# A Consistent Hybrid of Analytical Hierarchy Process (AHP) and Graph Theory Matrix Approach (GTMA) with Application to Selection of PET scan Machine Problem for Cancer Hospital

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## ABSTRACT
The principle purpose of this paper is Hybridization of Analytical Hierarchy Process (AHP) with Graph Theory Matrix Approach (GTMA). In this article Hybridization of AHP is being done in the matter of basic leadership. The proposed methodology is considered for a specific Cancer hospital that was required to choose Pet Scan Machine for Cancer patients. There are such a significant number of PET scan machines accessible in market and the choice to choose a correct machine more often than not includes various criteria. In this way with a specific end goal to survive vulnerabilities, hybridization of AHP-GTMA has been made. Since AHP has been an outstanding technique for mining the decision maker psyche and GTMA approach is utilized for data mining. Both of these mining forms are hybridized so as to choose the machine with best highlights. Furthermore theoretical consistency is also presented. Finally the results are verified with Soft Expert Set and stability of hybrid method is checked by Satty's Scale.

## Keywords
Hybridization, Soft Expert Set, AHP Technique, GTMA, Agree soft expert set, Disagree soft expert set, Machine

## 1. INTRODUCTION
Multi Criteria Decision Making (MCDM) strategies have developed to suit different kind of uses. Different techniques have been developed, with little varieties to existing strategies leading towards the development of new branches of research. In our day by day lives, normally different criteria are measure verifiably and one might be ok with the results of such choices that are made in light of just intuition [1]. On the other hand, when a lot is on the line, it's vital to legitimately structure the problem and unequivocally assess various criteria. Organizing the complex issues properly and accounting various criteria unequivocally induce more educated and better choices. Many advances have been made in this field since the beginning of the MCDM technique. An assortment of methodologies and techniques, many decision making software have been used to actualize numerous specific complex issues [2, 3].

MCDM was first developed by B.Franklin (1706-1790). Many techniques are related to MCDM but in this paper recent work done on the approaches of MCDM AHP-GTMA as well as the different fields where these approaches have been applied recently, is analyzed. Basic intention is to present the case that AHP-GTMA is more suitable method to solve the PET scan machine selection problem because it ensures theoretical systematically consistency.

The AHP is a composed methodology that deals with separating complicated selections, in the matter of math and mind sciences. T.L Saaty created it in 1970s that has been comprehensively considered and reformed by then and into the foreseeable future. Cooperative choice making [4] is one of its specific applications. Furthermore, it is widely used in grouping of decision situations [5], in fields of business, government, industry, shipbuilding [6], social protection and training. Prescribing "right" decision, AHP empowers boss to find best option for their goal and for appreciation of the problem [7]. It gives a good platform for sorting out a decision problem, for addressing and estimating its part, by relating those segments to general destinations, and for surveying elective game plans. Advantages of AHP includes straightforwardness and advantageous, effortlessness by utilizing pair-wise comparisons, consistency in assessment and flexibility. Whereas disadvantages are as per: change from verbal to numerical scale, irregularities forced by one to nine scale, disagreement between decision makers and choice maker ability.

GTMA is a legitimate and systematical approach which was begun from combinatorial science. Graph theory matrix approach examines and comprehends the system as an entire by distinguishing system and sub-system up to the segment level. It helps in choosing the most reasonable decision from among a substantial number of choices for a given issue.

Some of combined application of AHP and GTMA as listed as following: GTMA is used by Govindan et al [8] to analyze barriers in implementation of green supply chain management in India. Fathi et al [9] apply GTMA with fuzzy AHP (FAHP) for gear selection. Chaghoosi et al [10] integrate fuzzy GTMA (FGTMA) and FAHP to optimize the selection of locations for gas pressure reducing stations. Darviash et al [11] GTMA for the purpose of ranking contractors in India. Kulkarni [12] uses GTMA for performance evaluation of TQM in Indian business sector. Singh and Rao [13] apply AHP-GTMA in relation to problems of industrial environment. They call it "The hybrid multiple attribute decision making (MADM) technique".

Lanjewar et al [14] use a certain "Hybrid graph theory with AHP method", which can definitely be considered an application of AHP-GTMA, for assessment of different fuels of transportation. Rao et al [15] apply the "Utility concept" and use GTMA to improve the overall performance of machining by assigning weights to the different machining characteristics. Jain and Raj [16] model the performance variables in a Flexible Manufacturing System (FMS) using different methods like interpretive structural modelling, structural equation modeling and GTMA, and then they compare the results of all these models.

Chou and Ongkowijoyo [17] use “a stochastic graphical matrix model” to assay risk-based group decision making for decisions about different types of renewable energy schemes. Akaa et al [18] work on designing appropriate fire protection for steel structures using a grouped-decision-based AHP decision analysis. Edrogen et al [19] apply these approaches to construction management. In particular, they use AHP and expert choice decision-making approach. Dweiri et al [20] design an AHP based decision support system for selection of suppliers in the automotive industry. Szulecka and Zalazar [21] incorporate SWOT into AHP and research on forest plantations in their country. Bian et al [22] study complex networks and adopt AHP to identify the most influential nodes in such a network.

Sindhu et al [23] use AHP+TOPSIS, a hybrid MADM method, to evaluate the feasibility of solar farms deployment. Dong and Copper [24] designed the "Orders-of-magnitude AHP” (OM-AHP), which is a supply chain risk assessment framework that uses AHP. Hillerman et al [25] use clustering for analysis of seemingly bogus health care claims received by health care providers and then AHP is used for sorting of suspicious claims to be sent for audit. Taylan et al [26] is focused on applying the FAHP+FTOPSIS (fuzzy TOPSIS) method to study construction projects i.e. identifying and assessment of the risks involved and the selection of a project among an array of projects.

Mainly the paper emphasizes is to attain a verdict for a everyday life problem by using AHP-GTMA and the efficiency of the same issue is demonstrated. Finally, the results are verified by Soft Expert Set technique. In section 2 basic preliminaries and proposed methodologies are presented, section 3 comprises of hybridization of AHP with GTMA, section 4 includes the consistency analysis, in section 5 verification with soft expert set is presented. Finally, section 6 comprises the conclusion of whole research.

## 2. PRELIMINARIES:

**Definition: (Soft Expert Set)**

A pair (F, A) is called a soft expert set over U, where F is a mapping given by

$$F: A \to P(U)$$

Where P (U) denotes the power set of U.

**Definition: (Agree-Soft expert Set)**

An agree soft expert set $(F, A)_1$ over U is a soft expert subset of (F, A) is defined as

$$(F, A)_1 = \{F_1 e : e \in E * X * 1\}$$

**Definition: (Disagree-Soft Expert Set)**

An disagree soft expert set $(F, A)_0$ over U is a soft expert subset of (F, A) is defined as

$$(F, A)_0 = \{ F_0(e) : e \epsilon E * X * 0\}$$

**Definition: (AHP)**

The Analytic Hierarchy Process (AHP) is a multi-criteria decision making method allows decision makers to model a complex problem in a hierarchical structure, showing the relationships of the goal, objectives (criteria), and alternatives.

**Definition: (Data Mining)**

Data mining is the route toward managing considerable instructive accumulations to perceive plans additionally, develop associations with handle issues through data investigation. Data mining mechanical assemblies empower dares to anticipate future examples.

**Definition: (Mind Mining)**

The additional estimation of cutting edge information mining systems is their capacity to distinguish shrouded structures (obscure relations) in vast collections of information. Interestingly, the estimation of concealed signs from the mind and body with a specific end goal to light up the client’s cognizant and oblivious reasoning is mind mining

**Definition: (Graph Theory Matrix Approach)**

GTMA is a legitimate and systematical approach which was begun from combinatorial science. Graph theory matrix approach examines and comprehends the system as an entire by distinguishing system and sub-system up to the segment level. It helps in choosing the most reasonable decision from among a substantial number of choices for a given issue.

**Reason for Hybridization:**

Hybridization induces cementing two specific sections that made typical thing made out of various fragments or blended character. All things considered anything got from heterogeneous sources, or made out of parts of different or stirred up sorts is believed to be cross breed. The path toward mixing two parts is called hybridization.

AHP has been an outstanding technique for mining the DM’s brain as to have the pair wise comparison sentiments and in addition need of criteria. And, since GTMA is an ongoing prevalent technique for basic decision making it is utilized to mind the informational collections so by adopting the AHP GTMA strategy not just uses data from data mining yet in addition mind mining.

## 3. DESCRIPTION OF PROPOSED METHODOLOGY:

## 4. THE PROPOSED METHODOLOGY REQUIRES MAKING A DECISION FOR SELECTION OF THE MACHINE. THIS WAS INITIALLY DONE BY APPLYING AHP ON THE OPINIONS OF EXPERTS. STEPS OF AHP ARE AS FOLLOWS [27]:

- Make a pair wise comparison matrix for criteria. In this study, a three point scale has been used as previous studies. Because it has been demonstrated that most responders do not use more than three judgments (equal, more, much more) and lay users reported puzzled at a moment of using a more complex scale.
- Determine the relative normalized weight ($W_j$) of each criteria/sub-criterion by normalizing the geometric mean of the rows in the comparison matrix:

$$GM_j = \prod_{j=1}^{N} a_{ij}^{\;1/N}$$

$$W_j = {GM_j}\Big/{\sum_{j=1}^{N} GM_j}$$

After that AHP is hybridized with Game Theory Matrix Approach (GTMA). Steps of GTMA are as follows [28]:

- Alternative selection of attributes
- Permanent values of alternatives
- Index Score
- Ranking the alternatives

- Final outcome

Final results are also checked by consistency analysis. Furthermore Soft Expert Set is used to verify the results.
Steps of Soft Expert set are as follows [29]:

- Assigning weights to parameters
- Construction of Agree Matrix
- Construction of Average Time Dependent (ATD) Matrix
- Construction of Refined Time Dependent(RTD) Matrix
- Construction of CETD Matrix
- Sum of column Matrix
- Observation of outcome and arrive at result.

**Description of Proposed Problem:**
**The problem that is to be considered is as follows:**
The decision problem requires making a decision for PET scan machine for Cancer Hospital. The decision problem involves four machines alternatives to be considered from the universe

$U = \{s_1, s_2, s_3, s_4\}$.

Let $X = \{s, t, u\}$ be a set of expert members. These experts consider a set of parameters

$$E = \{e_1, e_2, e_3, e_4\}$$

Where the parameter $i, (i = 1, 2, 3,4)$ stands for efficiency, cost, result and time respectively. Here $e_1$ and $e_3$ are TMTB (The More The Better) and $e_2$ and $e_4$ are TLTB (The Less The Better).

**Decision Making Team:**
The expert team was formed by:

- s = Director of Hospital
- t = Head of required Department
- u = Head of HRM department.

**Description of Criteria:**

- $e_1$ = *Efficiency* (TMTB)
- $e_2$ =*Cost* (TLTB)
- $e_3$ =*Result* (TMTB)
- $e_4$ =*Time* (TLTB)

# 5. HYBRIDIZATION OF AHP-GTMA:

Hybridization of AHP-GTMA includes following steps:

## Implementation of AHP:

**Step 1: Source Data for Alternatives:**
Table 1 demonstrates the exhibition of choices in light of the four given criteria. The choice includes two the more the better (TMTB) criteria and comparatively two the less the better (TLTB) criteria.

**Table 1. Exhibition of Choices**

| | $e_1$ | $e_2$ | $e_3$ | $e_4$ |
|---|---|---|---|---|
| $s_1$ | 2 | 2 | 1 | 3 |
| $s_2$ | 1 | 1 | 1 | 2 |
| $s_3$ | 1 | 1 | 1 | 1 |
| $s_4$ | 3 | 2 | 2 | 1 |

This states that the decision makers s, t & u gave points to the machines $\{s_1, s_2, s_3, s_4\}$ by judging them on the criteria $\{e_1, e_2, e_3, e_4\}$. Table can be interpreted as:

- Machine $s_1$ got 2 recommendations on the criteria $e_1$ and $e_2$ , 1 recommendation on $e_3$and 3 recommendations on criteria $e_4$.
- Machine $s_2$ got 1 recommendation on the criteria $e_1, e_2, e_3$ and 2 recommendations on criteria $e_4$.
- Machine $s_3$ got 1 recommendation from each criterion.
- Machine $s_4$ got 3 recommendations on the criteria $e_1$ , 2 recommendations on $e_2$ , $e_3$ and 1 recommendation on criteria $e_4$.

**Step 2: Pairwise Comparison Matrix:**
This part explores the DM's preference structure over the 4 considered criteria by utilizing AHP. A DM from required hospital which is to buy a pet scam machine is interviewed, for the pairwise comparison information uncovering his inclination to preferred criteria. This matrix is appeared in Table 2.

**Table 2. Pairwise Comparison Matrix**

| **Pairwise Comparison** | $e_1$ | $e_2$ | $e_3$ | $e_4$ |
|---|---|---|---|---|
| $e_1$ | 1 | 7 | 1 | 3 |
| $e_2$ | 1/7 | 1 | 1/5 | 1/3 |
| $e_3$ | 1 | 5 | 1 | 5 |
| $e_4$ | 1/3 | 3 | 1/5 | 1 |

**Step 3: Normalized Performance of the Alternatives:**
Choice Matrix (in Table 1)is normalized utilizing the maximal value of column as the pivot for a the more the better quality and utilizing the negligible incentive as the pivot for the less the better property, Normalized decision matrix is appeared in Table 3

**Table 3. Normalized Matrix**

| | $e_1$ | $e_2$ | $e_3$ | $e_4$ |
|---|---|---|---|---|
| $e_1$ | 2/3 | ½ | 1/2 | 1/3 |
| $e_2$ | 1/3 | 1 | 1/2 | ½ |
| $e_3$ | 1/3 | 1 | 1/2 | 1 |
| $e_4$ | 1 | ½ | 1 | 1 |

**Implementation of GTMA:**

**Step 1: Alternative selections**

**Table 4. Alternative selection for $s_1$**

| | $e_1$ | $e_2$ | $e_3$ | $e_4$ |
|---|---|---|---|---|
| $e_1$ | 2/3 | 7 | 1 | 3 |
| $e_2$ | 1/7 | ½ | 1/5 | 1/3 |
| $e_3$ | 1 | 5 | 1/2 | 5 |
| $e_4$ | 1/3 | 3 | 1/5 | 1/3 |

**Table 5. Alternative selection for $s_2$**

| | $e_1$ | $e_2$ | $e_3$ | $e_4$ |
|---|---|---|---|---|
| $e_1$ | 1/3 | 7 | 1 | 3 |
| $e_2$ | 1/7 | 1 | 1/5 | 1/3 |
| $e_3$ | 1 | 5 | ½ | 5 |
| $e_4$ | 1/3 | 3 | 1/5 | 1/2 |

**Table 6. Alternative selection for $s_3$**

| | $e_1$ | $e_2$ | $e_3$ | $e_4$ |
|---|---|---|---|---|
| $e_1$ | 1/3 | 7 | 1 | 3 |
| $e_2$ | 1/7 | 1 | 1/5 | 1/3 |
| $e_3$ | 1 | 5 | ½ | 5 |
| $e_4$ | 1/3 | 3 | 1/5 | 1 |

**Table 7. Alternative selection for $s_4$**

| | $e_1$ | $e_2$ | $e_3$ | $e_4$ |
|---|---|---|---|---|
| $e_1$ | 1 | 7 | 1 | 3 |
| $e_2$ | 1/7 | ½ | 1/5 | 1/3 |
| $e_3$ | 1 | 5 | 1 | 5 |
| $e_4$ | 1/3 | 3 | 1/5 | 1 |

**Step 2: Permanent values of Alternatives:**

The permanent value of square matrix is characterized as the sum of results of (all) arrangements of matrix elements that show up in distinct rows and columns. The Permanent Value of a 3 × 3 square matrix, H, is ascertained naturally as follows:

$$H = \begin{pmatrix} d & e & f \\ g & h & i \\ j & k & l \end{pmatrix}$$

$$Permanent(k) = dhl + gkf + jie + fhj + ikd + lhe$$

**Permanent for alternative $s_1$**

Permanent for alternative $s_1$ will be calculated by using Table.4 such as:

$Permanent(s_1) =$

$$= \frac{2}{3}\begin{pmatrix} 1/2 & 1/5 & 1/3 \\ 5 & 1/2 & 5 \\ 3 & 1/5 & 1/3 \end{pmatrix} + \frac{1}{7}\begin{pmatrix} 7 & 1 & 3 \\ 5 & 1/2 & 5 \\ 3 & 1/5 & 1/3 \end{pmatrix}$$

$$+1\begin{pmatrix} 7 & 1 & 3 \\ 1/2 & 1/5 & 1/3 \\ 3 & 1/5 & 1/3 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 7 & 1 & 3 \\ 1/2 & 1/5 & 1/3 \\ 5 & 1/2 & 5 \end{pmatrix}$$

$$= \frac{2}{3}\left(\frac{19}{4}\right) + \frac{1}{7}\left(\frac{97}{3}\right) + 1\left(\frac{21}{5}\right) + \frac{1}{3}\left(\frac{193}{12}\right)$$

$$= 17.3468254$$

**Permanent for alternative $s_2$**

Permanent for alternative $s_2$ will be calculated by using Table.5 such as:

$Permanent(s_2) =$

$$= \frac{1}{3}\begin{pmatrix} 1 & 1/5 & 1/3 \\ 5 & 1/2 & 5 \\ 3 & 1/5 & 1/3 \end{pmatrix} + \frac{1}{7}\begin{pmatrix} 7 & 1 & 3 \\ 5 & 1/2 & 5 \\ 3 & 1/5 & 1/2 \end{pmatrix}$$

$$+1\begin{pmatrix} 7 & 1 & 3 \\ 5 & 1/5 & 1/3 \\ 3 & 1/5 & 1/2 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 7 & 1 & 3 \\ 1 & 1/5 & 1/3 \\ 5 & 1/2 & 5 \end{pmatrix}$$

$$= \frac{1}{3}\left(\frac{67}{12}\right) + \frac{1}{7}\left(\frac{135}{4}\right) + 1\left(\frac{76}{15}\right) + \frac{1}{3}\left(\frac{58}{3}\right)$$

$$= 18.19365079$$

**Permanent for alternative $s_3$**

Permanent for alternative $s_3$ will be calculated by using Table.6 such as:

$Permanent(s_3) =$

$$= \frac{1}{3}\begin{pmatrix} 1 & 1/5 & 1/3 \\ 5 & 1/2 & 5 \\ 3 & 1/5 & 1 \end{pmatrix} + \frac{1}{7}\begin{pmatrix} 7 & 1 & 3 \\ 5 & 1/2 & 5 \\ 3 & 1/5 & 1 \end{pmatrix}$$

$$+1\begin{pmatrix} 7 & 1 & 3 \\ 5 & 1/5 & 1/3 \\ 3 & 1/5 & 1 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 7 & 1 & 3 \\ 1 & 1/5 & 1/3 \\ 5 & 1/2 & 5 \end{pmatrix}$$

$$= \frac{1}{3}\left(\frac{19}{3}\right) + \frac{1}{7}\left(\frac{38}{1}\right) + 1\left(\frac{14}{3}\right) + \frac{1}{3}\left(\frac{58}{3}\right)$$

$$= 18.65079365$$

**Permanent for alternative $s_4$**

Permanent for alternative $s_4$ will be calculated by using Table.7 such as:

$Permanent(s_4) =$

$$= 1\begin{pmatrix} 1/2 & 1/5 & 1/3 \\ 5 & 1 & 5 \\ 3 & 1/5 & 1 \end{pmatrix} + \frac{1}{7}\begin{pmatrix} 7 & 1 & 3 \\ 5 & 1 & 5 \\ 3 & 1/5 & 1 \end{pmatrix}$$

$$+1\begin{pmatrix} 7 & 1 & 3 \\ 1/2 & 1/5 & 1/3 \\ 3 & 1/5 & 1 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 7 & 1 & 3 \\ 1/2 & 1/5 & 1/3 \\ 5 & 1 & 5 \end{pmatrix}$$

$$= 1\left(\frac{19}{3}\right) + \frac{1}{7}\left(\frac{46}{1}\right) + 1\left(\frac{82}{15}\right) + \frac{1}{3}\left(\frac{18}{1}\right)$$

$$= 24.37142857$$

**Step 3: Index Score:**

**Table 8. Index Score**

| | ***Index Score*** | ***Rank*** |
|---|---|---|
| $s_1$ | 17.3468254 | 4 |
| $s_2$ | 18.19365079 | 3 |
| $s_3$ | 18.65079365 | 2 |
| $s_4$ | 24.37142857 | 1 |

**Step 4: Ranking the Alternatives:**

Final rank orders with the help of permanent values are following:

$$s_4 > s_3 > s_2 > s_1$$

This concludes as $s_4$ being the best machine to be opted.

## 6. CONSISTENCY ANALYSIS:

It is often necessary to check for consistency of poll responses of decision makers. Formal Consistency Analysis of AHP is conducted for this purpose. There can be found many approaches to consistency analysis in the literature, but here Saaty's (2003) methodology is applied. This method uses Consistency Ratio, which is a sort of correlation between Consistency Index and Random Consistency Index, calculated as:

$$CR = \frac{CI}{RI}$$

Values of this ratio ranging from 0 to 10 percent imply significant inconsistency which needs to be remedied.

Start the process by first computing a Column Summation Vector (CSV) by summing the sections in Table 2. It has the following qualities:

$$CSV = [2.4762, \quad 16, \quad 2.4, \quad 9.3333]$$

**Table 9. Computation of CWV**

| **Attr ×Attr** | $e_1$ | $e_2$ | $e_3$ | $e_4$ | **CWV** |
|---|---|---|---|---|---|
| $e_1$ | 0.40384 | 0.4375 | 0.41666 | 0.3214 | 0.39485 |
| $e_2$ | 0.05769 | 0.0625 | 0.08333 | 0.03571 | 0.05980 |
| $e_3$ | 0.40384 | 0.3125 | 0.41666 | 0.53571 | 0.41717 |
| $e_4$ | 0.13461 | 0.1875 | 0.08333 | 0.10714 | 0.12814 |

$$CWV = [0.39485, \quad 0.05980, \quad 0.41717, \quad 0.12814]$$

At that point the progression that takes after is to distinguish the biggest Eigen value of pair-wise comparison matrix ($\lambda_{max}$). This should be possible to start by a column-wise multiplication, that is, by multiplying each column of pair wise matrix with the related component in $CWV^{tr}$ (the transpose vector of CWV).

**Table 10. Computation of Weighted Sum Vector (WSV)**

| **Attr ×Attr** | $e_1$ | $e_2$ | $e_3$ | $e_4$ | **WSV** |
|---|---|---|---|---|---|
| $e_1$ | 0.39485 | 0.4186 | 0.41717 | 0.38442 | 1.61504 |
| $e_2$ | 0.05640 | 0.05980 | 0.08343 | 0.04271 | 0.24235 |
| $e_3$ | 0.39485 | 0.299 | 0.41717 | 0.6407 | 1.75172 |
| $e_4$ | 0.13161 | 0.1794 | 0.08343 | 0.12814 | 0.52259 |

The above procedure is as what one may have done utilizing the simple additive weighting (SAW) technique to score the options. Be that as it may, here, the things to assess are the criteria. By dividing each column summed values in the WSV with the comparing component in the CWV, acquire the consistency vector (CV) as follows.

$$CV = [4.09026, \quad 4.05274, \quad 4.19905, \quad 4.07827]^{tr}$$

By computing the mean estimation of CV, the biggest Eigen value is obtained that is

$$\lambda_{max} = 4.10508$$

Since the size of problem is n = 4 the CI of the matrix can be computed as indicated by the formula.

$$CI = \frac{\lambda_{max} - N}{N - 1}$$

$$CI = \frac{4.10508 - 4}{4 - 1}$$

$$CI = 0.0350$$

Under the given size of problem that is N = 4 the Random Index (RI) is turned upward as.

$$RI = 0.8815$$

Hence the Consistency Ratio of the pair-wise comparison matrix is calculated as follows:

$$CR = \frac{CI}{RI}$$

$$CR = \frac{0.0350}{0.8815}$$

$$CR = 0.03981$$

The CR value calculated here as per Satty's scale is satisfactory to legitimize Pairwise comparison matrix as consistent.

## 7. VERIFICATION WITH SOFT EXPERT SET:

Here transpose of Table 1 is utilized. Now, in order to reach at a decision, i.e. to choose one machine out of four, the attributes are given some weight, say

$$e_1 = 0.8, \quad e_2 = 0.2, \quad e_3 = 1, \quad e_4 = 0.1$$

Agree matrix of decision makers was such as:

$$\begin{pmatrix} 2 & 1 & 1 & 3 \\ 2 & 1 & 1 & 2 \\ 1 & 1 & 1 & 2 \\ 3 & 2 & 1 & 1 \end{pmatrix}$$

**Step 1: Average Time Dependent (ATD) Matrix:**

Average Time Dependent Matrix is calculated as follow:

$$\begin{pmatrix} 2.5 & 1.25 & 1.25 & 3.75 \\ 10 & 5 & 5 & 10 \\ 1 & 1 & 1 & 2 \\ 30 & 20 & 10 & 10 \end{pmatrix}$$

**Step 2: Refined Time Dependent Data (RTD) Matrix:**

To obtain the Refined Time Dependent Data (RTD) Matrix [30] we use.

$$if\ a_{ij} \leq (\mu_j - \alpha * \sigma_j)\ Then\ e_{ij} = -1$$

$$if\ a_{ij} \in (\mu_j - \alpha * \sigma_j, \mu_j + \alpha * \sigma_j)\ Then\ e_{ij} = 0$$

$$if\ a_{ij} \geq (\mu_j + \alpha * \sigma_j)\ Then\ e_{ij} = +1$$

With values of $\alpha$ as follows 0.25, 0.50, 0.75 the following RTD's were obtained.

For $\alpha = 0.25$

$$\begin{pmatrix} -1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 1 \\ -1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

For $\alpha = 0.50$

$$\begin{pmatrix} -1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 1 \\ -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

For $\alpha = 0.75$

$$\begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & -1 & -1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

**Step 3: Combined Effective Time Dependent Matrix (CETD):**

The CETD matrix is formed by adding all the above obtained RTD is

$$\begin{pmatrix} -2 & -2 & -3 & -2 \\ 0 & 0 & 0 & 3 \\ -3 & -2 & -3 & -3 \\ 3 & 3 & 3 & 3 \end{pmatrix}$$

The sum of above column matrix is

$$\begin{pmatrix} -2 & -1 & -3 & 1 \end{pmatrix}$$

**Step 4: Final outcome:**

This matrix gives the outcome that machine at $s_1, s_2$ and $s_3$ positions cannot be opted for the manager post.

Hence $s_4$ is the best machine, which is the same answer as in AHP-GTMA technique.

## 8. CONCLUSION

This work applies the AHP-GTMA way dealing to the decision making with the assurance of single kind of machine for Cancer Hospital. Genuine information of the machines (alternative), which are required by AHP-GTMA are gathered from the heterogeneous information sources. Firstly, AHP is connected to mine the DM's psyche as to conclude the Pairwise comparison and need of criteria is built up. Secondly, hybridization of AHP with GTMA is utilized to mine the information which prompted the positioning/choice of option. By adopting the AHP-GTMA strategy this work used data from data mining as well as from mind mining. Theoretical consistency is also presented. Finally, the results were verified by Soft Expert strategy that affirms its impact on a definitive outcome. The basic purpose of this research is to build up the idea of hybridization of two basic leadership techniques i.e. AHP and GTMA to solve decision making problem. In future this technique can be used for the solution of further multi criteria decision making issues. Hybridization of AHP with other MCDM techniques can take place. The developed methods can be further checked for consistency.

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