International Journal of Computer Applications (0975 - 8887)
Volume 179 - No.39, May 2018

A Markovian Working Vacation Queue with Server State Dependent Arrival Rate and with Unreliable Server

R. Kalyanaraman
Professor of Mathematics
Annamalai University
Annamalainagar-608002, India

A. Sundaramoorthy
Research scholar of Mathematics
Annamalai University
Annamalainagar-608002, India

ABSTRACT

A single server Markovian queueing system with the system alternates between regular busy state, repair state and working vacation state has been considered. The system is busy, it functions as a single server Markovian queue. When it is on vacation, again it functions as a single server Markovian queue but with different arrival and service rates. The vacation policy is multiple vacation policy and the vacation period follows negative exponential. In addition, during service the server may break down, the repair of the server starts immediately. The repair period follows negative exponential. The steady state probability vector of number of customers in the queue and the stability condition are obtained using Matrix-Geometric method. Some illustrative examples are also provided.

Keywords

Working vacation, State dependent arrival rate, Matrix-Geometric method

1. INTRODUCTION

In many real life queueing situations , after service completion, if no customer in the queue, it can be seen that the server leaves the system for a random period of time such a system is called vacation queue. In the past Queueing systems with server vacations have been investigated by many researchers. The readers may refer the survey paper by Doshi(1986) and the monograph by Takagi(1991) for a detailed analysis of vacation system. Recent decads have seen an increasing interest in such a queueing systems due to their applications in telecommunication systems, manufacturing systems and computer systems.

In day to day life, it can be seen that the server works during his vacation period, if the necessity occur, called working vacation queue. In the working vacation queues, the server works with variable service rate, in particular reduced service rate, rather than completely stops the service during the vacation period. Servi and Finn(2002) have first analyzed an $M/M/1$ queue with multiple working vacation, in which the vacation times are exponentially distributed. Wu and Takagi(2006) extend the work to an $M/G/1$ queue. Kim et al.(2003) analyzed the queue length distribution of the $M/G/1$ queue with working vacations. Liu et al.(2007), examined stochastic decomposition structure of the queue length and waiting time in an $M/M/1$ working vacation queue. Xu et al.(2009) extended the $M/M/1$ working vacation queue to an $M^{[X]}/M/1$ working vacation queue. Ke et al.(2010) have given a short survey on vacation models in recent years.

In many real life situations, it can be observed that the arrival of customers as well as service of a customer depends on the number of customers in the system, etc., Yechiali and Naor(1971) have considered a single-server exponential queueing model with arrival rate depending on operational state or breakdown state of the server. Fond and Ross(1977) analyzed the same model with the assumption that any arrival finding the server busy is lost, and they obtained the steady-state proportion of customer's lost. Shogan(1979) has deals a single server queueing model with arrival rate dependent on server state. Shanthikumar(1982) has analyzed a single server Poisson queue with arrival rate depending on the state of the server. Jayaraman et al.(1994) analyzed a general bulk service queue with arrival rate dependent on server breakdowns. Tian and Yue(2002) discussed the queueing system with variable arrival rate. The author studied the model by using the principle of quasi-birth and death process(QBD). Furthermore, they calculated some performance measures, such as the number of customers in the system in steady-state, etc., Matrix-geometric method approach is a useful tool for solving the more complex queueing problems. Matrix-geometric method has been applied by many researchers to solve various queueing problems in different frameworks. Neuts(1981) explained various matrix geometric solutions of stochastic models. Matrix-geometric approach is utilized to develop the computable explicit formula for the probability distributions of the queue length and other system characteristic. Li et al.(2009) used the matrix analytic method to analyze an $M/G/1$ queue with exponential working vacation under a specific assumption.

In practice, it can be seen that, the service channel is subject to breakdowns or some other kinds of service interruption, which are beyond control of the server and the management. Many researchers have contributed on queue with unreliable server [Li et al.(1997), Wang(1995,1997)]. Some more works on queue with breakdown are Wang et al.(1999) and Ke(2005). Grey et al.(2000) incorporated the server breakdown on vacation queuing model. Haridass and Arumuganathan(2008) studied $M^{[X]}/G/1$ queuing system with unreliable server and with single vacation. Lin and Ke(2009) consider a multi server queue with single working vacation. Jain and Jain(2010) investigated a single working vacation model with server break down. Choudhury and Deka(2012) investigated an $M/G/1$ unreliable server, Bernoulli vacation queue with two phases of service. Kalyanaraman and Nagarajan(2016), have analyzed $M^{[X]}/G^K/1$ with vacation and unreliable server. Usually if the server break down the repair process starts immediately. It is quite natural that the server may have to wait for the repairs to start, this time is called delay time. In 2013, Choudhury and Deka(2013) studied a batch arrival unreliable server, Bernoulli vacation queue with two phases of service and delayed repair. Ke et al.(2012), analyzed $M^{[X]}/G/1$ queuing system with an unreliable server and repair, in which the server operates with a randomized vacation policy with multiple vacation. Kalyanaraman and Nagarajan(2017) analyzed a bulk arrival fixed batch service queue with unreliable server, compulsory vacation and delay time.

To the best of our knowledge, in the study of working vacation queue, the existing literatures focus mainly on queueing system with server state independent arrival rates, in this work we deviate from these work by assuming server state dependent arrival rates.

In this paper, we consider an $M/M/1$ queue with multiple working vacation and with breakdown. The arrival rate dependents on the server states. The model has been analyzed using matrix geometric method. The rest of this paper is organized as follows: In section 2, we give the model description. In section 3, we present the steady state solution using matrix geometric method . In section 4, we present some system performance measures. Section 5 gives some particular models. and In section 6, we carried out a numerical study. Finally, a conclusion has been given.

2. THE MODEL

We consider a single-server queueing system with the following characteristics:

(1) The system alternate between two states, up state and down state. In the up state it is either in regular state or in working vacation state. In the down state it is in the repair state.
(2) Arrival process follows Poisson.
(3) When the system is in regular busy period it serves customers based on exponential distribution with rate μ.
(4) During the regular busy period the arrival parameter is λ.
(5) The server takes vacation, if there are no customer in the queue at a service completion point.
(6) During vacation, the arrival rate is λ_1 ($\lambda_1 < \lambda$).
(7) Vacation period follows negative exponential with rate θ and the vacation policy is multiple vacation policy.
(8) When the server is in vacation, if customer arrives, the server serve the customers using exponential distribution with rate μ_1 ($\mu_1 < \mu$). As this vacation period ends, the server instantaneously switches over to the normal service rate μ, if there is at least one customer waiting for service. Upon completion of a service at a vacation period, the server will (i) Continue the current vacation if it is not finished and no customer is waiting for service; (ii) Continue the service with rate μ_1 if the vacation has not expired and if there is at least one customer waiting for service.
(9) The server may break down during a service and the break downs are assumed to occur according to a Poisson process with rate α.
(10) Once the system break downs, the customer whose service is interrupted goes to the head of the queue and the repair to server starts immediately.
(11) Duration of repaired period follows negative exponential with rate β.
(12) During repair period no service takes place but customers arrive according to Poisson process with rate λ_2 ($\lambda_2 < \lambda_1$).
(13) The first come first served (FCFS) service rule is followed to select the customer for service.

2.1 The quasi-birth-and-death(QBD) process

The model defined in this article can be studied as a QBD process. The following notations are necessary for the analysis:

Let $L(t)$ be the number of customers in the queue at time t and let

$$J(t) = \begin{cases} 0, \text{if the server is on working vacation} \\ 1, \text{if the server is busy} \\ 2, \text{if the server is on repaired period} \end{cases}$$

be the server state at time t.

Let $X(t) = (L(t), J(t))$, then $\{(X(t)) : t \geq 0\}$ is a Continuous time Markov chain (CTMC) with state space $S = \{(i,j) : i = 0,1,2; j \geq 0\}$, where j denotes the number of customer in the queue and i denotes the server state.

Using lexicographical sequence for the states, the rate matrix Q, is the infinitesimal generator of the Markov chain and is given by

$$Q = \begin{bmatrix} B_0 & A_0 & & & & & \\ A_2 & A_1 & A_0 & & & & \\ & A_2 & A_1 & A_0 & & & \\ & & A_2 & A_1 & A_0 & & \\ & & & \cdot & \cdot & \cdot & \\ & & & & \cdot & \cdot & \cdot \\ & & & & & \cdot & \cdot & \cdot \end{bmatrix}$$

where the sub-matrices $A_0,\ A_1$, and A_2 are of order 3×3 and are appearing as

$$A_0 = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda_2 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} -(\lambda_1+\mu_1+\theta) & \theta & 0 \\ 0 & -(\lambda+\mu+\alpha) & \alpha \\ 0 & \beta & -(\lambda_2+\beta) \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \mu_1 & 0 & 0 \\ 0 & \mu & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and the boundary matrix is defined by

$$B_0 = \begin{bmatrix} -(\lambda_1+\theta) & \theta & 0 \\ \mu & -(\lambda+\mu+\alpha) & \alpha \\ 0 & \beta & -(\lambda_2+\beta) \end{bmatrix}$$

We define the matrix $A = A_0 + A_1 + A_2$. This matrix A is a 3×3 matrix and it is of the form

$$A = \begin{bmatrix} -\theta & \theta & 0 \\ 0 & -\alpha & \alpha \\ 0 & \beta & -\beta \end{bmatrix}$$

3. THE STEADY STATE SOLUTION

Let $P = (p_0, p_1, p_2, \ldots)$ be the stationary probability vector associated with Q, such that $PQ = 0$ and $Pe = 1$, where e is a column vector of $1's$ of appropriate dimension. The sub probability vector p_0 and p_1 are defined as, $p_0 = (p_{00}, p_{10}, p_{20})$ and $p_i = (p_{0i}, p_{1i}, p_{2i})$ for $i \geq 1$.

If the steady state condition is satisfied, then the sub vectors p_i are such that

$$p_0 B_0 + p_1 A_2 = 0 \tag{1}$$

$$p_i A_0 + p_{i+1} A_1 + p_{i+2} A_2 = 0,\ i \geq 0 \tag{2}$$

$$p_i = p_0 R^i; i \geq 1 \tag{3}$$

where R is the rate matrix, is the minimal non-negative solution of the matrix quadratic equation (see Neuts(1981)).

$$R^2 A_2 + RA_1 + A_0 = 0 \tag{4}$$

Substituting the equation (3) in (1), we have

$$p_0(B_0 + RA_2) = 0 \tag{5}$$

and the normalizing condition is

$$p_0(I - R)^{-1} e = 1 \tag{6}$$

THEOREM 1. *The queueing system described in section 2 is stable if and only if* $\rho < 1$, *where* $\rho = \dfrac{(\lambda_2 \alpha + \lambda \beta)}{\mu \beta}$

PROOF. Consider the infinitesimal generator

$A = \begin{bmatrix} -\theta & \theta & 0 \\ 0 & -\alpha & \alpha \\ 0 & \beta & -\beta \end{bmatrix}$, which is a square matrix of order 3. Let the row vector $\pi = (\pi_1, \pi_2, \pi_3)$ satisfies the condition $\pi A = 0$ and $\pi e = 1$.

Following Neuts(1981), the system is stable if and only if $\pi A_0 e < \pi A_2 e$.

That is,

The system is stable if and only if $\dfrac{(\lambda_2 \alpha + \lambda \beta)}{\mu \beta} < 1$ □

THEOREM 2. *If* $\rho < 1$, *the matrix equation (4) has the minimal non-negative solution*

$$R = \begin{bmatrix} r_0 & r_{01} & r_{02} \\ 0 & r_1 & r_{12} \\ 0 & r_{21} & r_2 \end{bmatrix}$$

where

$$r_{01} = \frac{-\theta r_0}{\left[\mu r_0 + \mu r_1 - (\lambda + \mu + \alpha) + \dfrac{\alpha \mu}{\lambda_2 + \beta} r_{21} + \dfrac{\alpha \beta}{\lambda_2 + \beta}\right]}$$

$$r_{02} = \frac{\alpha r_{01}}{(\lambda_2 + \beta)}$$

$$r_{12} = \frac{\alpha r_1}{(\lambda_2 + \beta)}$$

$$r_{21} = \frac{(\lambda_2 + \beta) r_2 - \lambda_2}{\alpha}$$

$$r_0 = \frac{1}{2\mu_1}\left[(\lambda_1 + \mu_1 + \theta) - \sqrt{(\lambda_1 + \mu_1 + \theta)^2 - 4\lambda_1 \mu_1}\right]$$

$$r_1 = \frac{-S_1 - \sqrt{S_1^2 - 4\lambda\mu(\lambda_2 + \beta)^2}}{2\mu(\lambda_2 + \beta)}$$

$$r_2 = \frac{S_2 - \sqrt{S_2^2 - 4(\lambda_2 + \beta)[\lambda_2(\lambda + \mu + \alpha) - \lambda_2 \mu r_1]}}{2(\lambda_2 + \beta)}$$

PROOF. Let

$$R = \begin{bmatrix} r_0 & r_{01} & r_{02} \\ 0 & r_1 & r_{12} \\ 0 & r_{21} & r_2 \end{bmatrix} \tag{7}$$

$$R^2 A_2 = \begin{bmatrix} \mu_1 r_0^2 & \mu(r_{01}(r_0 + r_1) + r_{02} r_{21}) & 0 \\ 0 & \mu(r_1^2 + r_{12} r_{21}) & 0 \\ 0 & \mu(r_{21}(r_1 + r_2)) & 0 \end{bmatrix} \tag{8}$$

$$RA_1 = \begin{bmatrix} -V_0 r_0 & \theta r_0 - V_1 r_{01} + \beta r_{02} & \alpha r_{01} - (\lambda_2 + \beta) r_{02} \\ 0 & -V_1 r_1 + \beta r_{12} & \alpha r_1 - (\lambda_2 + \beta) r_{12} \\ 0 & -V_1 r_{21} + \beta r_2 & \alpha r_{21} - (\lambda_2 + \beta) r_2 \end{bmatrix} \tag{9}$$

where $V_0 = (\lambda_1 + \mu_1 + \theta)$, $V_1 = (\lambda + \mu + \alpha)$
Substituting (8), (9) and A_0 into (4), gives the following set of equations

$$\mu_1 r_0^2 - (\lambda_1 + \mu_1 + \theta) r_0 + \lambda_1 = 0 \tag{10}$$

$$\mu(r_{01}(r_0 + r_1) + r_{02} r_{21}) + \theta r_0 - (\lambda + \mu + \alpha) r_{01} + \beta r_{02} = 0 \tag{11}$$

$$\alpha r_{01} - (\lambda_2 + \beta) r_{02} = 0 \tag{12}$$

$$\mu(r_1^2 + r_{12} r_{21}) - (\lambda + \mu + \alpha) r_1 + \beta r_{12} + \lambda = 0 \tag{13}$$

$$\alpha r_1 - (\lambda_2 + \beta) r_{12} = 0 \tag{14}$$

$$\mu r_{21}(r_1 + r_2) - (\lambda + \mu + \alpha) r_{21} + \beta r_2 = 0 \tag{15}$$

$$\alpha r_{21} - (\lambda_2 + \beta) r_2 + \lambda_2 = 0 \tag{16}$$

From equations (10)-(16), we get

$$r_{01} = \frac{-\theta r_0}{\left[\mu r_0 + \mu r_1 - (\lambda + \mu + \alpha) + \dfrac{\alpha \mu}{\lambda_2 + \beta} r_{21} + \dfrac{\alpha \beta}{\lambda_2 + \beta}\right]}$$

$$r_{02} = \frac{\alpha r_{01}}{(\lambda_2+\beta)}$$

$$r_{12} = \frac{\alpha r_1}{(\lambda_2+\beta)}$$

$$r_{21} = \frac{(\lambda_2+\beta)r_2 - \lambda_2}{\alpha}$$

$$r_0 = \frac{1}{2\mu_1}\left[(\lambda_1+\mu_1+\theta) - \sqrt{(\lambda_1+\mu_1+\theta)^2 - 4\lambda_1\mu_1}\right]$$

$$r_1 = \frac{-S_1 - \sqrt{S_1^2 - 4\lambda\mu(\lambda_2+\beta)^2}}{2\mu(\lambda_2+\beta)}$$

$$r_2 = \frac{S_2 - \sqrt{S_2^2 - 4(\lambda_2+\beta)[\lambda_2(\lambda+\mu+\alpha) - \lambda_2\mu r_1]}}{2(\lambda_2+\beta)}$$

where,

$$S_1 = [\mu(\lambda_2+\beta)r_2 - \mu\lambda_2 - (\lambda+\mu+\alpha)(\lambda_2+\beta) + \alpha\beta]$$

$$S_2 = [\lambda_2 + (\lambda+\mu+\alpha)(\lambda_2+\beta) - \mu(\lambda_2+\beta)r_1 - \alpha\beta]$$

It is clear that the above equation have unique non-negative solution. Therefore, this non-negative solution must be the minimal. □

THEOREM 3. *If $\rho < 1$, the stationary probability vectors $p_0 = (p_{00}, p_{10}, p_{20})$ and $p_i = (p_{0i}, p_{1i}, p_{2i})$ are*

$$p_{00} = \frac{\mu(1-r_0)(\lambda_2+\beta)[(1-r_1)(1-r_2) - r_{21}r_{12}]}{\mu(\lambda_2+\beta)S_2 + (1-r_0)(\lambda_1+\theta-\mu_1 r_0)V_2}$$

$$p_{10} = \frac{1}{\mu}[(\lambda_1+\theta) - \mu_1 r_0]p_{00}$$

$$p_{20} = \frac{\alpha[(\lambda_1+\theta) - \mu_1 r_0]}{\mu(\lambda_2+\beta)}p_{00}$$

and $p_i = p_0 R^i;\ i \geq 1$
where $V_2 = [(r_{21} - r_2 + 1)(\lambda_2+\beta) + \alpha(r_{21} + 1 - r_1)]$

PROOF. p_{00}, p_{10} and p_{20} follows from the equations (1), (5), (6) and (7). □

REMARK 1. *Even though R in Theorem 2 has a nice structure which enables us to make use of the properties like $R^n = \begin{bmatrix} r_0^n & r_{01}\sum_{j=0}^{n-1} r_0^j r_1^{n-j-1} \\ 0 & r_1^n \end{bmatrix}$, for $n \geq 1$, due to the form of r_0 & r_{01}, it may not be easy to cary out the computation required to calculate the p_i and the performance measures. Hence, we explore the possibility of algorithmic computation of R. The computation of R can be carried out using a number of well-known methods. We use Theorem 1 of Latouche and Neuts(1980). The matrix R is computed by successive substitutions in the recurrence relation:*

$$R(0) = 0 \tag{17}$$

$$R(n+1) = -A_0A_1^{-1} - [R(n)]^2 A_2 A_1^{-1} \text{for} n \geq 0 \tag{18}$$

and is the limit of the monotonically increasing sequence of matrices $\{R(n), n \geq 0\}$.

4. PERFORMANCE MEASURES

Using straightforward calculations the following performance measures have been obtained:

(i) Mean queue length $E(L) = p_0R(I-R)^{-2}e$
(ii) Second moment of queue length= $p_0R(I+R)(I-R)^{-3}e$
(iii) Variance of queue length= $var(L)$
=$p_0R\{(I+R) - p_0R(I-R)^{-1}e\}(I-R)^{-3}e$
(iv) Probability that the server is ideal =p_0e
(v) Mean queue length when the server is an vacation period =$\sum_{i=0}^{\infty} ip_{0i}$
(vi) Mean queue length when the server is in regular busy period =$\sum_{i=0}^{\infty} ip_{1i}$
(vii) Probability that the server is in working vacation period= $pr\{J=0\} = \sum_{i=1}^{\infty} p_{0i}$
(viii) Probability that the server is in regular busy period =$pr\{J=1\} = \sum_{i=1}^{\infty} p_{1i}$

5. PARTICULAR MODEL

By taking particular values to the parameters, we have obtained a particular model related to the system discussed in this article. In the above model, we assume that $\lambda_1 = \lambda_2 = \lambda$, and $\mu_1 = \mu$. We get

$$R = \begin{bmatrix} r_0 & r_{01} & r_{02} \\ 0 & r_1 & r_{12} \\ 0 & r_{21} & r_2 \end{bmatrix}$$

$$p_{00} = \frac{\mu(1-r_0)(\lambda+\beta)[(1-r_1)(1-r_2) - r_{21}r_{12}]}{\mu(\lambda+\beta)S_5 + (1-r_0)(\lambda+\theta-\mu r_0)V_3}$$

$$p_{10} = \frac{1}{\mu}[(\lambda+\theta) - \mu r_0]p_{00}$$

$$p_{20} = \frac{\alpha[(\lambda+\theta) - \mu r_0]}{\mu(\lambda+\beta)}p_{00}$$

and $p_i = p_0R^i;\ i \geq 1$
where

$$r_{01} = \frac{-\theta r_0}{\left[\mu r_0 + \mu r_1 - (\lambda+\mu+\alpha) + \frac{\alpha\mu}{\lambda+\beta}r_{21} + \frac{\alpha\beta}{\lambda+\beta}\right]}$$

$$r_{02} = \frac{\alpha r_{01}}{(\lambda+\beta)}$$

$$r_{12} = \frac{\alpha r_1}{(\lambda+\beta)}$$

$$r_{21} = \frac{(\lambda+\beta)r_2 - \lambda}{\alpha}$$

$$r_0 = \frac{1}{2\mu}\left[(\lambda+\mu+\theta) - \sqrt{(\lambda+\mu+\theta)^2 - 4\lambda\mu}\right]$$

$$r_1 = \frac{-S_4 - \sqrt{S_4^2 - 4\lambda\mu(\lambda+\beta)^2}}{2\mu(\lambda+\beta)}$$

$$r_2 = \frac{S_5 - \sqrt{S_5^2 - 4(\lambda+\beta)[\lambda(\lambda+\mu+\alpha) - \lambda\mu r_1]}}{2(\lambda+\beta)}$$

$$S_4 = [(\lambda+\beta)(\mu r_2 - \lambda + \mu + \alpha) - \mu\lambda + \alpha\beta]$$

$$S_5 = [\lambda + (\lambda+\beta)(\lambda+\mu+\alpha-\mu r_1) - \alpha\beta]$$

$$V_3 = [(r_{21} - r_2 + 1)(\lambda+\beta) + \alpha(r_{21} + 1 - r_1)]$$

6. NUMERICAL STUDY

In this section, some numerical examples are given to show the effect of the parameters $\lambda,\ \lambda_1,\ \lambda_2,\ \mu,\ \mu_1,\ \theta,\ \alpha$ and β on the performance measures: mean queue length, second moment of queue length, variance of queue length, probability that the server is idle, mean queue length when the server is an vacation period, mean queue length when the server is in regular busy period, probability that the server is in working vacation period and probability that the server is in regular busy period for the model analyzed in this paper. The corresponding results are presented as case(1) and case(2).

Case(1): If$\lambda_1 = \lambda_2 = \lambda = 0.3,\ \mu_1 = \mu = 2,\ \theta = 4.2,\ \alpha = 0.3$ and $\beta = 0.6$, the matrix R is obtained using the equations (17) & (18)

$$R = \begin{bmatrix} 0.046829 & 0.103172 & 0.034391 \\ 0 & 0.15 & 0.05 \\ 0 & 0.15 & 0.383334 \end{bmatrix}$$

and the invariant probability vector is
$P = (p_0, p_1, p_2, \ldots)$ where
p_0=(0.147429, 0.324811, 0.200750)
and the remaining vectors p_i's are evaluated using the relation
$p_i = p_0 R^i$, for $i \geq 1$
p_1=(0.0069039529189, 0.0940446928143, 0.0982650816440)
p_2=(0.0003233051975, 0.0295587610453, 0.0426080152392)
p_3=(0.0000151400590, 0.0108583727851, 0.0178221575915)
p_4=(0.0000007089938, 0.0043036416172, 0.0073752785101)
p_5=(0.0000000332014, 0.0017519112443, 0.0030424015130)
p_6=(0.0000000015547, 0.0007191503536, 0.0012538527371)
p_7=(0.0000000000728, 0.0002959506236, 0.0005166019545)
p_8=(0.0000000000034, 0.0001218828983, 0.0002128286287)
p_9=(0.0000000000001, 0.0000502067305, 0.0000876785998)
p_{10}=(0.000000000000007, 0.0000206828008, 0.0000361205275)
p_{11}=(0.000000000000001, 0.0000085204992, 0.0000148803665)
p_{12}=(0, 0.0000035101300, 0.0000061301757)
p_{13}=(0, 0.0000014460458, 0.0000025254114)
p_{14}=(0, 0.0000005957186, 0.0000010403783)
For the chosen parameters $p_{14} \to 0$, and the sum of the steady state probabilities is found to be 0.993217
The performance measures are

(i) Mean queue length $E(L) = 0.523449$
(ii) Second moment of queue length =1.208776
(iii) Variance of queue length=$var(L) = 0.934778$
(iv) Probability that the server is ideal =0.67299
(v) Mean queue length when the server is an vacation period =0.007599
(vi) Mean queue length when the server is regular busy period =0.219899
(vii) Probability that the server is in working vacation period $=$ $pr\{J = 0\} = 0.007243$
(viii) Probability that the server is in regular busy period =$pr\{J = 1\} = 0.141743$

Case(2): In this case we fix the values $\lambda = 0.5,\ \lambda_1 = 0.3,$ $\lambda_2 = 0.1,\ \mu = 5,\ \mu_1 = 2,\ \alpha = 0.4,\ \beta = 0.8$ and we take $\theta = 2.6$ ans 3.0, the corresponding numerical results are generated. The probabilities are presented in Table-1. In Table-2, the performance measures are given.

Table 1. Probabilities.

	$\lambda = 0.5,\ \lambda_1 = 0.3,\ \lambda_2 = 0.1,\ \mu = 5,\ \mu_1 = 2,\ \alpha = 0.4$ and $\beta = 0.8$	
	$\theta = 2.6$	$\theta = 3.0$
p_0	(0.152432, 0.084579, 0.618117)	(0.150313, 0.095728, 0.531057)
p_1	(0.009578, 0.026134, 0.080295)	(0.008698, 0.025733, 0.070443)
p_2	(0.000601, 0.004553, 0.010945)	(0.000503, 0.004303, 0.009739)
p_3	(0.000038, 0.000695, 0.001525)	(0.000029, 0.000644, 0.001368)
p_4	(0.000002, 0.000101, 0.000214)	(0.000002, 0.000093, 0.000193)
p_5	(0.000000149, 0.0000145, 0.000030280)	(0.000000097, 0.000013207, 0.000027343)
p_6	(0.000000009, 0.0000021, 0.000004281)	(0.000000006, 0.000001871, 0.000003869)
p_7	(0, 0.00000029209934, 0.000000605461)	(0, 0.000000264725003, 0.000000547637)
p_8	(0, 0.00000004133980, 0.000000085647)	(0, 0.000000037437285, 0.000000077480)
p_9	(0, 0.00000000584821, 0.000000012115)	(0, 0.000000005294170, 0.000000010962)
p_{10}	(0, 0.0000000008272, 0.000000001714)	(0, 0.00000000174, 0.000000001553)
p_{11}	(0, 0.0000000001171, 0.000000000242)	(0, 0.000000000105, 0.000000000224)
p_{12}	(0, 0.0000000000165, 0.000000000034)	(0, 0.000000000015, 0.000000000031)
p_{13}	(0, 0.0000000000023, 0.000000000005)	(0, 0.0000000000021 0.000000000004)
p_{14}	(0, 0.0000000000003, 0.000000000001)	(0, 0.0000000000003, 0.000000000001)
Total probability	0.989865	0.898894

Table 2. Performance measures.

	$\lambda = 0.5,\ \lambda_1 = 0.3,\ \lambda_2 = 0.1,\ \mu = 5,$ $\mu_1 = 2,\ \alpha = 0.4$ and $\beta = 0.8$	
	$\theta = 2.6$	$\theta = 3.0$
Mean queue length	0.156527	0.141484
Second moment of queue length	0.207232	0.187301
Variance of queue length	0.182731	0.167283
Probability that the server is ideal	0.855128	0.777098
Mean queue length when the server is an vacation period	0.010906	0.009801
Mean queue length when the server is in regular busy period	0.037819	0.036715
Probability that the server is in working vacation period	0.010221	0.009231
Probability that the server is in regular busy period	0.031502	0.159546

7. CONCLUSION

In this article a single unreliable server Markovian queue with working vacation has been analyzed. The model can be extended by assuming the repair time follows general distribution.

8. ACKNOWLEDGMENTS

The work of the second author was supported by University Grants Commission, New Delhi through the BSR (Fellowship) Grant No. F.25-1/2014-15(BSR)/7-254/2009.

9. REFERENCES

[1] **Doshi, B.T.,** Queueing systems with vacations-a survey. *Queueing systems* 1, 29-66, 1986.

[2] **Fond, S. and Ross, S.,** A heterogeneous arrival and service queueing loss model. *Tech-Report ORC 77-12, Operations Research Center, University of California, Berkeley, CA*, 1977.

[3] **Choudhury, G. and Deka, M.,** A single server queueing system with two phases of service subject to server break down and Bernoulli vacation. *Applied mathematical modelling,* 36, 6050-6060, 2012.

[4] **Choudhury, G. and Deka, M.,** A batch arrival unreliable server Bernoulli vacation queue with two phases of service delayed repair. *International journal of operations research,* 10(3), 134-152, 2013.

[5] **Grey, W.L. Wang, M.K. and Scatt, M.K.,** A vacation queueing model with service break down. *Applied mathematical modelling,* 24, 391-400, 2000.

[6] **Haridass, M. and Arumuganathan, R.,** Analysis of a bulk queue with unreliable server and single vacation. *International journal of open problems compt.math,* 1(2), 130-148, 2008.

[7] **Jain, M. and Jain, A.,** Working vacation queueing model multiple type of server breakdown. *Appl. math. modelling,* 34(1), 1-13, 2010.

[8] **Jayaraman, D., R. Nadarajan and M.R. Sitrarasu,** A general bulk service queue with arrival rate dependent on server breakdowns, *Appl. math. modelling,* 18, 156-160, 1994.

[9] **Kalyanaraman, R. and Nagarajan, P.,** Bulk arrival, fixed batch service queue with unreliable server and with Bernoulli vacation. *International journal of applied engineering research,* 11(1), 421-429, 2016.

[10] **Kalyanaraman, R. and Nagarajan, P.,** Bulk arrival, fixed batch service queue with unreliable server, compulsory vacation and with delay time. *International journal of pure and applied mathematics,* 111(6), 20-28, 2017.

[11] **Ke, J.C,** Modified T vacation policy for an $M/G/1$ queuing system with an unreliable server and start up, *Mathematical and computer modelling,* 41, 1267-1277, 2005.

[12] **Ke, J.C. and Huang, K.B.,** Analysis of batch arrival queue with randomized vaation policy and an un-reliable server, *Journal of system science complexity,* 25(4), 759-777, 2012.

[13] **Ke, J.C, Wu, C.H, and Zhang, Z.G.,** Recent development in vacation queueing models: A short surrey, *Int. Jr. of Oper. Res,* 7(4), 3-8, 2010.

[14] **Kim, J.D., Choi, D.W., and Chae, K.C.,** Analysis of queue-length distribution of the $M/G/1$ with working vacation, $(M/G/1/WV)$. *In Proceeding of the International Conference on Statistics and related, Honolulu, Hawaii, USA,* 2003.

[15] **Latouche, G and Neuts, M.F.,** Efficient algorithmic solutions to exponential tandem queues with blocking, *SIAM J. Algebraic Discrete Math.,* 1, 93-106, 1980.

[16] **Li, W, Shi, D., and Chao, X.**, Reliability analysis of $M/G/1$ queueing system with server breakdown and vacations, *Journal of applied probability,* 34, 546-555, 1997.

[17] **Li, J.H, Tian, N., Zhang, Z.G., and Luh, H.P.**, Analysis of the $M/G/1$ queue with exponenetially working vacation-a matrix analytic approach, *Queueing systems,* 61, 139-166, 2009.

[18] **Lin, Ch.H., and Ke, J.Ch.,** Multi-server system with single working vacation, *Applied Mathematical Modelling,* 33, 2967-2977, 2009.

[19] **Liu, W., Xu, X. and Tian, N.**, Stochastic decompositions in the $M/M/1$ queue with working vacation, *Oper. Res. Lett.,* 35., 595-600, 2007.

[20] **Neuts, M.F,** Matrix-Geometric solution in stochastic models, *Vol 2 of John Hopkins series in the Mathematical Sciences, Johns Hopkins University press, Baltimore, md, USA,* 1981.

[21] **Servi, L.D., and Finn, S.G.,** $M/M/1$ queues with working vacations $(M/M/1/WV)$, *Perform. Eval.* 50, 41-52, 2002.

[22] **Shanthikumar, J.G.**, Analysis of a single server queue with time and operational dependent server failures, *Adv. in mgnt. studies,* 1, 339-359, 1982.

[23] **Shogan, A.W.**, A single server queue with arrival rate dependent on server breakdowns, *Naval Res. Log. Quart.* 26, 487-497, 1979.

[24] **Takagi, H.,** Queueing Analysis-A Foundation of Performance Evaluation vacation and Priority Systems, Vol. 1, North-Holland, New York, 1991.

[25] **Tian. N and Yue. D,** Quasi-birth and death process and the matrix geometric solution. *Beijing: Science Press*, 2002.

[26] **Wang, K.H.,** Optimal operation of a Markovian queuing system with a removable and non reliable server, *Micro electron Reliab.,* 35, 1131-1136, 1991.

[27] **Wang, K.H., Chang, K.W., and Sivazlian, B.D,** Optimal control of removable and non reliable server in an infinite and finite $M/H_z/1$ queueing system, *Applied mathematical modeling,* 23, 651-666, 1999.

[28] **Wu, D.A., and Takagi, H.,** $M/G/1$ queue with multiple working vacations, *Perform. Eval.* 63, 654-681, 2006.

[29] **Xu, X., Zhang, Z. and Tian, N.** Analysis for the $M^X/M/1$ working vacation queue, *Int. Jr. of Infor. & Manag. Sci.,* 20, 3, 379-394, 2009.

[30] **Yechiali, V and Naor. P.,** Queueing problems with heterogeneous arrivals and service, *Oprs. Res,* 19, 722-734, 1971