Dynamic Modelling with a Modified PID Controller of a Three Link Rigid Manipulator

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ABSTRACT
This paper presents the modelling of three links rigid manipulator (TRLM) deriving its dynamic equations depending on Lagrange/Euler (L-E) method, the manipulator design and implementation has a complexity, uncertainty and instability dynamic features which lead to a non-linear characteristics, so controlling the manipulator means controlling multi-body multi-input multi-output (MIMO) non-linear and coupled system, the second part of this paper introduce a precise modified Proportional Integral Derivative (PID) controller to control the manipulator under applying different scenarios for the reference signal according to manipulator applications.

Keywords
Dynamic modelling, Three link rigid manipulator, Lagrange-Euler, PID controller, Differential evolution

1. INTRODUCTION
 Recently robots have been considered as an indispensable part of advanced manufacturing field with the ability of robots to accomplish hazardous and risky jobs for human. In robotics, there are two main subjects which are kinematics and dynamics. Within the science of kinematics one studies the position, the velocity, the acceleration and all higher order derivatives of the position variables. Kinematics itself is divided into Forward and Inverse kinematics [1, 2].

However, the relationship between these motions and the forces and torques that cause them constitutes the problem of dynamics [3, 4]. The dynamic model of a robot studies the relation between the joint actuator torques and the resulting motion.

There are two problems related to the dynamics of a manipulator; the first; given the trajectory point [position (θ), velocity ($\dot{\theta}$), and acceleration ($\ddot{\theta}$)] and we wish to calculate the required vector of joint torques (τ). In other words, computation of the vector $\tau(t)$ necessary to obtain a desired trajectory $\ddot{q}$(t), $\dot{q}$(t), q(t) (q is the generalized joint coordinates), once the forces applied of the end-effector are known. This is the Inverse dynamic problem.

The second is to calculate how the mechanism will move under application of a set of joint torques, that is, given a torque vector τ calculate the resulting motion (position, velocity, acceleration) of the manipulator In other words, computation of the time evolution of $\ddot{q}$(t) then of $\dot{q}$(t) and q(t), given the vector of generalized forces (torques) $\tau(t)$ applied to the joints and, in case, the external forces applied to the end-effector, and the initial conditions q(t = t0) $\dot{q}$(t = t0) and

this is Direct dynamic model [5]. An accurate dynamic model of a robot manipulator has many benefits; for the design of motion control system, analysis of mechanical design, and simulation of manipulator motion [6]. In other words, to achieve a proper control, a valid dynamic model must be obtained. In Kostic et al. [7] approach for modeling and identification of a high-performance robot control, they highlighted a procedure for obtaining kinematic and dynamic models of a robot for the control design. The procedure involves deduction of the robot's kinematic and dynamic models, estimation of the model parameters experimentally, validation of the model and identification of the remaining robot dynamics, which should not be ignored if robustness and high-performance robot operation are required. There are many approaches for generating the dynamic equations of a mechanical system.

Two commonly methods are used for formulating the dynamics based on the specific geometric and inertial parameters of the robot, they are L-E formulation and the recursive Newton-Euler method [8]. Both are equivalent as both describe the dynamic behaviour of the robot motion, but are specifically useful for different purposes. We will use L-E method for our derivation which relies on the energy properties of mechanical systems to compute the equations of motion.

Serial linkage robot arms are a highly non-linear uncertain system with complicated interactions between each joint [9]. These interactions represent gravitational forces dependent on the position of the joint, effective inertia forces due to the acceleration of each joint where the deriving torque is acting reaction forces due to the accelerations of other joints, Coriolis forces generated by the velocities of the other joints and centrifugal forces generated by the angular velocity of each joint. Therefore, the robot arms are usually assumed as non-linear, uncertain, MIMO system.

The most used form of industrial controllers is the PID controller. Statistics shows that they constitute more than 90% of feedback controller used today. This is because it's low cost, simple in structure and robust in performance over a wide range of operation conditions [10].

This paper is organized as follows; section two introduces the mathematical model of the manipulator deriving its dynamic equations showing the response of the system without control. Section three introduces the design of PID controller representing some tuning parameters methods to have the optimum response; also it represents the simulation results. Finally, Section four represents conclusion and future work.

2. DYNAMICS OF THREE-LINKS MANIPULATOR

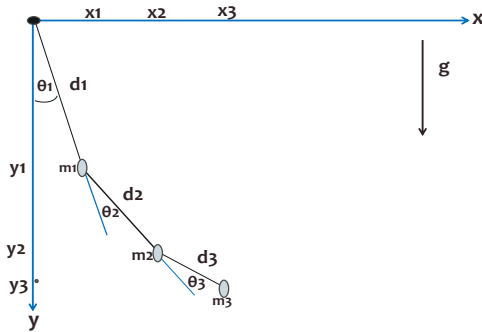We consider the three-link robotic manipulator. The physical system is shown in fig. 1

Fig.1 Three Link Manipulator

The system consists of three masses connected by weightless bars. The bars have length d1, d2, d3. The masses are denoted by m1, m2, m3, respectively.

Let $\theta 1$, $\theta 2$, $\theta 3$ denote the angles in which the first bar rotates about the origin and the second bar rotates about endpoint of the first bar.

The equations for the x-position and the y-position:

$X_1 = d_1 \sin \theta_1, Y_1 = - d_1 \cos \theta_1$ (1)

$X_2 = d_1 \sin \theta_1 + d_2 \sin (\theta_1 + \theta_2),$

$Y_2 = - [d_1 \cos \theta_1 + d_2 \cos (\theta_1 + \theta_2)]$ (2)

$X_3 = d_1 \sin \theta_1 + d_2 \sin (\theta_1 + \theta_2) + d_3 \sin (\theta_1 + \theta_2 + \theta_3)$

$Y_3 = - [d_1 \cos \theta_1 + d_2 \cos (\theta_1 + \theta_2) + d_3 \cos (\theta_1 + \theta_2 + \theta_3)]$ (3)

The joint velocity of each link,

$V_{12} = \dot{x}_{12} + \dot{y}_{12}, V_{22} = \dot{x}_{22} + \dot{y}_{22}, V_{32} = \dot{x}_{32} + \dot{y}_{32}$ (4)

The Kinetic energy (K.E) of each link,

$K_i = \frac{1}{2} m_i v_i^2$ (5)

i = 1, 2, 3 (no. of joints)

The total K.E of all links;

$K = K_1 + K_2 + K_3$ (6)

The potential energy (P.E) of each link;

$P_i = - m_i g Y_i$ (7)

The total P.E of all links;

$P = P_1 + P_2 + P_3$ (8)

2.1 Lagrange-Euler Formulation

The L-E equations are obtained as [2]:

$L = K - P$,

$\tau_i = d/dt [\delta L / \delta \dot{\theta}_i] – [\delta L / \delta \theta_i]$ (9)

Where:

K = Total kinetic energy of the robot arms system.

P = Total potential energy of the robot arms system

θ_i = Generalized coordinates of the robot arms.

$\dot{\theta}_i$ = First time derivative of the generalized coordinate.

τ_i = Generalized torques applied to the system at joint i

to drive link i.

By applying (9) to The Lagrange function of the robot arm yields the necessary generalized torque Ti for joint i actuator to drive the i-th link of the manipulator, for i = 1, 2, 3 which gives:

$\tau_1 = [(m_1 + m_2 + m_3) d_{12} + (m_2 + m_3) d_{22} + m_3 d_{32} + 2 (m_2 + m_3) d_1 d_2 \cos \theta_2 + 2 m_3 d_2 d_3 \cos \theta_3 + 2 m_3 d_1 d_2 \cos (\theta_2 + \theta_3)] \ddot{\theta}_1 + [(m_1 + m_2) d_{22} + m_3 d_{32} + (m_2 + m_3) d_1 d_2 \cos\theta_2 + 2 m_3 d_2 d_3 \cos\theta_3 + m_3 d_1 d_2 \cos(\theta_2 + \theta_3)] \ddot{\theta}_2 + [m_3 d_{32} + m_3 d_2 d_3 \cos\theta_3 + m_3 d_1 d_2 d_3 \cos (\theta_2 + \theta_3)] \ddot{\theta}_3 - [(m_2 + m_3) d_1 d_2 \sin \theta_2 + m_3 d_1 d_2 \sin (\theta_2 + \theta_3)] \dot{\theta}_{22} - [m_3 d_2 d_3 \sin \theta_3 + m_3 d_1 d_2 \sin (\theta_2 + \theta_3)] \dot{\theta}_{32} - [2 (m_2 + m_3) d_1 d_2 \sin \theta_2 + 2 m_3 d_1 d_2 \sin (\theta_2 + \theta_3)] \dot{\theta}_1 \dot{\theta}_2 - [2 m_3 d_2 d_3 \sin \theta_3 + 2 m_3 d_1 d_2 \sin (\theta_2 + \theta_3)] \dot{\theta}_1 \dot{\theta}_3 - [2 m_3 d_2 d_3 \sin \theta_3 + 2 m_3 d_1 d_2 \sin (\theta_2 + \theta_3)] \dot{\theta}_2 \dot{\theta}_3 + [(m_1 + m_2 + m_3) d_1 g \sin \theta_1 + (m_2 + m_3) d_2 g \sin (\theta_2 + \theta_1) + m_3 d_3 g \sin (\theta_1 + \theta_2 + \theta_3)$ (10)

$\tau_2 = [(m_1 + m_2) d_{22} + m_3 d_{32} + (m_2 + m_3) d_1 d_2 \cos \theta_2 + 2 m_3 d_2 d_3 \cos \theta_3 + m_3 d_1 d_2 \cos (\theta_1 + \theta_3)] \ddot{\theta}_1 + [(m_2 + m_3) d_{22} + m_3 d_{32} + 2 m_3 d_1 d_2 \cos \theta_3] \ddot{\theta}_2 + [m_3 d_{32} + m_3 d_2 d_3 \cos \theta_3] \ddot{\theta}_3 + [(m_2 + m_3) d_1 d_2 \sin \theta_2 + m_3 d_1 d_2 \sin (\theta_2 + \theta_3)] \dot{\theta}_{12} - [m_2 d_2 d_3 \sin \theta_3] \dot{\theta}_{32} - [2 m_3 d_2 d_3 \sin \theta_3] \dot{\theta}_1 \dot{\theta}_3 - [2 m_3 d_1 d_2 \sin \theta_3] \dot{\theta}_2 \dot{\theta}_3 + [(m_2 + m_3) d_1 g \sin (\theta_1 + \theta_2) + m_3 d_3 g \sin (\theta_1 + \theta_2 + \theta_3)]$ (11)

$\tau_3 = [m_3 + d_{32} + m_3 d_2 d_3 \cos \theta_3 + m_3 d_1 d_2 \cos (\theta_2 + \theta_3)] \ddot{\theta}_1 + [m_3 d_{32} + m_3 d_2 d_3 \cos \theta_3] \ddot{\theta}_2 + m_3 d_{32} \ddot{\theta}_3 + [m_3 d_2 d_3 \sin \theta_3 + m_3 d_1 d_2 \sin (\theta_2 + \theta_3)] \dot{\theta}_{12} + [m_3 d_1 d_2 \sin \theta_3] \dot{\theta}_{22} - [2 m_3 d_2 d_3 \sin \theta_3] \dot{\theta}_1 \dot{\theta}_2 + m_3 d_3 g \sin (\theta_1 + \theta_2 + \theta_3)$ (12)

Let us rewrite the dynamic equations in the general form:

$\tau_i = D_{ii} \ddot{\theta}_i$ "*Effective torque*"

$+ D_{ik} \ddot{\theta}_k + D_{im} \ddot{\theta}_m$ "*Coupling torque*"

$+ D_{iii} \dot{\theta}_{i2} + D_{ikk} \dot{\theta}_{k2} + D_{imm} \dot{\theta}_{m2}$ "*Centrifugal torque*"

$+ D_{iik} \dot{\theta}_i \dot{\theta}_k + D_{iki} \dot{\theta}_k \dot{\theta}_i$

$+ D_{iim} \dot{\theta}_i \dot{\theta}m + D_{imi} \dot{\theta}_m \dot{\theta}_i$ "*Coriolis torque*"

$+ D_{ikm} \dot{\theta}_k \dot{\theta}_m + D_{imk} \dot{\theta}_m \dot{\theta}_k + D_i$
"*Gravity torque*"

Unfortunately, the computation of these coefficients requires a large amount of arithmetic operations. Thus, the L-E equations are very difficult to utilize for real-time control purposes unless they are simplified [11].

These dynamic motion equations of a manipulator are coupled, nonlinear, and second order ordinary differential equations. The use of these equations to compute the joints torques from the given joint positions, velocities and accelerations for each trajectory set point in real time has been a computational bottleneck. In order to perform real-time control, a simplified robot arm dynamic model is proposed which ignores the Coriolis and centrifugal forces [12].

2.2 State Space Representation
The derivation of the dynamic model of a manipulator based on the L-E formulation is simple and systematic. The L-E equations of motion can be utilized to solve for the forward dynamics problem.

Next, we transform the dynamic equations (second order differential equation) into the state space representation [13]. The coupling inertias are neglected since they are very small compared with the effective joint inertia. It is noted that, the design of the variable structure controller does not require accurate mathematical dynamic model of the manipulator, thus the bounds of the model parameters are sufficient to construct the controller. This property is desirable since the complexity of the manipulator dynamics make the exact calculation of the dynamics infeasible if not impossible [5, 11].

Define the state variables as:

$\theta_1 = x_1 \qquad , \qquad \dot\theta_1 = x_2$

$\theta_2 = x_3 \qquad , \qquad \dot\theta_2 = x_4$

$\theta_3 = x_5 \qquad , \qquad \dot\theta_3 = x_6$

$\dot\theta_1 = \dot x_1 = x_2$

$\ddot\Theta_1 = \dot x_2 = [\tau_1 + D_{122}\, x_{42} + D_{133}\, x_{62} + D_{112}\, x_2\, x_4 + D_{113}\, x_2\, x_6 + D_{123}\, x_4\, x_6 - C_1] / D_{11}$

$\dot\theta_2 = \dot x_3 = x_4$

$\ddot\Theta_2 = \dot x_4 = [\tau_2 + D_{233}\, x_{62} + D_{231}\, x_2\, x_6 + D_{223}\, x_4\, x_6 - D_{211}\, x_{22} - C_2] / D_{22}$

$\dot\theta_3 = \dot x_5 = x_6$

$\ddot\Theta_3 = \dot x_6 = [\tau_3 + D_{311}\, x_{22} + D_{322}\, x_{42} + D_{312}\, x_2\, x_4 - C_3] / D_{33}$

By applying Rung-Kutta forth order (ode-45) method with a sample rate (0.001) to solve the above equations, assuming the following parameters,

  m1 = 1 kg,   m2 = 0.8 kg,   m3 = 3 kg,

  d1 = 1 m,   d2 = 0.8 m,   d3 = 0.6 m

With initial conditions:

$x\,(0) = [0.175, 0, 0.25, 0, 0.275, 0]^T,$

 The torques τi are set to Zero,

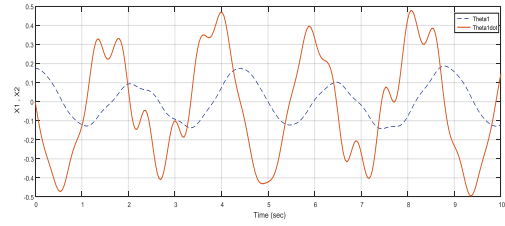That is there is no control on the links. These leads to the results shown in the figures 2, 3 and 4 below:
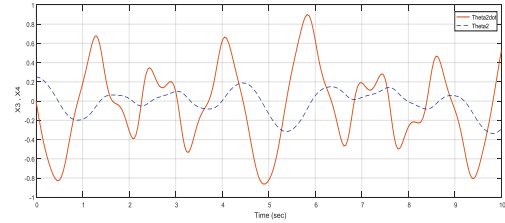

Fig. 2 state versus time for first link

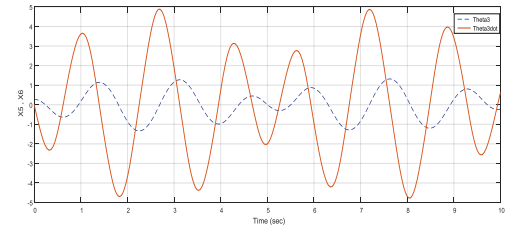**Fig. 3 state versus time for second link**

Fig. 4 State versus time for third link

As shown in the above figures, the system performance is oscillatory under no torques applied (without control). The velocity of the link decreases as it goes to the point of changing direction till it becomes zero at the point of changing direction then, it starts to increase again.

In general, the motion control problem consists of obtaining dynamic model of the manipulator and validates this model, then using this model to design a robust controller that achieves the desired response and performance that resists the environmental characteristics.

3 CONTROLLER DESIGN
The control problem for robot manipulators is the problem of determining the time history of joint inputs required to hold the system of three links in a particular position on X-Y plane or to cause the end-effector to execute a command motion. Having a robust controller gives us the ability to hold each link at a particular angle θi with respect to X-axis. Our algorithm used works by defining an error variable which is the difference between the desired position (target position) and the real position of the manipulator with the effect of the controller.

The general form of equations of motion can be written as

$\tau = M(q)\ddot q + V(q,\dot q) + G(q)$ (13)

Where, q is the generalized joint coordinates

M (q) is the mass matrix (inertia matrix)

V $(q,\dot q)$ is the centrifugal & Coriolis forces

G (q) is the gravity forces

τ is the generalized forces (torques applied to the robot)

$$M(q) = \begin{bmatrix} D_{11} & 0 & 0 \\ 0 & D_{22} & 0 \\ 0 & 0 & D_{33} \end{bmatrix} \qquad (14)$$

$$V(q,\dot{q}) = \begin{bmatrix} -D_{122}\,\dot{\theta}_2^2 - D_{133}\,\dot{\theta}_3^2 - D_{122}\,\dot{\theta}_1\dot{\theta}_2 - D_{133}\,\dot{\theta}_1\dot{\theta}_3 - D_{123}\,\dot{\theta}_2\dot{\theta}_3 \\ D_{233}\,\dot{\theta}_3 - D_{213}\,\dot{\theta}_1\dot{\theta}_3 - D_{223}\,\dot{\theta}_2\dot{\theta}_3 + D_{211}\,\dot{\theta}_1^2 \\ D_{311}\,\dot{\theta}_1^2 + D_{322}\,\dot{\theta}_2^2 + D_{312}\,\dot{\theta}_1\dot{\theta}_2 \end{bmatrix}$$
(15)

$$G(q) = [D_{311}\,(\dot{\theta}_1)^2 + D_{322}\,(\dot{\theta}_2)^2 + D_{312}\,\dot{\theta}_1\,\dot{\theta}_2] \qquad (16)$$

By solving the function $\ddot{q}$, we get

$$\ddot{q} = M(q)^{-1}\,[-V(\ddot{q},\dot{q}) - G(q)] + \tau \qquad (17)$$

The controller for any joint would be

$$F_i = K_{pi}\,e + K_{di}\,\dot{e} + K_{ii}\int e\,dt \qquad \text{where } i = 1, 2, 3 \quad (18)$$

The error (e) signals general form is

$$e(\boldsymbol{\theta}_i) = \boldsymbol{\theta}_{if} - \boldsymbol{\theta}_i \quad \text{where } \boldsymbol{\theta}_{if} \text{ is the target position.} \quad (19)$$

By assuming that:

$$x_i = \int e(\boldsymbol{\theta_i})\,dt, \text{ then } \dot{x}_i = \boldsymbol{\theta}_{if} - \boldsymbol{\theta}_i \quad ; \quad (20)$$

$$\boldsymbol{\theta}_0 = [\boldsymbol{\theta}_1(0) \quad \boldsymbol{\theta}_2(0) \quad \boldsymbol{\theta}_3(0)]^T$$

$$\begin{bmatrix} \ddot{\theta}_{m1} \\ \ddot{\theta}_{m2} \\ \ddot{\theta}_{m3} \end{bmatrix} = M(q)^{-1} * [-V(\dot{q},q) - G(q)] +$$

$$\begin{bmatrix} k_{p1}\,(\theta_{1f} - \theta_1) - k_{d1}\,\dot{\theta}_1 + k_{i1}\,x_1 \\ k_{p2}\,(\theta_{2f} - \theta_2) - k_{d2}\,\dot{\theta}_2 + k_{i2}\,x_2 \\ k_{p3}\,(\theta_{3f} - \theta_3) - k_{d3}\,\dot{\theta}_3 + k_{i3}\,x_3 \end{bmatrix} \qquad (21)$$

This is a second order non-linear system; we have to convert it to a first order system by using state space previously explained. By choosing a proper set of state variables, complex systems may be brought to a more convenient form (state-space form), which only requires solving first order ODE's in matrix form [14].

Defining the state variables as:

$$z_1 = x_1 \ , \ \theta_1 = z_4 \ , \ \dot{\theta}_1 = z_7 \ , \ \ddot{\theta}_1 = \dot{z}_7$$

$$z_2 = x_2 \ , \ \theta_2 = x_5 \ , \ \dot{\theta}_2 = z_8 \ , \ \ddot{\theta}_2 = \dot{z}_8$$

$$z_3 = x_3 \ , \ \theta_3 = x_6 \ , \ \dot{\theta}_3 = z_9 \ , \ \ddot{\theta}_3 = \dot{z}_9$$

The initial conditions are given by

$$Z_0 = [z_{10}\ z_{20}\ z_{30}\ z_{40}\ z_{50}\ z_{60}\ z_{70}\ z_{80}\ z_{90}].$$

Now, we have a system of first order nonlinear differential equations.

3.1 PID Tuning methods

There are many methods for tuning PID parameters, Ziegler Nichols [15], Cohen-Coon [16] and Differential Evolution method [17]. PID tuning using Ziegler Nichols method is based on the frequency response of the closed-loop system. Another method for tuning PID parameters is the DE algorithm which is known to be another effective global optimizer. The performance of the DE depends on three main operations: mutation, generation (reproduction) and selection [18].

3.2 Simulation and results

By applying that:

$\theta_{1f} = 90^o$, $\theta_{2f} = 30^o$, and $\theta_{3f} = 30^o$ and

$\theta_{1i} = 45^o$, $\theta_{2i} = 90^o$, and $\theta_{3i} = -90^o$

The simulation results show that the PID controller gets the links of masses M1, M2 and M3 to the desired positions determined by the angles in a short time (less than 4 seconds) that shows also a rapid response for the system with no steady state error. As shown in the following figures 5, 6 and 7.
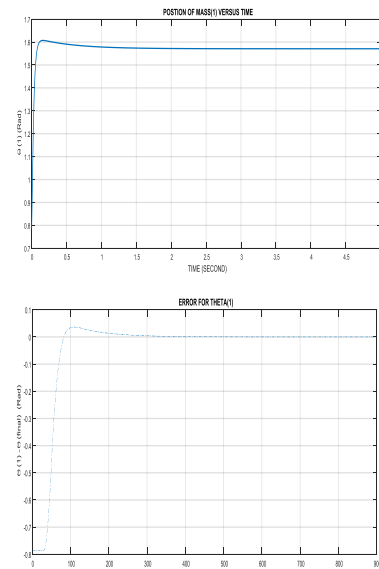

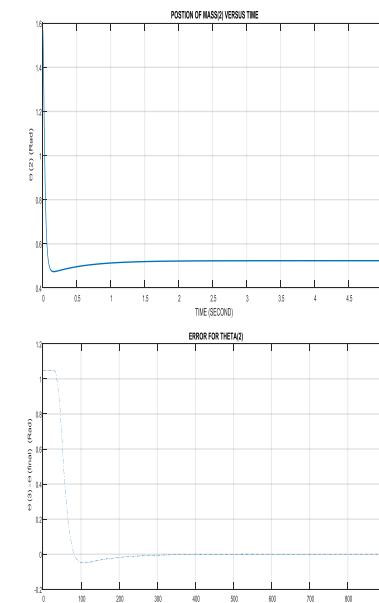**Fig. 5 (a) the time response of θ_1 of mass M_1, (b) the error of θ1**

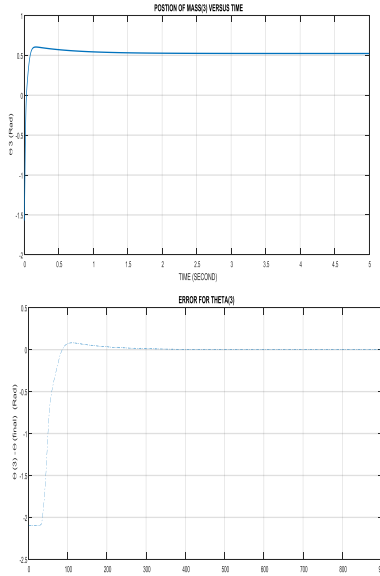**Fig. 6 (a) the time response of θ_2 of mass M_2, (b) the error of θ_2**

Fig. 7 (a) the time response of θ_3 of mass M_3, (b) the error of θ_3

The second part of the simulation procedures is a multi-step response which is achieved by comparing the response of [19] with our results. Figures 8, 9 and 10 show the closed-loop transient response of the controller. Our modified PID controller achieved the target positions for the masses M1, M2 and M3 which is determined by the target angles θ1f, θ2f and θ3f.

The rise time for θ1, θ2 and θ3 approximately equals (0.6, 0.6 and 0.6) seconds respectively with very small overshoots (0.2, 0.2 and 0.1 %) and with almost no steady state error. The comparison between figures 11, 12 in [19] and our modified controller figures 8, 9 and 10 showed that the transient response of our controller is more convenient than transient response of [19].
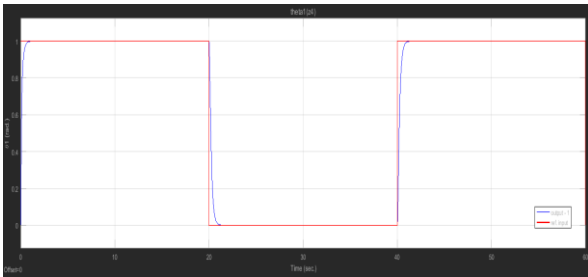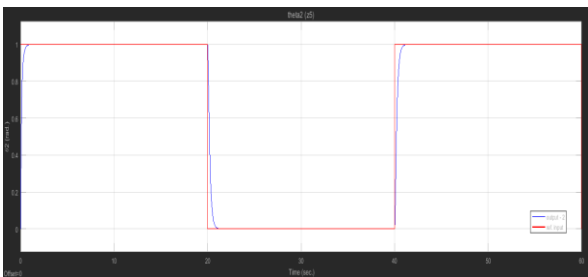

Fig. 8 Transient Response of θ_1

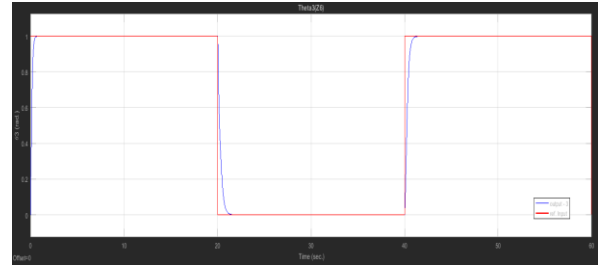**Fig. 9 Transient Response of θ_2**

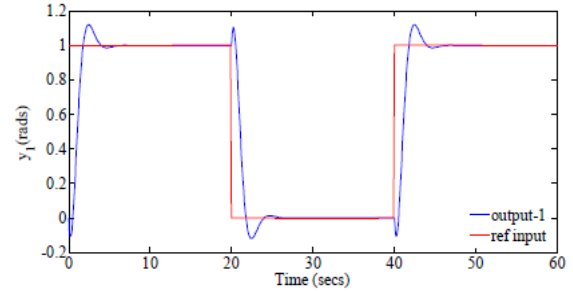**Fig. 10 Transient Response of θ_3**

Fig. 11 Transient Response of θ_1 of [19]
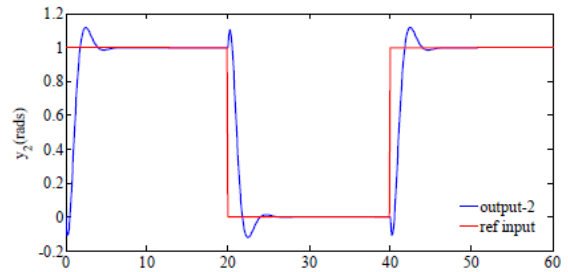

Fig. 12 Transient Response of θ_2 of [19]

4. CONCLUSION

In this paper the dynamic equations of a TRLM were derived using L-E method. The system behaviours without control are shown. We also introduced a robust PID controller applying to the manipulator to control the position of its end-effector. The controller was tested within Matlab/Simulink environment. From the results, we can conclude that obtaining the correct dynamic model of a robot is a necessary step for its control. In other words, to achieve a proper control, a valid dynamic model must be obtained. The PID controller can control the position of manipulator well, but this is dependent on setting the values of PID parameters with proper values. As shown in the comparisons; we can get a better system response. As a future work, we would use neural PID controller instead of the conventional PID controller. Applying disturbance to the system showing the disturbance rejection of the controller it would give better accuracy.

5. APPENDIX. A

$$D_{11} = (m_1 + m_2 + m_3)\, d_{12} + (m_2 + m_3)\, d_{22} + m_3\, d_{32}$$
$$+\, 2\, m_3\, d_2\, d_3\, \cos x_5 + 2\, (m_2 + m_3)\, d_1\, d_2\, \cos x_3$$
$$+\, 2\, m_3\, d_1\, d_2\, \cos (x_3 + x_5)$$

$$D_{122} = (m_2 + m_3)\, d_1\, d_2\, \sin x_3 + m_3 d_1 d_2\, \sin (x_3 + x_5)$$

$$D_{133} = m_3\, d_2\, d_3\, \sin x_5 + m_3\, d_1\, d_2\, \sin (x_3 + x_5)$$

$$D_{112} = 2\, (m_2 + m_3)\, d_1\, d_2\, \sin x_3 + 2\, m_3\, d_1\, d_2\, \sin (x_3 + x_5)$$

$$D_{113} = 2\, m_3\, d_2\, d_3\, \sin x_5 + 2\, m_3\, d_1\, d_2\, \sin (x_3 + x_5)$$

$D_{123} = 2\,m_3\,d_2\,d_3\,sin\,x_5 + 2\,m_3\,d_1\,d_2\,sin\,(x_3 + x_5)$

$C_1 = (m_1 + m_2 + m_3)\,g\,d_1\,sin\,x_1 + (m_2 + m_3)\,g\,d_2\,sin\,(x_1 + x_3) + m_3\,g\,d_3\,sin\,(x_1 + x_3 + x_5)$

$D_{22} = (m_2 + m_3)\,d_{22} + m_3\,d_{32} + 2\,m_3\,d_1\,d_2\,cos\,x_5$

$D_{211} = (m_2 + m_3)\,d_1\,d_2\,sin\,x_3 + m_3\,d_1\,d_2\,sin\,(x_3 + x_5)$

$D_{233} = m_3\,d_2\,d_3\,sin\,x_5$

$D_{213} = 2\,m_3\,d_2\,d_3\,sin\,x_5$

$D_{223} = 2\,m_3\,d_1\,d_2\,sin\,x_5$

$C_2 = (m_2 + m_3)\,d_2\,g\,sin\,(x_1\,x_3) + m_3\,d_3\,g\,sin\,(x_1 + x_3 + x_5)$

$D_{33} = m_3\,d_{32}$

$D_{311} = m_3\,d_2\,d_3\,sin\,x_5 + m_3\,d_1\,d_2\,sin\,(x_3 + x_5)$

$D_{322} = m_3\,d_1\,d_2\,sin\,x_5$

$D_{312} = 2\,m_3\,d_2\,d_3\,sin\,x_5$

$C_3 = m_3\,d_3\,g\,sin\,(x_1 + x_5 + x_3)$

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