

# Performance Analysis of DE-QPSK and $\pi/4$ QPSK with MRC Receiver over $\kappa - \mu$ fading Channel

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## ABSTRACT

This paper presents simple and highly accurate approximated expression for the average symbol error rate of Differential Encoded Quadrature Phase Shift Keying (DEQPSK) and  $\pi/4$  QPSK with MRC (Maximal Ratio Combining) diversity in  $\kappa - \mu$  fading channel which is better suited for Line of Sight applications. Further the solution for average symbol error rate (ASER) of DEQPSK and  $\pi/4$  QPSK is derived. ASER is plotted for different values of fading parameter and for Lth order MRC receiver. Furthermore, error performance of both modulation techniques has been analyzed for Nakagami-m, Rician, One sided Gaussian and Rayleigh fading as a special case.

## General Terms

Wireless communication

## Keywords

ASER, DE-QPSK,  $\pi/4$  QPSK

## 1. INTRODUCTION

Fading channel modelling is important to realize, think and analyses different issues in wireless communication. Different distributions are proposed in literature to model the fading channels like Rayleigh, Rice, Nakagami-m etc. The  $\kappa - \mu$  fading distribution proposed by Yacoub in [1] is used to represent the small- scale fading in a line of sight condition. This model better fits to experimental data than other fading distributions like the Rice, the Rayleigh, the Nakagami-m etc. even in tail regions. Some commonly used fading models like the Rice and the Nakagami-m can be implemented using  $\kappa - \mu$  fading distributions.

Error performance is an important criterion to evaluate quality of service (QoS) of any wireless communication system. Lot of research work has been done to analyse the error performance of wireless communication systems. In [2] symbol error rate of MRC receiver is calculated for Quadrature amplitude modulation in  $\kappa - \mu$  fading channel. Steps suggested in [2] to evaluate symbol error rate of DE-QPSK and  $\pi/4$ -QPSK are used in this paper. ASER of L-branch MRC receiver for the same modulations for  $\kappa - \mu$  fading channel is calculated. In [4] ASER of MPSK, with MRC receiver, is derived using MGF based approach over  $\kappa - \mu$  fading channel. Effect of outdated CSI (Channel State Information) is also considered in this paper.

Similarly in [5] MGF (Moment Generating Function) approach is used to calculate AER (Average Error Rate) of BPSK( Binary Phase Shift Keying) modulation for  $\kappa - \mu$  fading channels. In [6] ABER expressions are derived using MGFs of the  $\eta - \mu$  and  $\kappa - \mu$  fading channels.

## 2. CHANNEL MODEL

The fading channel is modelled as slow, flat and with  $\kappa - \mu$  statistics corrupted by additive white Gaussian noise (AWGN). Nonhomogenous environment is assumed for modelling the physical model. In this, signal is considered to be formed of clusters of multipath waves. The received signal at the lth branch of the MRC receiver with L antennas, is

$$r_l(t) = \alpha_l s(t) \alpha^{j\phi_l} + n_l(t), \quad (1)$$

where  $s(t)$  is the transmitted signal and  $n_l(t)$  is the complex Gaussian noise with zero mean. The instantaneous phase  $\phi_l$  of the random variable.  $\alpha_l$  is the amplitude of the  $\kappa - \mu$  fading channel and pdf of this is given by [1]

$$p_{\alpha_{\kappa-\mu}}(\alpha_l) = \frac{2^{\mu}(1+\kappa)^{\frac{\mu+1}{2}} \alpha_l^{\mu} e^{-\frac{\mu(1+\kappa)}{\Omega} \alpha_l^2} I_{\mu-1} \left( 2^{\mu} \sqrt{\frac{\kappa(1+\kappa)}{\Omega}} \alpha_l \right)}{\kappa^{\frac{\mu-1}{2}} e^{+\frac{\mu\kappa}{\Omega}} \Omega^{\frac{\mu+1}{2}}} \quad (2)$$

Where  $\kappa > 0$  and  $\mu > 0$  and  $I_\nu(\cdot)$  is the modified Bessel function of the first kind and of  $\nu$ th order.  $\mu = \frac{1}{V(\alpha^2)} \frac{1+2\kappa}{(1+\kappa)^2}$  is the number of clusters for  $\kappa - \mu$  distribution. The parameter  $\kappa$  represent the ratio of the power of the dominant component to the power of the scattered component.

## 3. PERFORMANCE ANALYSIS

In MRC diversity, received signal from all the branches are co-phased, proportionally weighted and are algebraically added together as  $\gamma_t = \sum_{l=1}^L \gamma_l = \frac{E_b}{N_o} (\alpha_1^2 + \alpha_2^2 + \dots + \alpha_L^2)$  where  $\gamma_l = \frac{E_b}{N_o} \alpha_l^2$  is the instantaneous SNR at the  $l^{th}$  input branch [3]. The PDF of MRC output SNR( $\gamma_t$ ) is taken from [2 eq. 12]

$$p_{\gamma_{\kappa-\mu}}(\gamma_t) = L\mu \left[ \frac{1+\kappa}{\bar{\gamma}_t} \right]^{\frac{L\mu+1}{2}} \left[ \frac{\gamma_t}{\kappa} \right]^{\frac{L\mu-1}{2}} e^{-L\mu \left( \frac{(1+\kappa)\gamma_t}{\bar{\gamma}_t} + \kappa \right)} I_{L\mu-1} \left( 2L\mu \sqrt{\kappa(1+\kappa)} \frac{\gamma_t}{\bar{\gamma}_t} \right) \quad (3)$$

Where  $\bar{\gamma}_t$  is the average SNR and  $I_\alpha(\cdot)$  is the modified Bessel function of the first kind of the order  $\alpha$ .

**Average Symbol Error Rate:** Conditional SER expression for Coherent DE-QPSK and  $\pi/4$  QPSK is same as given in [3 (8.39)]

$$P_s(e|\gamma_t) = 8 \left[ \frac{Q(\sqrt{\gamma_t})}{2} - Q^2(\gamma_t) + Q^3(\sqrt{\gamma_t}) - \frac{Q^4(\sqrt{\gamma_t})}{2} \right], \quad (4)$$

The ASER can be calculated by averaging the conditional symbol error rate over the PDF of the output SNR for additive white Gaussian noise, which is as:

$$ASER = P(e) = \int_0^\infty P(e|\gamma_t) P_\gamma(\gamma_t) d\gamma_t, \quad (5)$$

Where  $P(e|\gamma_t)$  the Conditional Symbol Error Rate for the given modulation is scheme and  $P_\gamma(\gamma_t)$  is the PDF of the output SNR. We know the relation between Q-function and complemented error function  $erfc(x)$  as  $Q(x) = \frac{1}{2}erfc\left(\frac{x}{\sqrt{2}}\right)$ , A upper bound for  $erfc$  function is given in [2 eq. 6]

$$erfc(x) \approx \frac{1}{6}e^{-x^2} + \frac{1}{2}e^{-\frac{4x^2}{3}}, \quad x > 0.5 \quad (6)$$

Put (3), (4) and (6) in (5) and solving the equation,

Equation for ASER can be written as equation (7), where

$$\gamma_2(\rho) = \left[ \frac{L\mu(1+\kappa)}{L\mu(1+\kappa)+\rho\gamma_t} \right]^{L\mu} e^{-\frac{L\mu\kappa\rho\gamma_t}{L\mu(1+\kappa)+\rho\gamma_t}} \text{ as given in [7]}$$

### 3. NUMERICAL RESULTS AND DISCUSSIONS

The expression for average SER is numerically evaluated and plotted for different values of fading parameters of  $\kappa$ - $\mu$  fading channel. The effect of L on the performance of the wireless system is studied, as expected, when L is increased performance is improved. Effect of  $\mu$  i.e. number of clusters is also investigated. When  $\kappa$  remains constant and  $\mu$  increases, then SER decreases as shown in Figure 1,2,3. Since large value of  $\mu$  means more number of multipath clusters at the receiver, resulting the received signals tend to be more deterministic than those having less multipath clusters. Increase in  $\mu$  results gain of SNR. It is observed from the table that for  $\mu=2$ , SER is improved than other values of  $\mu$ . For  $\mu=1$ , SER is approximately  $10^{-8}$  at  $E_b/N_0=30$ , while it is approximately  $10^{-15}$  for  $\mu=2$ . Figures 4 to 7 show the average SER for DEQPSK and  $\pi/4$  QPSK modulation for different well known fading distributions obtained from  $\kappa$ - $\mu$  fading distribution as special cases such as Nakagami -m, Rayleigh, One sided Gaussian, Rician fading distributions. To get the desired results, suitable combination of L and  $\mu$  can be selected.

### 4. CONCLUSION

ASER expression for DE-QPSK and  $\pi/4$ -QPSK schemes for  $\kappa$ - $\mu$  fading distribution are presented and plotted the results. ASER is improved with MRC combining. ASER for different modulations and different combining techniques can be calculated for future work.

### 5. ACKNOWLEDGEMENTS

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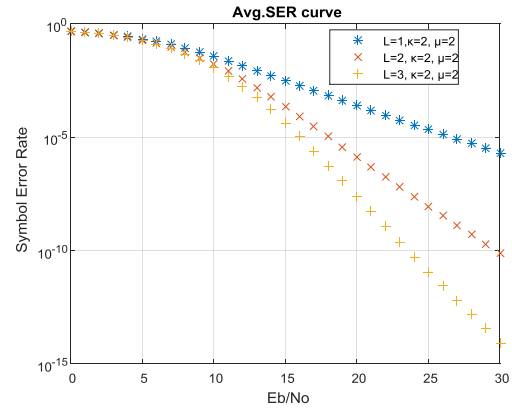


Figure 1. ASER for DE-QPSK and  $\pi/4$ -QPSK for  $\kappa=2, \mu=2$

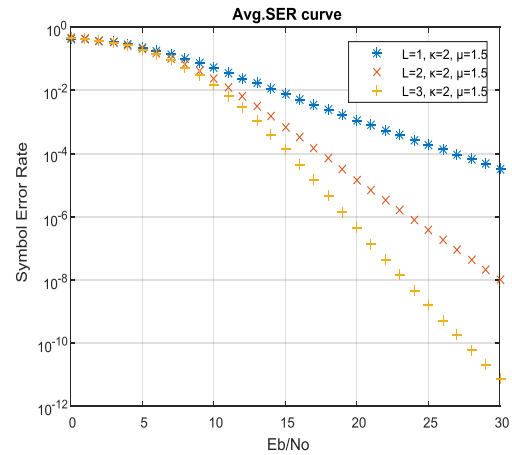


Figure 2. ASER for DE-QPSK and  $\pi/4$ -QPSK for  $\kappa=2, \mu=1.5$

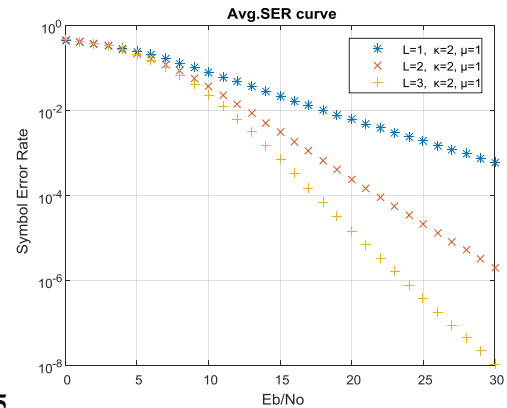
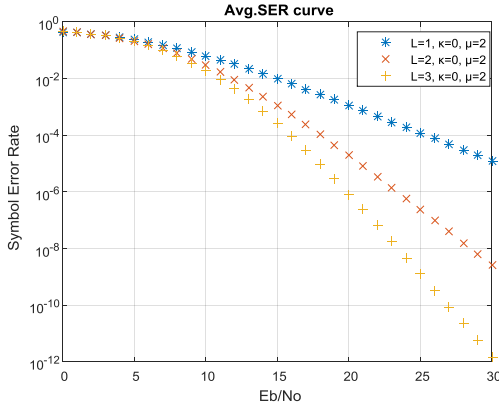
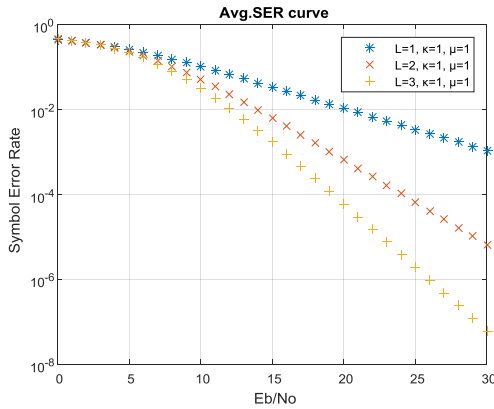


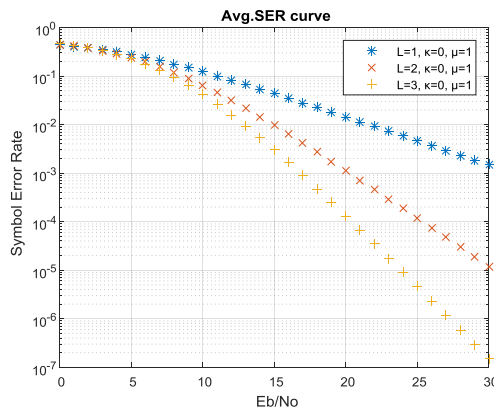
Figure 3. ASER for DE-QPSK and  $\pi/4$ -QPSK for  $\kappa=2, \mu=1$



**Figure 4. Special case: ASER for DE-QPSK and  $\pi/4$ -QPSK for  $\kappa=0, \mu=2$  (Nakagami-m Fading)**



**Figure 5. Special case: ASER for DE-QPSK and  $\pi/4$ -QPSK for  $\kappa=1, \mu=1$  (Rician Fading)**

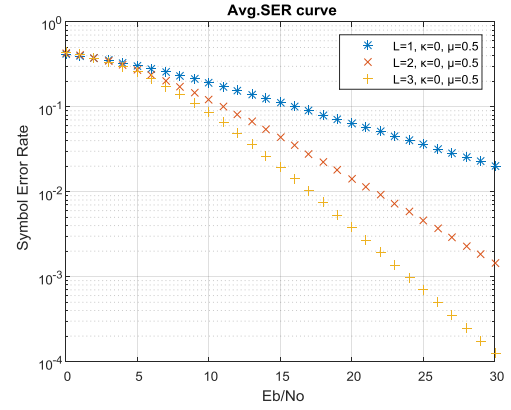


**Figure 6. Special case: ASER for DE-QPSK and  $\pi/4$ -QPSK for  $\kappa=0, \mu=1$  (Rayleigh Fading)**

$$P_s(e) = \frac{1}{3} Y_2\left(\frac{1}{2}\right) + Y_2\left(\frac{2}{3}\right) - \frac{1}{18} Y_2(1) - \frac{1}{3} Y_2\left(\frac{7}{6}\right) - \frac{1}{2} Y_2\left(\frac{4}{3}\right) + \frac{1}{216} Y_2\left(\frac{3}{2}\right) + \frac{1}{24} Y_2\left(\frac{5}{3}\right) + \frac{1}{8} Y_2\left(\frac{11}{6}\right) + \frac{644}{5184} Y_2(2) - \frac{1}{432} Y_2\left(\frac{13}{6}\right) - \frac{1}{96} Y_2\left(\frac{7}{3}\right) - \frac{1}{48} Y_2\left(\frac{5}{2}\right) - \frac{1}{64} Y_2\left(\frac{8}{3}\right) \quad (7)$$

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**Figure 7. Special case: ASER for DE-QPSK and  $\pi/4$ -QPSK for  $\kappa=0, \mu=0.5$  (One sided Gaussian Fading)**

**Table 1. Values of ASER for different values of  $\kappa, \mu$**

Value of $\kappa, \mu$	L	ASER
		Approx. SNR(Db)=30
$\kappa = 2, \mu=2$	1	$2.02 * 10^{-6}$
	2	$7.56 * 10^{-11}$
	3	$8.19 * 10^{-15}$
$\kappa = 2, \mu=1.5$	1	$3.24 * 10^{-5}$
	2	$1.05 * 10^{-8}$
	3	$7.08 * 10^{-12}$
$\kappa = 2, \mu=1$	1	0.000588
	2	$2.02 * 10^{-6}$
	3	$1.055 * 10^{-8}$

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