.3 & 0.2 \end{pmatrix}$$

Step.4
Obtain the object of Maximum Score whereas this is the optimal solution of given problem.

$$\mathcal{S}(\ddot{A}, Ƀ) = \begin{pmatrix} 0.2 & 0.2 & 0.6 \\ 0.6 & -0.1 & -0.5 \\ 0.3 & -0.7 & -0.1 \end{pmatrix}$$

$$\text{Total score} = \begin{pmatrix} 1.0 \\ 0.0 \\ -0.5 \end{pmatrix}$$

Step.5
So that laptop $\mathcal{S}_1$ is selected by two friends.

11. CONCLUSION

This paper discusses some redefined operations of Intuitionistic Fuzzy Matrices. And use of these operations in decision

making issues such as in laptop selection by using Score Function of IS-matrices. So we have found which laptop will be selected. And as a result of Intuitionistic Fuzzy Matrices comes out to be more applicable and efficient than fuzzy soft matrices

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