Total Edge Dominating Functions of Corona Product Graph of a Cycle with a Star

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ABSTRACT
Graph Theory has been realized as one of the most flourishing branches of Mathematics of recent origin with wide applications to combinatorial problems and to classical algebraic problems. The theory of domination in graphs is an emerging area of research in graph theory today. It has been studied extensively and finds applications to various branches of Science & Technology.

Frucht and Harary [6] introduced a new product on two graphs G_1 and G_2, called corona product denoted by $G_1 \odot G_2$. The object is to construct a new and simple operation on two graphs G_1 and G_2 called their corona, with the property that the group of the new graph is in general isomorphic with the wreath product of the groups of G_1 and of G_2.

In this paper some results on minimal total edge dominating sets and functions of corona product graph of a cycle with a star are discussed.

Keywords
Corona Product, total edge dominating set, total edge domination number, total edge dominating function.

1. INTRODUCTION
Domination Theory has a wide range of applications to many fields like Engineering, Communication Networks, Social sciences, linguistics, physical sciences and many others. Allan, R.B. and Laskar, R.[1], Cockayne, E.J. and Hedetniemi, S.T. [4] have studied various domination parameters of graphs. An introduction and an extensive overview on domination in graphs and related topics is surveyed and detailed in the two books by Haynes et al [7, 8].

Products are often viewed as a convenient language with which one can describe structures, but they are increasingly being applied in more substantial ways. Every branch of mathematics employs some notion of product that enables the combination or decomposition of its elemental structures.

The concept of edge domination was introduced by Mitchell and Hedetniemi [11] and it is explored by many researchers. Arumugam and Velammal [3] have discussed the edge domination in graphs while the fractional edge domination in graphs is discussed in Arumugam and Jerry [2]. The complementary edge domination in graphs is studied by Kulli and Soner [10] while Jayaram [9] has studied the line dominating sets and obtained bounds for the line domination number. The bipartite graphs with equal edge domination number and maximum matching cardinality are characterized by Dutton and Klostermeyer [5] while Yannakakis and Gavril [14] have shown that edge dominating set problem is NP-complete even when restricted to planar or bipartite graphs of maximum degree. The edge domination in graphs of cubes is studied by Zelinka [15].

2. CORONA PRODUCT GRAPH $C_n \odot K_{1,m}$
The **corona product** of a cycle C_n wirh a star graph $K_{1,m}$ for m ≥ 2, is a graph obtained by taking one copy of a n-vertex graph C_n and n copies of $K_{1,m}$ and then joining the i[th] vertex of C_n to all vertices of i[th] copy of $K_{1,m}$. This graph is denoted by $C_n \odot K_{1,m}$.

The vertices in C_n are denoted by$v_1, v_2, \dots\dots\dots, v_n$ and the edges in C_n by $e_1, e_2, \dots\dots, e_n$ where e_i is the edge joining the vertices v_i and v_{i+1}, i≠n. For i = n, e_n is the edge joining the vertices $v_n$and v_1.

The vertex in the first partition of $i^{th} copy of K_{1,m}$ is denoted by u_i and the vertices in the second partition of $i^{th} copy of K_{1,m}$ are denoted by $w_{i1}, w_{i2}, \dots\dots\dots\dots, w_{im}$. The edges in the $i^{th} copy of K_{1,m}$ are denoted by l_{ij} where l_{ij} is the edge joining the vertex u_i to the vertex w_{ij}. There are another type of edges, denoted by h_i, h_{ij}. Here h_i is the edge joining the vertex v_i in C_n to the vertex u_i in the $i^{th} copy of K_{1,m}$. The edge h_{ij} is the edge joining the vertex v_i in C_n to the vertex w_{ij} in the $i^{th} copy of K_{1,m}$.

The edge induced sub graph on the set of edges

$E_i = \{h_i, h_{ij}, l_{ij} : j = 1,2, \dots\dots\dots, m\}$is denoted by H_i, for i = 1,2,.......,n.

Some graph theoretic properties of corona product graph $C_n \odot K_{1,m}$ and edge dominating sets, edge domination number of this graph are studied by Sreedevi, J [13]. Some results on edge dominating functions of $C_n \odot K_{1,m}$ are presented in Sreedevi, J [12].

3. TOTAL EDGE DOMINATING SETS AND TOTAL EDGE DOMINATING FUNCTIONS
First we discuss total edge dominating sets (TEDSs), total edge domination number of the graph $G = C_n \odot K_{1,m}$. Further some results on minimal total edge dominating functions of this graph are obtained.

Theorem 3.1: The total edge domination number of

$$G = C_n \odot K_{1,m} = \begin{cases} \dfrac{3n}{2}, & \text{if } n \text{ is even,} \\ \dfrac{3n+1}{2}, & \text{if } n \text{ is odd.} \end{cases}$$

Proof: Consider the graph $G = C_n \odot K_{1,m}$.

Case 1: Suppose n is even.

Let T = $\{h_1, h_2 \dots \dots \dots \dots \dots, h_n; e_2, e_4, \dots \dots, e_n\}$.

Then the edge h_i is adjacent to m edges l_{ij}in H_i, j = 1,2,......,m; two edges e_{i-1}, e_i when i≠1, e_1, e_n when i = 1; m edges h_{ij} in H_i, j = 1,2,.......,m. Since this is true for all i = 1,2,......,n, it follows that T dominates all the edges of G. Also the edges of T dominate among themselves.

Thus T becomes a Total Edge Dominating Set (TEDS) of G. This set is also minimal because

if we delete any edge say h_i from T, then the edges in H_i are not dominated by any edge in $T - \{h_i\}$. Again if we delete any edge say e_i from T, then the edges h_i, h_{i+1} are not adjacent to any edge in $T - \{e_i\}$.

Therefore T is a minimal total edge dominating set.

Now we have chosen n edges h_i into T and $\frac{n}{2}$ edges e_i into T.

Therefore $|\mathrm{T}| = n + \frac{n}{2} = \frac{3n}{2}$, $if n is even.$

We could easily see that any other choice of selection of edges in $C_n \odot K_{1,m}$less than $n + \frac{n}{2}$ when n is even into T, cannot make T a TEDS.

Hence total edge domination number of G is$\frac{3n}{2}$, if n is even.

Case 2: Suppose n is odd.

Let T = $\{h_1, h_2 \dots \dots \dots \dots \dots, h_n; e_1, e_3, \dots \dots, e_n\}$.

Then as in Case 1, we can easily verify that T is a total edge dominating set of G.

Further $|\mathrm{T}| = n + \frac{n+1}{2} = \frac{3n+1}{2}$.

Then we could easily see that for any other choice of selection of edges in $C_n \odot K_{1,m}$ less

than $\frac{3n+1}{2}$ into T, if n is odd, cannot make T a TEDS.

$$\text{Therefore}\gamma'_e(G) = \begin{cases} \frac{3n}{2}, & if n is even, \\ \frac{3n+1}{2}, & if n is odd. \end{cases} \blacksquare$$

Theorem 3.2: Let T be a MTEDS of $G = C_n \odot K_{1,m}$.

Then the function f: E → [0,1] defined by

$$f(e) = \begin{cases} 1, & \text{if} e \in T, \\ 0, & \text{otherwise}, \end{cases}$$

is a MTEDF of $G = C_n \odot K_{1,m}$.

Proof: Consider the graph $G = C_n \odot K_{1,m}$.

Case I: Suppose n is even.

Let T = $\{h_1, h_2 \dots \dots \dots \dots \dots, h_n; e_2, e_4, \dots \dots, e_n\}$.

Then we have seen in Theorem 3.2.1 that T is a MTEDS.

We now show that f is a MTEDF.

Now the summation value taken over N(e) of $e \in E$ is as follows:

Case 1: Let $e_i \in C_n$ where i=1,2,........,n.

Then $adj(e_i) = 2m + 4$.

If i is even, then

$$\sum_{e \in \mathrm{N}(e_i)} f(e) = (0+0) + \left(1 + \underbrace{0 + 0 + \cdots + 0}_{m-times}\right) + \left(1 + \underbrace{0 + 0 + \cdots + 0}_{m-times}\right) = 2 > 1$$

and if i is odd, then

$$\sum_{e \in \mathrm{N}(e_i)} f(e) = (1+1) + \left(1 + \underbrace{0 + 0 + \cdots + 0}_{m-times}\right) + \left(1 + \underbrace{0 + 0 + \cdots + 0}_{m-times}\right) = 4 > 1.$$

Case 2: Let $l_{is} \in H_i$ where i=1,2,......n;s=1,2,.......,m.

Then $adj(l_{is}) = m + 1$.

$$\text{Now} \sum_{e \in \mathrm{N}(l_{is})} f(e) = \left(\underbrace{0 + 0 + \cdots \dots \dots + 0}_{(m-1)-times}\right) + (1+0) = 1.$$

Case 3: Let $h_i \in H_i$ where i=1,2,........n.

Then $adj(h_i) = 2m + 2$.

$$\text{Now} \sum_{e \in \mathrm{N}(h_i)} f(e) = (1+0) + \left(\underbrace{0 + 0 + \cdots + 0}_{m-times}\right) + \left(\underbrace{0 + 0 + \cdots + 0}_{m-times}\right) = 1.$$

Case 4: Let $h_{is} \in H_i$ where i=1,2,........n;s=1,2,.....,m.

Then $adj(h_{is}) = m + 3$.

$$\text{Now} \sum_{e \in \mathrm{N}(h_{is})} f(e) = (0+1) + \left(\underbrace{0 + 0 + \cdots \dots \dots + 0}_{(m-1)-times}\right) + 1 + 0 = 2 > 1.$$

Then we have proved that

$$\sum_{e \in \mathrm{N}(h)} f(e) \geq 1, \forall h \in E.$$

So f is a TEDF.

Now we check for the minimalityof f.

Define $g: E \to [0,1]$ by

$$g(e) = \begin{cases} r, & \text{if } e = h_k \in T \text{ for some k}, \\ 1, & \text{if } e \in T - \{h_k\}, \\ 0, & \text{otherwise}, \end{cases}$$

where$0 < r < 1$.

Since strict inequality holds at the edge $h_k \in E$, it follows that g < f.

The summation value taken over N(e) of e ∈ E is as follows:

Case (**i**): Let $e_i \in C_n$ where i=1,2,.......,n.

Sub case 1: Let $h_k \in N(e_i)$. Then k = i or i + 1, if$i \neq 1$ and k = 1 or n, if i = n.

If i $\neq 1, n$, then

$$\sum_{e\in N(e_i)} g(e) = (0+0) + \Big(r + \underbrace{0+0+\cdots+0}_{m-times}\Big) + \Big(1 + \underbrace{0+0+\cdots+0}_{m-times}\Big)$$

$$= r+1 > 1, \text{if } i \text{ is even}$$

and $$\sum_{e\in N(e_i)} g(e) = (1+1) + \Big(r + \underbrace{0+0+\cdots+0}_{m-times}\Big) + \Big(1 + \underbrace{0+0+\cdots+0}_{m-times}\Big)$$

$$= r+3 > 1, \text{if } i \text{ is odd.}$$

If i = 1, then

$$\sum_{e\in N(e_i)} g(e) = (1+1) + \Big(r + \underbrace{0+0+\cdots+0}_{m-times}\Big) + \Big(1 + \underbrace{0+0+\cdots+0}_{m-times}\Big) = r+3 > 1.$$

If i = n, then we can show that

$$\sum_{e\in N(e_i)} g(e) = r+1 > 1.$$

Sub case 2:Let $h_k \notin N(e_i)$.

If i $\neq 1$, n, then

$$\sum_{e\in N(e_i)} g(e) = (0+0) + \Big(1 + \underbrace{0+0+\cdots+0}_{m-times}\Big) + \Big(1 + \underbrace{0+0+\cdots+0}_{m-times}\Big)$$

$$= 2 > 1, \text{if } i \text{ is even}$$

and $$\sum_{e\in N(e_i)} g(e) = (1+1) + \Big(1 + \underbrace{0+0+\cdots+0}_{m-times}\Big) + \Big(1 + \underbrace{0+0+\cdots+0}_{m-times}\Big)$$

$$= 4 > 1, \text{if } i \text{ is odd.}$$

If i = 1, then

$$\sum_{e\in N(e_i)} g(e) = (1+1) + \Big(1 + \underbrace{0+0+\cdots+0}_{m-times}\Big) + \Big(1 + \underbrace{0+0+\cdots+0}_{m-times}\Big) = 4 > 1.$$

If i = n, then we can show that

$$\sum_{e\in N(e_i)} g(e) = 2 > 1.$$

Case (**ii**): Let $l_{ij} \in H_i$ where i=1,2,......n;j=1,2,.......,m.

Sub case 1:Let $h_k \in N(l_{ij})$. Then k = i.

Now $$\sum_{e\in N(l_{ij})} g(e) = \Big(\underbrace{0+0+\cdots+0}_{(m-1)-times}\Big) + (r+0) = r < 1.$$

Sub case 2: Let $h_k \notin N(l_{ij})$.

Then $$\sum_{e\in N(l_{ij})} g(e) = \Big(\underbrace{0+0+\cdots+0}_{(m-1)-times}\Big) + (1+0) = 1.$$

Case (**iii**): Let $h_i \in H_i$ where i=1,2,........n.

If i $\neq 1$, then

$$\sum_{e\in N(h_i)} g(e) = (1+0) + \Big(\underbrace{0+0+\cdots+0}_{m-times}\Big) + \Big(\underbrace{0+0+\cdots+0}_{m-times}\Big) = 1.$$

If i = 1, then we can show that

$$\sum_{e\in N(h_i)} g(e) = 1.$$

Case ($\boldsymbol{iv}$): Let $h_{ij} \in H_i$ where i=1,2,........n;j=1,2,.....,m.

Sub case 1: Let $h_k \in N(h_{ij})$. Then k = i.

If i $\neq 1$, then

$$\sum_{e\in N(h_{ij})} g(e) = (0+1) + \Big(\underbrace{0+0+\cdots\ldots\ldots+0}_{(m-1)-times}\Big) + r + 0$$

$$= r+1 > 1.$$

If i = 1, then we can show that

$$\sum_{e\in N(h_{ik})} g(e) = r+1 > 1.$$

Sub case 2: Let $h_k \notin N(h_{ij})$.

If i $\neq 1$, then

$$\sum_{e\in N(h_{ij})} g(e) = (0+1) + \Big(\underbrace{0+0+\cdots\ldots\ldots+0}_{(m-1)-times}\Big) + 1 + 0$$

$$= 2 > 1.$$

If i = 1, then we can show that

$$\sum_{e\in N(h_{ik})} g(e) = 2 > 1.$$

Thus we have seen that $\sum_{e\in N(h)} g(e) < 1$ for some edge h $\in$ E.

So g is not a TEDF.

Since g is defined arbitrarily, it follows that there exists no g $<$ f such that g is a TEDF.

Thus f is a minimal total edge dominating function (MTEDF).

Case II: Suppose n is odd.

Let T = $\{h_1, h_2 \ldots\ldots\ldots\ldots\ldots, h_n; e_1, e_3, \ldots\ldots, e_n\}$.

Then we have seen in Theorem 3.2.1 that T is a MTEDS.

Proceeding in similar lines as in Case I, we can show that

$$\sum_{e\in N(h)} f(e) \geq 1, \forall h \in E.$$

ie., f is a TEDF.

Now we check for the minimalityof f.

Define $g: E \to [0,1]$ by

$$g(e) = \begin{cases} r, & \text{if} e = h_k \in T \text{for some k}, \\ 1, & \text{if} e \in T - \{h_k\}, \\ 0, & \text{otherwise}, \end{cases}$$

where $0 < r < 1$.

Since strict inequality holds at the edge $h_k \in E$, it follows that g < f.

The functional values of g(e), e ∈ E are as follows.

As in Case I, for Case (ii), except for i = 1 in Case (iii), except for i = 1in Case (iv),

the functional values of

$\sum_{e \in N(h)} g(e)$ for h = l_{ij}, h_i, h_{ij} respectively are same.

Now for Case(i), we have

if $h_k \in N(e_i)$, then

$$\sum_{e \in N(e_i)} g(e) = (1+1) + \Big(r + \underbrace{0+0+\cdots+0}_{m-times}\Big) + \Big(1 + \underbrace{0+0+\cdots+0}_{m-times}\Big)$$
$$= r + 3 > 1, \text{ if i is even},$$

$$\sum_{e \in N(e_i)} g(e) = (0+0) + \Big(r + \underbrace{0+0+\cdots+0}_{m-times}\Big) + \Big(1 + \underbrace{0+0+\cdots+0}_{m-times}\Big),$$
$$= r + 1 > 1, \text{if i is odd and if } i \neq 1, n.$$

Again if $h_k \in N(e_i)$ and i = 1,n, then

$$\sum_{e \in N(e_i)} g(e) = (0+1) + \Big(r + \underbrace{0+0+\cdots+0}_{m-times}\Big) + \Big(1 + \underbrace{0+0+\cdots+0}_{m-times}\Big)$$
$$= r + 2 > 1.$$

If $h_k \notin N(e_i)$, then

$$\sum_{e \in N(e_i)} g(e) = (1+1) + \Big(1 + \underbrace{0+0+\cdots+0}_{m-times}\Big) + \Big(1 + \underbrace{0+0+\cdots+0}_{m-times}\Big), \text{if i is even},$$
$$= 4 > 1,$$

$$\sum_{e \in N(e_i)} g(e) = (0+0) + \Big(1 + \underbrace{0+0+\cdots+0}_{m-times}\Big) + \Big(1 + \underbrace{0+0+\cdots+0}_{m-times}\Big),$$
$$= 2 > 1, \text{if i is odd and } i \neq 1, n.$$

If $i = 1, n$, then

$$\sum_{e \in N(e_i)} g(e) = (0+1) + \Big(1 + \underbrace{0+0+\cdots+0}_{m-times}\Big) + \Big(1 + \underbrace{0+0+\cdots+0}_{m-times}\Big)$$
$$= 3 > 1.$$

Again for i = 1 in Case(iii), we have

$$\sum_{e \in N(h_i)} g(e) = (1+1) + \Big(\underbrace{0+0+\cdots+0}_{m-times}\Big) + \Big(\underbrace{0+0+\cdots+0}_{m-times}\Big) = 2.$$

Again for i = 1 in Case(iv), we can show for $h_{ij} \in H_i$ that

if $h_k \in N(h_{ij})$, then

$$\sum_{e \in N(h_{ij})} g(e) = (1+1) + \Big(\underbrace{0+0+\cdots+0}_{(m-1)-times}\Big) + (r+0)$$
$$= 2 + r$$

and if $h_k \notin N(h_{ij})$, then

$$\sum_{e \in N(h_{ij})} g(e) = (1+1) + \Big(\underbrace{0+0+\cdots+0}_{(m-1)-times}\Big) + (1+0)$$
$$= 3.$$

We have seen in Case (ii) that $\sum_{e \in N(l_{ij})} g(e) < 1$ for some edge $l_{ij} \in$ E.

So, g is not a TEDF.

Since g is defined arbitrarily, it follows that there exists no $g < f$ such that g is a TEDF.

Thus f is a minimal total edge dominating function (MTEDF).■

Theorem 3.3: A function $f: E \to [0,1]$ defined by $f(e) = \frac{1}{q}, \forall e \in E$ is a total edge dominating function of $G = C_n \odot K_{1,m}$ if ≤ 3 . It is a minimal total edge dominating function if q = 3.

Proof: Let f be the function defined as in the hypothesis.

Case I: Suppose $q < 3$.

The summation value taken over $N(e)$ of $e \in E$ is as follows:

Case 1: Let $e_i \in C_n$ where i=1,2,........,n.

Then $adj(e_i) = 2m + 4$.

Now
$$\sum_{e \in N(e_i)} f(e) = \left(\frac{1}{q} + \frac{1}{q}\right) + \Bigg(\underbrace{\frac{1}{q} + \frac{1}{q} + \cdots + \frac{1}{q}}_{(m+1)-times}\Bigg) + \Bigg(\underbrace{\frac{1}{q} + \frac{1}{q} + \cdots + \frac{1}{q}}_{(m+1)-times}\Bigg) = \frac{2m+4}{q} > 1,$$

since m≥ 2, $q < 3$.

Case 2: Let $l_{ij} \in H_i$ where i=1,2,......n;j=1,2,.......,m.

Then $adj(l_{ij}) = m + 1$.

$$\text{Now} \sum_{e\in N(l_{ij})} f(e) = \left(\underbrace{\frac{1}{q}+\frac{1}{q}+\cdots+\frac{1}{q}}_{(m-1)-times}\right)+\frac{1}{q}+\frac{1}{q}=\frac{m+1}{q} > 1, \text{since } m \ge 2, q < 3.$$

Case 3: Let $h_i \in H_i$ where i = 1,2,........n.

Then $adj(h_i) = 2m + 2$.

$$\text{Now} \sum_{e\in N(h_i)} f(e) = \left(\frac{1}{q}+\frac{1}{q}\right)+\left(\underbrace{\frac{1}{q}+\frac{1}{q}+\cdots+\frac{1}{q}}_{m\ -times}\right)+\left(\underbrace{\frac{1}{q}+\frac{1}{q}+\cdots+\frac{1}{q}}_{m\ -times}\right)$$

$$=\frac{2m+2}{q} > 1, \text{since } m \ge 2, q < 3.$$

Case 4: Let $h_{ij} \in H_i$ where i = 1,2,........ n ; j = 1,2,.....,m.

Then $adj(h_{ij}) = m + 3$.

$$\text{Now} \sum_{e\in N(h_{ij})} f(e) = \left(\frac{1}{q}+\frac{1}{q}\right)+\left(\underbrace{\frac{1}{q}+\frac{1}{q}+\cdots+\frac{1}{q}}_{m\ -times}\right)+\frac{1}{q} = \frac{m+3}{q} > 1,$$

since $m \ge 2, q < 3$.

Therefore for all possibilities of $e \in E$, we get

$$\sum_{e\in N(h)} f(e) \ge 1, \forall h \in E.$$

This implies that f is a TEDF.

Now we check for the minimalityof f.

Define $g: E \to [0,1]$ by

$$g(e) = \begin{cases} r, & \text{if } e = h_k \in E \text{for some k}, \\ \frac{1}{q}, & \text{otherwise,} \end{cases}$$

where $0 < r < \frac{1}{q}$.

Since strict inequality holds at the edge $h_k \in E$, it follows that g< f.

The summation value taken over $N(e)$ of e ∈E is as follows :

Case (**i**): Let $e_i \in C_n$ where i=1,2,.......,n.

Sub case 1: Let $h_k \in N(e_i)$. Then k = i or i + 1, if i≠1and k = 1 or n, if i = n.

$$\text{Now} \sum_{e\in N(e_i)} g(e) = \left(\frac{1}{q}+\frac{1}{q}\right)+\left(r+\underbrace{\frac{1}{q}+\frac{1}{q}+\cdots+\frac{1}{q}}_{m\ -times}\right)+\left(\underbrace{\frac{1}{q}+\frac{1}{q}+\cdots+\frac{1}{q}}_{(m+1)\ -\ times}\right)$$

$$= r + \frac{2m+3}{q} > 1, \text{since} m \ge 2, q < 3.$$

Sub case 2: Let $h_k \notin N(e_i)$.

$$\text{Then} \sum_{e\in N(e_i)} g(e) = \left(\frac{1}{q}+\frac{1}{q}\right)+\left(\underbrace{\frac{1}{q}+\frac{1}{q}+\cdots+\frac{1}{q}}_{(m+1)\ -\ times}\right)+\left(\underbrace{\frac{1}{q}+\frac{1}{q}+\cdots+\frac{1}{q}}_{(m+1)\ -\ times}\right)$$

$$= \frac{2m+4}{q} > 1, \text{since} m \ge 2, q < 3.$$

Case(**ii**): Let $l_{ij} \in H_i$ where i=1,2,......n;j=1,2,.......,m.

Sub case 1: Let $h_k \in N(l_{ij})$. Then k= i.

$$\text{Now} \sum_{e\in N(l_{ij})} g(e) = \left(\underbrace{\frac{1}{q}+\frac{1}{q}+\cdots+\frac{1}{q}}_{(m-1)\ -times}\right)+r+\frac{1}{q} = r+\frac{m}{q} > 1, \text{since} m \ge 2, q < 3.$$

Sub case 2: Let $h_k \notin N(l_{ij})$.

$$\text{Then} \sum_{e\in N(l_{ij})} g(e) = \left(\underbrace{\frac{1}{q}+\frac{1}{q}+\cdots+\frac{1}{q}}_{(m-1)\ -times}\right)+\frac{1}{q}+\frac{1}{q}=\frac{m+1}{q} > 1, \text{since} m \ge 2, q < 3.$$

Case(**iii**): Let $h_i \in H_i$ where i=1,2,........n.

$$\text{Then} \sum_{e\in N(h_i)} g(e) = \left(\frac{1}{q}+\frac{1}{q}\right)+\left(\underbrace{\frac{1}{q}+\frac{1}{q}+\cdots+\frac{1}{q}}_{m\ -times}\right)+\left(\underbrace{\frac{1}{q}+\frac{1}{q}+\cdots+\frac{1}{q}}_{m\ -times}\right)$$

$$=\frac{2m+2}{q} > 1, \text{since } m \ge 2, q < 3.$$

Case($\boldsymbol{iv}$): Let $h_{ij} \in H_i$ where i=1,2,........n; j=1,2,.....,m.

Sub case1: Let $h_k \in N(h_{ij})$. Then k=i.

$$\text{Now} \sum_{e\in N(h_{ij})} g(e) = \left(\frac{1}{q}+\frac{1}{q}\right)+\left(r+\underbrace{\frac{1}{q}+\frac{1}{q}+\cdots+\frac{1}{q}}_{(m-1)-times}\right)+\frac{1}{q}$$

$$= r + \frac{m+2}{q} > 1, \text{since } m \ge 2, q < 3.$$

Sub case 2: Let $h_k \notin N(h_{ij})$.

$$\text{Then} \sum_{e\in N(h_{ij})} g(e) = \left(\frac{1}{q}+\frac{1}{q}\right)+\left(\underbrace{\frac{1}{q}+\frac{1}{q}+\cdots+\frac{1}{q}}_{m-\ times}\right)+\frac{1}{q} = \frac{m+3}{q} > 1,$$

since$m \ge 2, q < 3$.

Hence it follows that

$$\sum_{e\in N(h)} g(e) > 1, \forall h \in E.$$

Since g is defined arbitrarily for all possibilities of defining a function g< f, we see that g

is a TEDF.

This implies that f is not a MTEDF.

Case II: Suppose q = 3.

Substituting q=3 in all the Cases 1,2,3,4 of Case I, we have for $e_i \in C_n$,

$$\sum_{e\in N(e_i)} f(e) = \frac{2m+4}{q} = \frac{2m+4}{3} = \frac{2m+1}{3} + 1 > 1.$$

For $l_{ij} \in H_i$,

$$\sum_{e\in N(l_{ij})} f(e) = \frac{m+1}{q} = \frac{m+1}{3} \geq 1, \text{since} m \geq 2.$$

Now for $h_i \in H_i$,

$$\sum_{e\in N(h_i)} f(e) = \frac{2m+2}{q} = \frac{2m+2}{3} > 1, \text{since} m \geq 2.$$

For$h_{ij} \in H_i$,

$$\sum_{e\in N(h_{ij})} f(e) = \frac{m+3}{q} = \frac{m+3}{3} = \frac{m}{3} + 1 > 1.$$

Therefore for all possibilities of $e \in E$, we get

$$\sum_{e\in N(h)} f(e) \geq 1, \forall h \in \text{E}.$$

This implies that f is a TEDF.

Now we check for theminimalityof f.

Define g: $E \to [0,1]$ by

$$\text{g}(e) = \begin{cases} r, & \text{if } e = h_k \in E \text{ for some k}, \\ \frac{1}{q}, & \text{otherwise}, \end{cases}$$

where $0 < r < \frac{1}{q}$.

Since strict inequality holds at the edge $h_k \in E$, it follows that g < f.

Then we can show as in Case (i) that for $e_i \in C_n$,

$$\sum_{e\in N(e_i)} g(e) = r + \frac{2m+3}{q} = \text{r} + \frac{2m+3}{3} = \text{r} + \frac{2m}{3} + 1 > 1, \text{if} h_k \in N(e_i)$$

and $$\sum_{e\in N(e_i)} g(e) = \frac{2m+4}{q} = \frac{2m+4}{3} = \frac{2m+1}{3} + 1 > 1, \text{if} h_k \notin N(e_i).$$

Again we can see as in Case(ii) that for $l_{ij} \in H_i$,if$h_k \in N(l_{ij})$, then

$$\sum_{e\in N(l_{ij})} g(e) = r + \frac{m}{q} = r + \frac{m}{3},$$

$$< 1 \ \left(\because \text{if} m = 2, r = 0.1, \text{then} r + \frac{m}{3} = 0.1 + \frac{2}{3} = \frac{2.3}{3} < 1\right)$$

and $$\sum_{e\in N(l_{ij})} g(e) = \frac{m+1}{q} = \frac{m+1}{3} \geq 1, \text{if} h_k \notin N(l_{ij}).$$

Again we can see as in Case(iii) that for $h_i \in H_i$,

$$\sum_{e\in N(h_i)} g(e) = \frac{2m+2}{q} = \frac{2m+2}{3} > 1, \text{since} m \geq 2.$$

Similarly we can see as in Case (iv) that for $h_{ij} \in H_i$,if$h_k \in N(h_{ij})$, then

$$\sum_{e\in N(h_{ij})} g(e) = r + \frac{m+2}{q} = r + \frac{m+2}{3} > 1, \text{since m} \geq 2$$

and $$\sum_{e\in N(h_{ij})} g(e) = \frac{m+3}{q} = \frac{m+3}{3} = \frac{m}{3} + 1 > 1,$$

if $h_k \notin N(h_{ij})$.

Thus we have seen that $\sum_{e\in N(h)} g(e) < 1$ for some edge

h ∈ E.

So, g is not a TEDF.

Since g is defined arbitrarily, it follows that for all possibilities of defining a function g < f,

g is not a TEDF.

Therefore f is a MTEDF.■

Theorem 3.4 : A function $f: E \to [0,1]$ defined by

$f(e) = \frac{p}{q}, \forall e \in E$ where $p = \min\{m,n\}, q = \max\{m,n\}$is a total edge dominating function of $G = C_n \odot K_{1,m}$ if $\frac{p}{q} \geq \frac{1}{3}$. It becomes a minimal total edge dominating function if

$\frac{1}{3} \leq \frac{p}{q} < \frac{1}{2}$.

Proof: Consider the graph$G = C_n \odot K_{1,m}$.

Let $f: E \to [0,1]$ be defined by $f(e) = \frac{p}{q}, \forall e \in E$ where

$p = \min\{m,n\}, q = \max\{m,n\}$.

Clearly $\frac{p}{q} > 0$.

Case I: Suppose $\frac{p}{q} \geq \frac{1}{3}$.

The summation value taken over $N(e)$ of $e \in E$ is as follows:

Case 1: Let $e_i \in C_n$ where i=1,2,........,n.

Then $$\sum_{e\in N(e_i)} f(e) = \left(\frac{p}{q} + \frac{p}{q}\right) + \left(\underbrace{\frac{p}{q} + \frac{p}{q} + \cdots + \frac{p}{q}}_{(m+1)-times}\right) + \left(\underbrace{\frac{p}{q} + \frac{p}{q} + \cdots + \frac{p}{q}}_{(m+1)-times}\right) = (2m+4)\left(\frac{p}{q}\right)$$

$$\geq (2m+4)\left(\frac{1}{3}\right)\left(\because \frac{p}{q} \geq \frac{1}{3}\right)$$

$$> 1. (\because \text{m} \geq 2)$$

Case 2: Let $l_{ij} \in H_i$ where i=1,2,......n;j=1,2,.......,m.

Then $\sum_{e\in N(l_{ij})} f(e) = \left(\underbrace{\frac{p}{q}+\frac{p}{q}+\cdots+\frac{p}{q}}_{(m-1)-times}\right)+\frac{p}{q}+\frac{p}{q}$

$$= (m+1)\left(\frac{p}{q}\right)$$

$$\geq (m+1)\left(\frac{1}{3}\right)\left(\because \frac{p}{q}\geq\frac{1}{3}\right)$$

$$\geq 1. \ (\because m \geq 2)$$

Case 3: Let $h_i \in H_i$ where i = 1,2,........n.

Then $\sum_{e\in N(h_i)} f(e) = \left(\frac{p}{q}+\frac{p}{q}\right)+\left(\underbrace{\frac{p}{q}+\frac{p}{q}+\cdots+\frac{p}{q}}_{m-\ times}\right)$

$$+\left(\underbrace{\frac{p}{q}+\frac{p}{q}+\cdots+\frac{p}{q}}_{m-times}\right) = (2m+2)\left(\frac{p}{q}\right)$$

$$\geq (2m+2)\left(\frac{1}{3}\right)\left(\because \frac{p}{q}\geq\frac{1}{3}\right)$$

$$> 1. \ (\because m \geq 2)$$

Case 4: Let $h_{ij} \in H_i$ where i = 1,2,........ n; j = 1,2,.....,m.

Then $\sum_{e\in N(h_{ij})} f(e) = \left(\frac{p}{q}+\frac{p}{q}\right)+\left(\underbrace{\frac{p}{q}+\frac{p}{q}+\cdots+\frac{p}{q}}_{m-times}\right)+\frac{p}{q}$

$$= (m+3)\left(\frac{p}{q}\right)$$

$$\geq (m+3)\left(\frac{1}{3}\right)\left(\because \frac{p}{q}\geq\frac{1}{3}\right)$$

$$> 1. \ (\because m \geq 2)$$

Therefore for all possibilities of $e \in E$, we get

$\sum_{e\in N(h)} f(e) > 1, \forall h \in$ E.

This implies that f is a TEDF.

Case II: Suppose $\frac{p}{q} \geq \frac{1}{2}$.

Clearly f is a TEDF.

Now we check for the minimalityof f.

Define g: $E \to [0,1]$ by

$$g(e) = \begin{cases} r, & \text{if } e = h_k \in E \text{ for some k}, \\ \frac{p}{q}, & \text{otherwise}, \end{cases}$$

where $0 < r < \frac{p}{q}$.

Since strict inequality holds at the edge $h_k \in E$, it follows that g < f.

The summation value taken over N(e) of e ∈ E is as follows:

Case (**i**): Let $e_i \in C_n$ where i=1,2,.......,n.

Sub case 1: Let $h_k \in N(e_i)$. Then k = i or i + 1, if i≠1 and k = 1 or n, if i = n.

Then $\sum_{e\in N(e_i)} g(e) = \left(\frac{p}{q}+\frac{p}{q}\right)+\left(r+\underbrace{\frac{p}{q}+\frac{p}{q}+\cdots+\frac{p}{q}}_{m-times}\right)$

$$+\left(\underbrace{\frac{p}{q}+\frac{p}{q}+\cdots+\frac{p}{q}}_{(m+1)-times}\right)$$

$$= r+(2m+3)\left(\frac{p}{q}\right)$$

$$\geq r+(2m+3)\left(\frac{1}{2}\right)\left(\because \frac{p}{q}\geq\frac{1}{2}\right)$$

$$> 1. \ (\because m \geq 2)$$

Sub case 2: Let $h_k \notin N(e_i)$.

Then $\sum_{e\in N(e_i)} g(e) = \left(\frac{p}{q}+\frac{p}{q}\right)+\left(\underbrace{\frac{p}{q}+\frac{p}{q}+\cdots+\frac{p}{q}}_{(m+1)-times}\right)$

$$+\left(\underbrace{\frac{p}{q}+\frac{p}{q}+\cdots+\frac{p}{q}}_{(m+1)-times}\right) = (2m+4)\left(\frac{p}{q}\right)$$

$$\geq (2m+4)\left(\frac{1}{2}\right)\left(\because \frac{p}{q}\geq\frac{1}{2}\right)$$

$$= m+2 > 1. \ (\because m \geq 2)$$

Case(**ii**): Let $l_{ij} \in H_i$ where i=1,2,......n;j=1,2,.......,m.

Sub case 1: Let $h_k \in N(l_{ij})$. Then k= i.

Then $\sum_{e\in N(l_{ij})} g(e) = \left(\underbrace{\frac{p}{q}+\frac{p}{q}+\cdots+\frac{p}{q}}_{(m-1)-times}\right)+ r+\frac{p}{q}$

$$= r+m\left(\frac{p}{q}\right) \geq r+m\left(\frac{1}{2}\right)\left(\because \frac{p}{q}\geq\frac{1}{2}\right)$$

$$> 1. \ (\because m \geq 2)$$

Sub case 2: Let $h_k \notin N(l_{ij})$.

Then $\sum_{e\in N(l_{ij})} g(e) = \left(\underbrace{\frac{p}{q}+\frac{p}{q}+\cdots+\frac{p}{q}}_{(m-1)-times}\right)+\frac{p}{q}+\frac{p}{q}$

$$= (m+1)\left(\frac{p}{q}\right)$$

$$\geq (m+1)\left(\frac{1}{2}\right)\left(\because \frac{p}{q}\geq\frac{1}{2}\right)$$

$$> 1. \ (\because m \geq 2)$$

Case (iii): Let $h_i \in H_i$ where i=1,2,........n.

Then $\sum_{e\in N(h_i)} g(e) = \left(\frac{p}{q}+\frac{p}{q}\right)+\left(\underbrace{\frac{p}{q}+\frac{p}{q}+\cdots+\frac{p}{q}}_{m-times}\right)$

$$+\left(\underbrace{\frac{p}{q}+\frac{p}{q}+\cdots+\frac{p}{q}}_{m-times}\right) = (2m+2)\left(\frac{p}{q}\right)$$

$$\geq (2m+2)\left(\frac{1}{2}\right)\left(\because \frac{p}{q}\geq\frac{1}{2}\right)$$

$$= m + 1 > 1. \quad (\because m \geq 2)$$

Case($\boldsymbol{iv}$): Let $h_{ij} \in H_i$ where i=1,2,........n;j=1,2,.....,m.

Sub case 1: Let $h_k \in N(h_{ij})$. Then k= i.

$$\text{Now} \sum_{e \in \mathrm{N}(h_{ij})} g(e) = \left(\frac{p}{q} + \frac{p}{q}\right) + \left(r + \underbrace{\frac{p}{q} + \frac{p}{q} + \cdots + \frac{p}{q}}_{(m-1)-times}\right) + \frac{p}{q}$$

$$= r + (m+2)\left(\frac{p}{q}\right)$$

$$\geq r + (m+2)\left(\frac{1}{2}\right)\left(\because \frac{p}{q} \geq \frac{1}{2}\right)$$

$$= r + \frac{m}{2} + 1 > 1. \quad (\because m \geq 2)$$

Sub case 2: Let $h_k \notin N(h_{ij})$.

$$\text{Then} \sum_{e \in \mathrm{N}(h_{ij})} g(e) = \left(\frac{p}{q} + \frac{p}{q}\right) + \left(\underbrace{\frac{p}{q} + \frac{p}{q} + \cdots + \frac{p}{q}}_{m\ -times}\right) + \frac{p}{q}$$

$$= (m+3)\left(\frac{p}{q}\right)$$

$$\geq (m+3)\left(\frac{1}{2}\right)\left(\because \frac{p}{q} \geq \frac{1}{2}\right)$$

$$> 1. \quad (\because m \geq 2)$$

Thus for all possibilities,

$$\sum_{e \in \mathrm{N}(h)} g(e) > 1, \forall h \in \mathrm{E}.$$

Since g is defined arbitrarily, it follows that for all possibilities of defining g < f, g becomes a

TEDF.This implies that f is not a MTEDF.

Case III: Suppose$\frac{1}{3} \leq \frac{p}{q} < \frac{1}{2}$.

As in Case 1, we can show that for$e_i \in C_n$,

$$\sum_{e \in \mathrm{N}(e_i)} f(e) = (2m+4)\left(\frac{p}{q}\right).$$

$$\text{Now } \frac{1}{3} \leq \frac{p}{q} < \frac{1}{2}$$

By multiplying with 2m+4,

$$\frac{2m+4}{3} \leq (2m+4)\left(\frac{p}{q}\right) < \frac{2m+4}{2}$$

$$ie., \frac{2m+1}{3} + 1 \leq \sum_{e \in \mathrm{N}(e_i)} f(e) < m + 2$$

$$\text{Therefore} \sum_{e \in \mathrm{N}(e_i)} f(e) > 1.$$

As in Case 2, we can show that for$l_{ij} \in H_i$,

$$\sum_{e \in \mathrm{N}(l_{ij})} f(e) = (m+1)\left(\frac{p}{q}\right).$$

By multiplying with m+1,

$$1 \leq \frac{m+1}{3} \leq (m+1)\left(\frac{p}{q}\right) < \frac{m+1}{2} > 1$$

$$ie., 1 \leq \frac{m+1}{3} \leq \sum_{e \in \mathrm{N}(l_{ij})} f(e) < \frac{m+1}{2} > 1$$

$$\text{Therefore} \sum_{e \in \mathrm{N}(l_{ij})} f(e) > 1.$$

As in Case 3, we can show that for$h_i \in H_i$,

$$\sum_{e \in \mathrm{N}(h_i)} f(e) = (2m+2)\left(\frac{p}{q}\right).$$

By multiplying with 2m+2,

$$\frac{2m+2}{3} \leq (2m+2)\left(\frac{p}{q}\right) < \frac{2m+2}{2}$$

$$ie., \frac{2m-1}{3} + 1 \leq \sum_{e \in \mathrm{N}(h_i)} f(e) < m + 1$$

$$\text{Therefore} \sum_{e \in \mathrm{N}(h_i)} f(e) > 1.$$

As in Case 4, we can show that for$h_{ij} \in H_i$,

$$\sum_{e \in \mathrm{N}(h_{ij})} f(e) = (m+3)\left(\frac{p}{q}\right).$$

By multiplying with m+3,

$$\frac{m+3}{3} \leq (m+3)\left(\frac{p}{q}\right) < \frac{m+3}{2}.$$

$$ie., \frac{m}{3} + 1 \leq \sum_{e \in \mathrm{N}(h_{ij})} f(e) < \frac{m+1}{2} + 1.$$

$$\text{Therefore} \sum_{e \in \mathrm{N}(h_{ij})} f(e) > 1.$$

Thus for all possibilities of e∈ E, we get

$$\sum_{e \in \mathrm{N}(h)} f(e) > 1, \forall h \in \mathrm{E}.$$

This implies that f is a TEDF.

Now we check for the minimalityof f.

Define g: E→ [0,1] by

$$\mathrm{g(e)} = \begin{cases} r, & \text{if } e = h_k \in E \text{ for some k,} \\ \frac{p}{q}, & \text{otherwise,} \end{cases}$$

where $0 < r < \frac{p}{q}$.

Since strict inequality holds at the edge $h_k \in E$, it follows that g < f.

Then as in Case (i) of Case II, we can show for $e_i \in C_n$ that

$$\sum_{e \in \mathrm{N}(e_i)} g(e) = r + (2m+3)\left(\frac{p}{q}\right), \text{if } h_k \in N(e_i)$$

$$\geq r + (2m+3)\left(\frac{1}{3}\right)\left(\because \frac{p}{q} \geq \frac{1}{3}\right)$$

$$= \mathrm{r} + \frac{2\mathrm{m}}{3} + 1 > 1$$

and $\sum_{e \in \mathrm{N}(e_i)} g(e) = (2m+4)\left(\frac{p}{q}\right)$, if $h_k \notin N(e_i)$.

$$\geq (2m+4)\left(\frac{1}{3}\right)\left(\because \frac{p}{q} \geq \frac{1}{3}\right)$$

$$= \frac{2\mathrm{m}+1}{3} + 1 > 1.$$

Again as in Case (ii) of Case II , we can show for $l_{ij} \in H_i$ that

$$\sum_{e \in \mathrm{N}(l_{ij})} g(e) = r + m\left(\frac{p}{q}\right), \text{if } h_k \notin N(l_{ij})$$

$$< 1, if\ \frac{p}{q} = 0.4, m = 2, r = 0.1$$

and $\sum_{e \in \mathrm{N}(l_{ij})} g(e) = (m+1)\left(\frac{p}{q}\right)$, if $h_k \notin N(l_{ij})$

$$\geq (m+1)\left(\frac{1}{3}\right)\left(\because \frac{p}{q} \geq \frac{1}{3}\right)$$

$$\geq 1.\ (\because \mathrm{m} \geq 2)$$

Again as in Case (iii) of Case II , we can show for $h_i \in H_i$ that

$$\sum_{e \in \mathrm{N}(h_i)} g(e) = (2m+2)\left(\frac{p}{q}\right) \geq (2m+2)\left(\frac{1}{3}\right)\left(\because \frac{p}{q} \geq \frac{1}{3}\right)$$

$$> 1. \quad (\because \mathrm{m} \geq 2)$$

Again as in Case (iv) of Case II , we can show for $h_{ij} \in H_i$ that

$$\sum_{e \in \mathrm{N}(h_{ij})} g(e) = r + (m+2)\left(\frac{p}{q}\right), \text{if } h_k \in N(h_{ij})$$

$$\geq r + (m+2)\left(\frac{1}{3}\right)\left(\because \frac{p}{q} \geq \frac{1}{3}\right)$$

$$> 1 \quad (\because \mathrm{m} \geq 2)$$

and $\sum_{e \in \mathrm{N}(h_{ij})} g(e) = (m+3)\left(\frac{p}{q}\right)$, if $h_k \notin N(h_{ij})$

$$\geq (m+3)\left(\frac{1}{3}\right)\left(\because \frac{p}{q} \geq \frac{1}{3}\right)$$

$$= \frac{m}{3} + 1 > 1.$$

Thus we have seen that $\sum_{e \in \mathrm{N}(h)} g(e) < 1$ for some h ∈ E.

So g is not a TEDF.

Since g is defined arbitrarily, it follows that there exists no g< f such that g is a TEDF.

Thus f is a MTEDF. ■

4. GRAPHS

MINIMAL TOTAL EDGE DOMINATING SET

Theorem 3.1

Case **1**

The edges with blue colour and pink colour are the edges of minimal total edge dominating set.

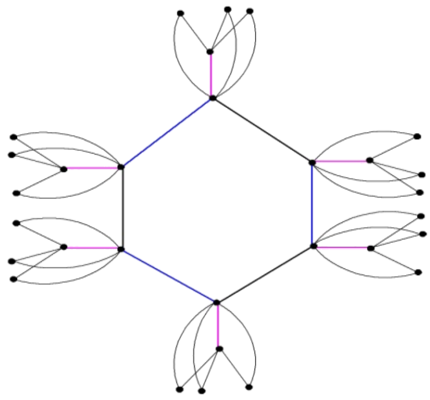


Figure 1

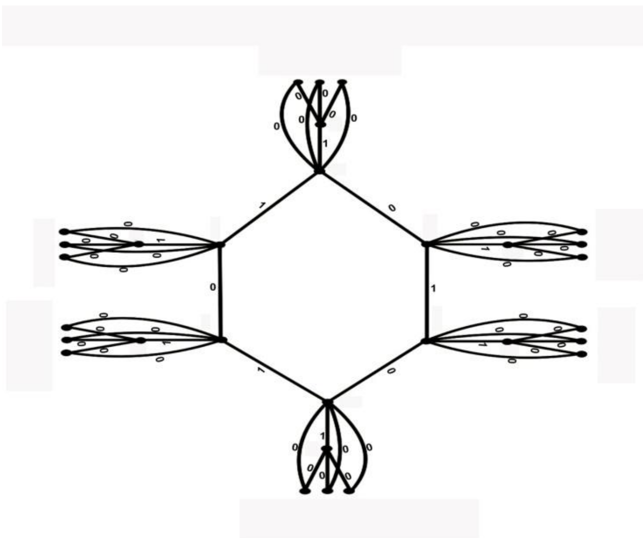
$G = C_6 \odot K_{1,3}$

MINIMAL TOTAL EDGE DOMINATING FUNCTION

Theorem 3.2

Case I

The functional values are given at each edge of the graph G.



$\boldsymbol{G = C_6 \odot K_{1,3}}$

Figure 2

5. CONCLUSIONS

Edge dominating functions is a new conceptintroduced in recent years and receiving much attention. Corona product graphs is another new concept and in this paper corona of a cycle with a star is considered.The edge dominating sets and edge dominating functions of this graph is studied by the authors. The results on total edge dominating functions are presented here. This study gives scope for further research on various other edge dominating functions such as signed, Roman etc. and the authors are working on that and the results obtained are communicated for publication.

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