Minimal Dominating Functions of Corona Product Graph of a Path with a Cycle

M. S. Vimala Devi
Sr. Asst, TTD
Tirupati, Andhra Pradesh,
India

B. Maheswari
Professor
Sri Padmavathi Mahila Viswa Vidyalayam, Tiruapti,
Andhra Pradesh, India

ABSTRACT

Graph Theory has been realized as one of the most useful branches of Mathematics or recent origin, finding widest applications in all most all branches of Sciences, Engineering and Computer Science. An introduction and an extensive overview on domination in graphs and related topics are given by Haynes et al. [1, 2].

Recently, dominating functions in domination theory have received much attention. They give rise to important classes of graphs and deep structural problems. In this paper it is a discussion on some results on minimal dominating functions of corona product graph of a Path with a Cycle.

Keywords

Corona Product, Path, Cycle, Dominating Function

1. INTRODUCTION

The Domination theory is an important branch of Graph Theory and the concept of domination number of a graph is first introduced by Berge [9] in his book on graph theory. The concept of Total dominating sets are introduced by C.J.Cockayane, and Hedetniemi,S.T[4] and the concept of dominating functions introduced by Hedetniemi et al.[5].

Frutch and Harary [7] introduced a new product on two graphs G_1 and G_2, called corona product denoted by $G_1 \oplus G_2$. The object is to construct a new and simple operation on two graphs G_1 and G_2 called their corona, with the property that the group of the new graph is in general isomorphic with the wreath product of the groups of G_1 and G_2.

Here some basic properties of corona product graph of a Path with a Cycle and some results on minimal dominating functions are presented.

2. CORONA PRODUCT OF $P_n \odot C_m$

The corona product of a Path P_n with a Cycle C_m, is a graph obtained by taking one copy of a n-vertex graph P_n and n copies of C_m and then joining the ith vertex of P_n to all vertices of ith copy of C_m and it is denoted by $\boldsymbol{P_n \odot C_m}$. Now we present some of the properties of corona product graph $\boldsymbol{P_n \odot C_m}$ without proofs.

Theorem 2.1: The graph $G = P_n \odot C_m$ is a connected graph.

Theorem 2.2: The degree of a vertex v in $G = P_n \odot C_m$ is given by

$$d(v_i) = \begin{cases} m+2, & if \quad v_i \in P_n \; and \; 2 \le i \le (n-1), \\ m+1, & if \quad v_i \in P_n \quad and \quad i = 1 \; or \; n, \\ 3, & if \quad v_i \in C_m . \end{cases}$$

Theorem 2.3: The number of vertices and edges in $G = P_n \odot C_m$ is given respectively by

1. $|V(G)| = n(m+1)$,
2. $|E(G)| = (2mn + n - 1)$.

Theorem 2.4: The graph $G = P_n \odot C_m$ is non hamiltonian.

Theorem 2.5: The graph $G = P_n \odot C_m$ is not eulerian.

Theorem 2.6: The graph $G = P_n \odot C_m$ is not bipartite.

3. DOMINATING SETS AND DOMINATING FUNCTIONS

In this section we study dominating sets, dominating functions of the graph $G = P_n \odot C_m$ and we present some results related to minimal dominating functions of this graph.

Theorem 3.1: The domination number of $G = P_n \odot C_m$ is n.

Proof: Let D denote a dominating set of G.

Case 1: Suppose D contains the vertices of P_n in G.

By the definition of G, the i^{th} vertex in P_n is adjacent to all vertices of i^{th}copy of C_m. That is the vertices in P_n dominate the vertices in all copies of C_m respectively. Therefore the vertices of D dominate all vertices of G. Thus D becomes a DS of G. This set is also minimal, because, if we delete one vertex say v_i from D, then the vertices in the i^{th} copy of C_m are not dominated by any vertex in D. Hence $\gamma(G) = n$.

Case2: Suppose D contains any one vertex of C_m in each copy of G.

That is $|D| = n$. Obviously every vertex in C_m dominates every other vertex in C_m and also a single vertex of P_n to which it is associated.

Therefore the vertices in D dominate all vertices of G. Further this set is also minimal. Therefore $\gamma(G) = n$. ■

Theorem 3.2: Let D be a MDS of $G = P_n \odot C_m$. Then a function $f : V \to [0, 1]$ defined by

$$f(v) = \begin{cases} 1, & \text{if v} \in D, \\ 0, & \text{otherwise.} \end{cases}$$

becomes a MDF of $G = P_n \odot C_m$.

Proof: we have seen in Theorem 3.1, that the DS of G contains all the vertices of P_n and this set is also minimum. Also the set of vertices whose degree is m in each copy of C_m

form a minimal DS of G. Let D be a MDS of G. For definiteness let D contain the vertices of P_n in G.

In P_n, there are two end vertices of degree m+1 and there are n-2 intermediate vertices of degree $m+2$ respectively in G. In C_m, there are m vertices of degree 3 respectively in G.

The summation value taken over $N[v]$ of $v \in V$ is as follows:

Case 1: Let $v \in P_n$ be such that $d(v) = m+2$ in G.

Then $N[v]$ contains m vertices of C_m and three vertices of P_n in G.

So $\sum_{u\in N[v]} f(u) = 1+1+1+\underbrace{0+.......+0}_{m-times} = 3.$

Case 2: Let $v \in P_n$ be such that $d(v) = m$+1 in G.

Then $N[v]$ contains m vertices of C_m and two vertices of P_n in G.

So $\sum_{u\in N[v]} f(u) = 1+1+\underbrace{0+.......+0}_{m-times} = 2.$

Case 3: Let $v \in C_m$ be such that $d(v) = 3$ in G.

Then $N[v]$ contains 3 vertices of C_m and one vertex of P_n in G.

So $\sum_{u\in N[v]} f(u) = 1+0+0+0 = 1.$

Therefore for all possibilities, we get $\sum_{u\in N[v]} f(u) \geq 1,$ $\forall$ v $\in$ V.

This implies that f is a DF.

Now we check for the minimality of f.

Define $g : V \to [0,1]$ by

$$g(v) = \begin{cases} r, & \text{if } v = v_k \in D \text{ with } d(v_k) = m+1, \\ 1, & \text{if } v \in D - \{v_k\}, \\ 0, & \text{otherwise.} \end{cases}$$

where $0 < r < 1$.

Since strict inequality holds at the vertex $v_k \in D$, it follows that $g < f$.

Now the following cases arise.

Case (i): Let $v \in P_n$ be such that $d(v) = m+2$ in G.

Sub case 1: Let $v_k \in N[v]$.

Then $\sum_{u\in N[v]} g(u) = r+1+1+\underbrace{0+.......+0}_{m-times} = r+2 > 1.$

Sub case 2: Let $v_k \notin N[v]$.

Then $\sum_{u\in N[v]} g(u) = 1+1+1+\underbrace{0+.......+0}_{m-times} = 3.$

Case (ii): Let $v \in P_n$ be such that $d(v) = m$+1 in G.

Sub case 1: Let $v_k \in N[v]$.

Then $\sum_{u\in N[v]} g(u) = r+1+\underbrace{0+.......+0}_{m-times} = r+1 > 1.$

Sub case 2: Let $v_k \notin N[v]$.

Then $\sum_{u\in N[v]} g(u) = 1+1+\underbrace{0+.......+0}_{m-times} = 2.$

Case (iii): Let $v \in C_m$ be such that $d(v) = 3$ in G.

Sub case 1: Let $v_k \in N[v]$.

Then $\sum_{u\in N[v]} g(u) = r+0+0+0 = r < 1.$

Sub case 2: Let $v_k \notin N[v]$.

Then $\sum_{u\in N[v]} g(u) = 1+0+0+0 = 1$

This implies that $\sum_{u\in N[v]} g(u) < 1,$ for some $v \in V$.

So g is not a DF.

Since g is taken arbitrarily, it follows that there exists no $g < f$ such that g is a DF.

Thus f is a MDF. ■

Theorem 3.3: A function $f : V \to [\,0,\,1\,]$ defined by $f(v) = \frac{1}{q}$, $\forall\, v \in V$ is a DF of $G = P_n \odot C_m$ if $q \leq 4$. It is a MDF if $q = 4$.

Proof: Let f be a function defined as in the hypothesis.

We know that in P_n, there are two end vertices of degree m+1 and there are $n-2$ intermediate vertices of degree $m+2$ respectively in G. In C_m, there are m vertices of degree 3 respectively in G.

Case I: Suppose $0 < q < 4$.

The summation value taken over $N[v]$ of $v \in V$ is as follows:

Case 1: Let $v \in P_n$ be such that $d(v) = m+2$ in G.

Then $N[v]$ contains m vertices of C_m and three vertices of P_n in G.

So $\sum_{u\in N[v]} f(u) = \underbrace{\frac{1}{q}+\frac{1}{q}+.......+\frac{1}{q}}_{(m+3)-times} = \frac{m+3}{q}.$

Since $q < 4$, it follows that $\frac{m+3}{q} > 1.$

Case 2: Let $v \in P_n$ be such that $d(v) = m$+1 in G.

Then $N[v]$ contains m vertices of C_m and two vertices of P_n in G.

So $$\sum_{u\in N[v]} f(u) = \underbrace{\frac{1}{q}+\frac{1}{q}+.......+\frac{1}{q}}_{(m+2)-times} = \frac{m+2}{q}.$$

Since $q < 4$, it follows that $\frac{m+2}{q} > 1.$

Case 3: Let $v \in C_m$ be such that $d(v) = 3$ in G.

Then $N[v]$ contains 3 vertices of C_m and one vertex of P_n in G.

So $$\sum_{u\in N[v]} f(u) = \frac{1}{q}+\frac{1}{q}+\frac{1}{q}+\frac{1}{q} = \frac{4}{q}.$$

Since $q < 4$, it follows that $\frac{4}{q} > 1.$

Therefore for all possibilities, we get $\sum_{u\in N[v]} f(u) > 1,$

$\forall \; v \in V.$

This implies that f is a DF.

Now we check for the minimality of f.

Define $g : V \to [0,1]$ by

$$g(v) = \begin{cases} \mathrm{r,\ if\ v = v_k \in D\ with\ d\,(v_k) = m+1,} \\ \frac{1}{\mathrm{q}},\ \mathrm{otherwise.} \end{cases}$$

where $0 < r < \frac{1}{q}$.

Since strict inequality holds at a vertex v_k of V, it follows that $g < f$.

Case (i): Let $v \in P_n$ be such that $d(v) = m + 2$ in G.

Sub case 1: Let $v_k \in N[v]$.

Then $$\sum_{u\in N[v]} g(u) = r + \underbrace{\frac{1}{q}+\frac{1}{q}+.......+\frac{1}{q}}_{(m+2)-times}$$

$$< \frac{1}{q}+\frac{m+2}{q} = \frac{m+3}{q}.$$

Since $q < 4$, it follows that $\frac{m+3}{q} > 1.$

Sub case 2: Let $v_k \notin N[v]$.

Then $$\sum_{u\in N[v]} g(u) = \underbrace{\frac{1}{q}+\frac{1}{q}+.......+\frac{1}{q}}_{(m+3)-times} = \frac{m+3}{q} > 1,$$

since $q < 4$.

Case (ii): Let $v \in P_n$ be such that $d(v) = m$+1 in G.

Sub case 1: Let $v_k \in N[v]$.

Then $$\sum_{u\in N[v]} g(u) = r + \underbrace{\frac{1}{q}+\frac{1}{q}+.......+\frac{1}{q}}_{(m+1)-times}$$

$$< \frac{1}{q}+\frac{m+1}{q} = \frac{m+2}{q} > 1,$$

since q < 4.

Sub case 2: Let $v_k \notin N[v]$.

Then $$\sum_{u\in N[v]} g(u) = \underbrace{\frac{1}{q}+\frac{1}{q}+.......+\frac{1}{q}}_{(m+2)-times} = \frac{m+2}{q} > 1,$$

since q < 4.

Case (iii): Let $v \in C_m$ be such that $d(v) = 3$ in G.

Sub case 1: Let $v_k \in N[v]$.

Then $$\sum_{u\in N[v]} g(u) = r + \frac{1}{q}+\frac{1}{q}+\frac{1}{q}$$

$$< \frac{1}{q}+\frac{3}{q} = \frac{4}{q} > 1,$$ since q < 4.

Sub case 2: Let $v_k \notin N[v]$.

Then $$\sum_{u\in N[v]} g(u) = \frac{1}{q}+\frac{1}{q}+\frac{1}{q}+\frac{1}{q} = \frac{4}{q} > 1.$$

Hence it follows that $\sum_{u\in N[v]} g(u) \geq 1, \; \forall \; v \in V.$

Thus g is a DF.

This implies that f is not a MDF.

Case II: Suppose $q = 4$.

The summation value taken over $N[v]$ of $v \in V$ is as follows:

Case 1: Let $v \in P_n$ be such that $d(v) = m + 2$ in G.

Then $N[v]$ contains m vertices of C_m and three vertices of P_n in G.

So

$$\sum_{u\in N[v]} f(u)=\underbrace{\frac{1}{q}+\frac{1}{q}+.......+\frac{1}{q}}_{(m+3)-times}=\frac{m+3}{q}=\frac{m+3}{4}>1,\ \text{since } q=4.$$

Case 2: Let $v \in P_n$ be such that $d(v) = m+1$ in G.

Then $N[v]$ contains m vertices of C_m and two vertices of P_n in G.

So

$$\sum_{u\in N[v]} f(u)=\underbrace{\frac{1}{q}+\frac{1}{q}+.......+\frac{1}{q}}_{(m+2)-times}=\frac{m+2}{q}=\frac{m+2}{4}>1,\ \text{since } q=4.$$

Case 3: Let $v \in C_m$ be such that $d(v) = 3$ in G.

Then $N[v]$ contains 3 vertices of C_m and one vertex of P_n in G.

So

$$\sum_{u\in N[v]} f(u)=\frac{1}{q}+\frac{1}{q}+\frac{1}{q}+\frac{1}{q}=\frac{4}{q}=\frac{4}{4}=1,\ \text{since } q=4.$$

Therefore for all possibilities, we get $\sum_{u\in N[v]} f(u)\geq 1$, $\forall\ v \in V$.

This implies that f is a DF.

Now we check for the minimality of f.

Define $g : V \to [0,1]$ by

$$g(v) = \begin{cases} r, \text{ if } v = v_k \in D \text{ with } d(v_k) = m+1, \\ \frac{1}{q}, \text{ otherwise.} \end{cases}$$

where $0 < r < \frac{1}{q}$.

Since strict inequality holds at a vertex v_k of V, it follows that $g < f$.

Case (i): Let $v \in P_n$ be such that $d(v) = m + 2$ in G.

Sub case 1: Let $v_k \in N[v]$.

Then $\sum_{u\in N[v]} g(u)=r+\underbrace{\frac{1}{q}+\frac{1}{q}+.......+\frac{1}{q}}_{(m+2)-times}$

$$<\frac{1}{q}+\frac{m+2}{q}=\frac{m+2}{q}=\frac{m+2}{4}>1.$$

Sub case 2: Let $v_k \notin N[v]$.

Then $\sum_{u\in N[v]} g(u)=\underbrace{\frac{1}{q}+\frac{1}{q}+.......+\frac{1}{q}}_{(m+3)-times}$

$$=\frac{m+3}{q}=\frac{m+3}{4}>1.$$

Case (ii): Let $v \in P_n$ be such that $d(v) = m+1$ in G.

Sub case 1: Let $v_k \in N[v]$.

Then $\sum_{u\in N[v]} g(u)=r+\underbrace{\frac{1}{q}+\frac{1}{q}+.......+\frac{1}{q}}_{(m+1)-times}$

$$<\frac{1}{q}+\frac{m+1}{q}=\frac{m+2}{q}=\frac{m+2}{4}>1.$$

Sub case 2: Let $v_k \notin N[v]$.

Then

$$\sum_{u\in N[v]} g(u)=\underbrace{\frac{1}{q}+\frac{1}{q}+.......+\frac{1}{q}}_{(m+2)-times}=\frac{m+2}{q}=\frac{m+2}{4}>1.$$

Case (iii): Let $v \in C_m$ be such that $d(v) = 3$ in G.

Sub case 1: Let $v_k \in N[v]$.

Then $\sum_{u\in N[v]} g(u)=r+\frac{1}{q}+\frac{1}{q}+\frac{1}{q}$

$<\frac{1}{q}+\frac{3}{q}=\frac{4}{4}=1,$ since $q = 4$.

Sub case 2: Let $v_k \notin N[v]$.

Then $\sum_{u\in N[v]} g(u)=\frac{1}{q}+\frac{1}{q}+\frac{1}{q}+\frac{1}{q}=\frac{4}{4}=1.$

This implies that $\sum_{u\in N[v]} g(u)<1$, for some $v \in V$.

So g is not a DF.

Since g is defined arbitrarily, it follows that there exists no $g < f$ such that g is a DF.

Thus f is a MDF. ■

Theorem 3.4: A function $f : V \to [0, 1]$ defined by $f(v) = \frac{p}{q}$, $\forall v \in V$ where $p = min\,(m,n)$ and $q = max\,(m,n)$ is a DF of $G = P_n \odot C_m$ if $\frac{p}{q} \geq \frac{1}{4}$. Otherwise it is not a DF. Also it becomes a MDF if $\frac{p}{q} = \frac{1}{4}$.

Proof: Let $f : V \to [0, 1]$ be defined by $f(v) = \frac{p}{q}$, $\forall v \in V$, where $p = min\,(m,n)$ and $q = max\,(m,n)$.

Clearly $\frac{p}{q} > 0$.

The summation value taken over $N[v]$ of $v \in V$ is as follows:

Case 1: Let $v \in P_n$ be such that $d(v) = m + 2$ in G.

Then $N[v]$ contains m vertices of C_m and three vertices of P_n in G.

So $$\sum_{u \in N[v]} f(u) = \underbrace{\frac{p}{q} + \frac{p}{q} + \dots\dots + \frac{p}{q}}_{(m+3)-times} = (m+3)\frac{p}{q}.$$

Case 2: Let $v \in P_n$ be such that $d(v) = m+1$ in G.

Then $N[v]$ contains m vertices of C_m and two vertices of P_n in G.

So $$\sum_{u \in N[v]} f(u) = \underbrace{\frac{p}{q} + \frac{p}{q} + \dots\dots + \frac{p}{q}}_{(m+2)-times} = (m+2)\frac{p}{q}.$$

Case 3: Let $v \in C_m$ be such that $d(v) = 3$ in G.

Then $N[v]$ contains 3 vertices of C_m and one vertex of P_n in G.

So $$\sum_{u \in N[v]} f(u) = \frac{p}{q} + \frac{p}{q} + \frac{p}{q} + \frac{p}{q} = 4\left(\frac{p}{q}\right).$$

From the above three cases, we observe that f is a DF if $\frac{p}{q} \geq \frac{1}{4}$.

Otherwise f is not a DF.

Case 4: Suppose $\frac{p}{q} > \frac{1}{4}$.

Clearly f is a DF.

Now we check for the minimality of f.

Define $g : V \to [0, 1]$ by

$$g(v) = \begin{cases} \text{r}, & \text{if } \text{v} = \text{v}_\text{k} \in \text{D with d}(\text{v}_\text{k}) = \text{m}+1, \\ \dfrac{\text{p}}{\text{q}}, & \text{otherwise.} \end{cases}$$

where $0 < r < \frac{p}{q}$.

Since strict inequality holds at a vertex v_k of V, it follows that $g < f$.

Case (i): Let $v \in P_n$ be such that $d(v) = m + 2$ in G.

Sub case 1: Let $v_k \in N[v]$.

Then $$\sum_{u \in N[v]} g(u) = r + \underbrace{\frac{p}{q} + \frac{p}{q} + \dots\dots + \frac{p}{q}}_{(m+2)-times}$$

$$< \frac{p}{q} + (m+2)\frac{p}{q} = (m+3)\frac{p}{q} > 1,$$

since $\frac{p}{q} > \frac{1}{4}$.

Sub case 2: Let $v_k \notin N[v]$.

Then
$$\sum_{u \in N[v]} g(u) = \underbrace{\frac{p}{q} + \frac{p}{q} + \dots\dots + \frac{p}{q}}_{(m+3)-times} = (m+3)\frac{p}{q} > 1.$$

Case (ii): Let $v \in P_n$ be such that $d(v) = m + 1$ in G.

Sub case 1: Let $v_k \in N[v]$.

Then $$\sum_{u \in N[v]} g(u) = r + \underbrace{\frac{p}{q} + \frac{p}{q} + \dots\dots + \frac{p}{q}}_{(m+1)-times}$$

$$< \frac{p}{q} + (m+1)\frac{p}{q} = (m+2)\frac{p}{q} > 1,$$

since $\frac{p}{q} > \frac{1}{4}$.

Sub case 2: Let $v_k \notin N[v]$.

Then
$$\sum_{u \in N[v]} g(u) = \underbrace{\frac{p}{q} + \frac{p}{q} + \dots\dots + \frac{p}{q}}_{(m+2)-times} = (m+2)\frac{p}{q} > 1.$$

Case (iii): Let $v \in C_m$ be such that $d(v) = 3$ in G.

Sub case 1: Let $v_k \in N[v]$.

Then $$\sum_{u \in N[v]} g(u) = r + \frac{p}{q} + \frac{p}{q} + \frac{p}{q}$$

$$< \frac{p}{q} + 3\left(\frac{p}{q}\right) = 4\left(\frac{p}{q}\right) > 1,$$ since $\frac{p}{q} > \frac{1}{4}$.

Sub case 2: Let $v_k \notin N[v]$.

Then $$\sum_{u \in N[v]} g(u) = \frac{p}{q} + \frac{p}{q} + \frac{p}{q} + \frac{p}{q} = 4\left(\frac{p}{q}\right) > 1.$$

Hence, it follows that $\sum_{u\in N[v]} g(u) > 1, \quad \forall\ v \in V.$

Thus g is a DF.

This implies that f is not a MDF.

Case 5: Suppose $\frac{p}{q} = \frac{1}{4}$.

As in Case 1 and 2, we have that

$$\sum_{u\in N[v]} f(u) = \underbrace{\frac{p}{q}+\frac{p}{q}+\ldots\ldots+\frac{p}{q}}_{(m+3)-times} = (m+3)\frac{p}{q} = (m+3)\frac{1}{4} > 1,$$

$if\ \ v \in P_n$.

and

$$\sum_{u\in N[v]} f(u) = \underbrace{\frac{p}{q}+\frac{p}{q}+\ldots\ldots+\frac{p}{q}}_{(m+2)-times} = (m+2)\frac{p}{q} = (m+2)\frac{1}{4} > 1,$$

$if\ \ v \in P_n$.

Again as in Case 3, we have $if\ \ v \in C_m$ then

$$\sum_{u\in N[v]} f(u) = \frac{p}{q}+\frac{p}{q}+\frac{p}{q}+\frac{p}{q} = 4\left(\frac{p}{q}\right) = 1.$$

Therefore for all possibilities, we get $\sum_{u\in N[v]} f(u) \geq 1,$

$\forall\ \ v \in V.$

This implies that f is a DF.

Now we check for the minimality of f.

Define $g : V \to [0, 1]$ by

$$g(v) = \begin{cases} r, & \text{if } v = v_k \in D \text{ with } d(v_k) = m+1, \\ \frac{p}{q}, & \text{otherwise.} \end{cases}$$

where $0 < r < \frac{p}{q}$.

Since strict inequality holds at a vertex v_k of V, it follows that $g < f$.

Then we can show as in Case (i) of Case 4 that

$$\sum_{u\in N[v]} g(u) = r + \underbrace{\frac{p}{q}+\frac{p}{q}+\ldots\ldots+\frac{p}{q}}_{(m+2)-times} > 1,$$

if $v \in P_n$ and $v_k \in N[v]$.

And $$\sum_{u\in N[v]} g(u) = \underbrace{\frac{p}{q}+\frac{p}{q}+\ldots\ldots+\frac{p}{q}}_{(m+3)-times} > 1,$$

if $v \in P_n$ and $v_k \notin N[v]$.

Again as in Case (ii) of Case 4, we can show that

$$\sum_{u\in N[v]} g(u) = r + \underbrace{\frac{p}{q}+\frac{p}{q}+\ldots\ldots+\frac{p}{q}}_{(m+1)-times} > 1,$$

if $v \in P_n$ and $v_k \in N[v]$.

And

$$\sum_{u\in N[v]} g(u) = \underbrace{\frac{p}{q}+\frac{p}{q}+\ldots\ldots+\frac{p}{q}}_{(m+2)-times} = (m+2)\frac{p}{q} = (m+2)\frac{p}{q} > 1,$$

if $v \in P_n$ and $v_k \notin N[v]$.

Again as in Case (iii) of Case 4, we can show that

$$\sum_{u\in N[v]} g(u) = r + \frac{p}{q}+\frac{p}{q}+\frac{p}{q}$$ if $v \in C_m$ and $v_k \in N[v]$.

$$< \frac{p}{q} + 3\left(\frac{p}{q}\right) = 4\left(\frac{p}{q}\right) = 1,$$ since $\frac{p}{q} = \frac{1}{4}$.

And $$\sum_{u\in N[v]} g(u) = \frac{p}{q}+\frac{p}{q}+\frac{p}{q}+\frac{p}{q} = 4\left(\frac{p}{q}\right) = 1,$$

if $v \in C_m$ and $v_k \notin N[v]$.

This implies that $\sum_{u\in N[v]} g(u) < 1,$ for some $v \in V$.

So g is not a DF.

Since g is defined arbitrarily, it follows that there exists no $g < f$ such that g is a DF.

Thus f is a MDF. ■

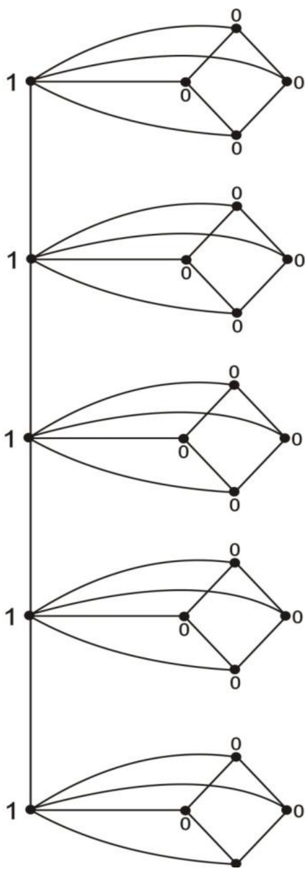

Fig.1

$G = P_5 \odot C_4$

Fig.1: The function f takes the value 1 for vertices of P_n and value 0 for vertices in each copy of C_m

4. CONCLUSION

It is interesting to study the minimal dominating functions of the corona product graph of a path with a cycle. This work gives the scope for the study of convexity of these minimal dominating functions and the authors have also studied this concept.

5. REFERENCES

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